# STUDY OF THE STABILITY REGION FOR SATELLITES OF EXTRASOLAR PLANETS OF SUN - JUPITER TYPE 

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#### Abstract

Many of the recently discovered extrasolar planets, present mass ratio similar to Jupiter-Sun system. The aim of this paper is to infer regions of stability of satellites in the systems where the primary body has the mass of the Sun and the secondary body has the mass of Jupiter. Possible stable regions where satellites can survive for long time are presented in $(a, e)$ plane. Our model is based on the classical planar elliptic restricted problem. An empirical formula which gives the limit of stable region is derived.


## 1 - INTRODUCTION

In the last years the discovery of planets in other solar systems led to the question of whether these planets also have satellites. A remarkable feature of the giant planets of our solar system is the general architecture in the population of their satellites: all of the giant planets have at least two distinct groups. Very close to the planet, there is a class of regular satellites (almost planar and circular). The second group is formed with small objects with high eccentricity and high inclination (usually in retrograde orbits). However it is important to emphasize that all of the giant planets of our solar system are rather far from the Sun (the closest is Jupiter which is about 5.2 AU far). Now, the question that arises is related to the stability or possibility to a giant planet host a satellite, in the case that the planet is very close to the star. For the time being, a significant number of extrasolar planets were discovered very near to the star, let's say, sometimes their perihelia ( $a_{P}$ ) are less than 0.1 $A U$. These planets are almost circular. In opposition, there are some interesting cases where for larger semimajor axis, the eccentricities tend to be high, reaching a maximum of about 0.93 (Murray et al., 2002). In Figure 1 we present the histogram of the number of extrasolar planets (detected until the moment) as a function of the semimajor axis, eccentricity and planet's mass. The planet mass $M_{P}$ $\sin i_{P}\left(i_{P}\right.$ is the inclination of the orbit with the observer's line of sight) is given in terms of Jupiter mass and the semimajor axis in $A U$. These data were obtained from: http://exoplanets.org/planet (California \& Carnegie Planet Search).


Figure 1: Histogram of the number of extrasolar planets (detected until March, 2003) as a function of semimajor axis $a_{P}$ (a and b), eccentricity $e_{P}$ (c) and planet's mass $M_{P} \sin i_{P}(\mathrm{~d})$.

Up to now, satellites of extrasolar planets were not detected. This is due mainly to the instrumental limitations and the adopted techniques. Several missions to search for extrasolar planet transits by highprecision space-based photometry are in the planning stages and will, have the capacity of detecting satellites (Sartoretti \& Schneider, 1999).

As mentioned before, due to the characteristics of the detected planetary systems it is natural to question which would be the possible conditions for the formation of satellites in this context. An aspect that can be explored without the need of having a closed theory on the formation of satellites is the stability of satellites in advanced stages of evolution, where the formation process is almost ended. In
this context, the main goal of the present work is to infer the stability regions of hypothetical satellites of extrasolar planets.

Holman \& Wiegert (1999) investigated in what regions around a binary system, can a body orbit the center of mass of the stars (or one of the stars) for long time. These authors investigated numerically, the orbital stability in the frame of the elliptic restricted three-bodies problem. They considered a circular orbit for the third body. Empirical expressions that give the critical semimajor axis $\left(a_{c}\right)$ as a function of the eccentricity $(e)$ and mass ratio $(\mu)$ of the binaries are developed. Such expressions are derived for binary systems with 0.0 e 0.8 and mass ratio $0.1 \quad 0.9$. The formula for bodies orbiting one of the stars is not predicted for the case 0 , however simulations at $\mathrm{e}=0$ and in range $0.9 \quad 1.0$ were made and a plot of $a_{c}$ as a function of is shown.

In the present work we studied only the case $=10^{-3}$ and it differs from Holman's work in the sense that here we consider a wide range of eccentricities of both bodies, that is, the secondary mass and also the particle's eccentricity. Moreover, from our results we derive an expression for the critical stable semimajor axis of the particle as a function of the eccentricity of the secondary mass.

## 2 - LIMIT OF THE STABILITY REGION

Very roughly speaking, the idea of the limit of stability can be posed in the following way: consider a particle orbiting a planet which in its turn orbits a star. If the particle is far enough from the planet, the perturbation caused by the star becomes so important, that the particle cannot remain orbiting the planet. The region around the planet where the particle can survive, for any initial condition, for any time, defines a stable boundary and therefore a limit of stability (see for instance Hamilton \& Burns, 1991 and Holman \& Wiegert, 1999)

We can determine this limit by looking at the outermost regions where the majority of orbits are stable. In agreement with Holman \& Wiegert (1999), in the systems with high mass ratio, the stability limit of the body in circular orbit is $r_{\text {stable }} \propto f R_{H}$, where $f$ is a constant and $R_{H}=(/ 3)^{1 / 3} r$ is the Hill's radius of the planet.

In the numeric study presented by Hamilton \& Burns (1991) the boundary of the stability region about asteroids for the case of prograde orbits was found to be approximately $0.5 R_{H}$.

In the present study we obtain a numeric estimate of the value of $f$, considering the following values for the planet: semimajor axis $\left(a_{p}\right)$ : 0.1 AU , eccentricity $e_{p}$ from 0.0 to 0.8 with $\Delta e=0.1$. For the hypothetical satellites, we take: semimajor axis $\left(a_{\text {sat }}\right)$ from 1.1 to $100 R_{P}$ ( $R_{P}$ is the Jupiter's radius) with $\Delta a=0.1 R_{P}$; eccentricity ( $e_{\text {sat }}$ ) from 0.0 to 0.5 with $\Delta e=0.01$.

## 3 - NUMERICAL RESULTS

The numeric simulations were made for an interval of $10^{4}$ planet's orbital periods. The integration was interrupted whenever one of the situations appeared: collision between the satellite and the planet, the satellite collided with the Sun or satellite's planetocentric energy became positive (escape). The initial conditions of the survived satellites for full integration time were stored in a file (plotted in figures 2 to 7). These satellites were considered stable.

The numerical results are presented in Figures 2 to 7 . There we illustrate $a_{\text {sat }}$ up to $20 R_{P}$ for better graphic visualization of the stable region. In those figures the time of escape is represented by a gray scale. The collisions are represented by the symbol + .

## 3.1 - CIRCULAR AND ELLIPTIC CASES

In Figure 2 the orbit of the planet was assumed to be circular. The border to the right of the light gray region corresponds to the zero
velocity curve associated to $L_{1}$. Usually satellites escape in a maximum time of 320 orbital periods of the planet. Satellites that don't escape for this time and they don't collide, have orbits considered stables. It is noticed that the stability region extends to about 7 planet's radius what corresponds to approximately half the radius of the planet's Hill's sphere. Therefore, the stability region is approximately given by $r_{\text {stability }}=R_{H} / 2$.


Figure 2: Time of stability of hypothetical satellites in the space of initial conditions of $a_{\text {sat }}$ versus $e_{s a t}$, for the orbit of a circular planet. The unit of time is represented by gray scale. The symbol $(+)$ refers to collision of the satellite. The border corresponds to the right of the light gray region to the zero velocity curve associated to $\mathrm{L}_{1}$.

In Figures 3 and 4 we present the results for elliptic cases where $e_{P}$ assume values from 0.0 to 0.8 with $\Delta e=0.1$. In such cases the stability region reduces with the increase of the planet's eccentricity, as expected. The relation $r_{\text {stability }}=R_{H} / 2$ continues being valid, but now $R_{H}$ has to be calculated considering the planet at the pericenter, $r=a_{P}\left(1-e_{P}\right)$. This is presented in Figure 5, considering the values of $a_{c}$ for $e_{\text {sat }}=0$, we found an expression for the stability limit given by

$$
\begin{equation*}
R_{\text {stability }}=0.4947-0.6805 e_{P}+0.1135 e_{P}^{2} \tag{1}
\end{equation*}
$$

This expression is given in terms of Hill's radius units.


Figure 3: Time of stability of hypothetical satellites in the space of initial conditions $a_{s a t}$ versus $e_{s a t}$, for the orbit of a planet with $e_{P}=0.1$ to 0.4 . The unit of time is represented by gray scale. The symbol $(+)$ refers to a collision of the satellite.
Plane1's periods
0-320 Escape
320-10 Escape
$10^{3}-10^{4}$ Escape
Stales

+ Collisions





Figure 4: Time of stability of hypothetical satellites in the space of initial conditions $a_{s a t}$ versus $e_{s a t}$, for the orbit of a planet with $e_{P}=0.5$ to 0.8 . The unit of time is represented by gray scale. The symbol ( + ) refers to a collision of the satellite.


Figure 5: Radial limit of the stability region of hypothetical satellites as a function of $e_{P}$. The full line is given by equation (1).

It can be noticed in Figure 5 that for the circular case the stability region extends to about $R_{H} / 2$, while for the elliptic case it reduces with the increase of $e_{P}$ as expected. Our results suggest that for $e_{P}>0.8$ the stability region tends to zero.

These numeric results can be considered for other values of larger planet's semimajor axis and masses, the difference of the results will be just a scale factor. In Figure 6 we present the limit of the stability region of satellites in terms of $a_{P}$. The limits of stability are presented for planets with masses $0.1 M_{J}, 1 M_{J}$ and $10 M_{J}$. The horizontal lines, dash and solid, represent the Roche limit of the planet and the surface of the planet, respectively. As expected, the larger is the planet's mass or semimajor axis, the larger is the stability region. For instance, if the planet has mass $M_{P}=0.1 M_{J}$ and $a_{P}=0.06 \mathrm{AU}$, its stability region is confined to Roche's region. The planet could have satellite, but of limited sizes due to the planetary tidal effects. On the other hand if this same planet had a mass 10 times larger, its stability region would be about twice as larger and this planet could have satellites whose size limit would be larger than in the case with smaller mass of the planet.


Figure 6: Stability regions limits of satellites in terms of $a_{P}$. These stability limits are for planets with masses $0.1 M_{J}, 1 M_{J}$ and $10 M_{J}$. The horizontal lines, dashed and solid, represent the Roche's limit of the planet and surface of the planet, respectively.

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