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INCLINATION TYPE RESONANCE IN THE PHOBOS PROBLEM

T. Yokoyama
M. R. Mana, C. Nascimento and M. T. Santos
Instituto de Geociências e Ciências Exatas, UNESP-IGCE-DEMAC
C.Postal 178 CEP: 13.500-970, Rio Claro, Brasil

ABSTRACT

Since Phobos' orbit is spiralling in toward Mars, it will face several secular resonances. A preliminary study of this problem was already given in Yokoyama (2002), when Mars was considered in a circular orbit. Here we consider more general conditions, extending the integration to much longer time. We include Mars eccentricity and we discuss some situations related to escape and changes in the amplitude of libration due to the appearance of new additional resonances. The inclusion of planetary perturbations and variation of Mars'equator is also pointed. In this paper we only give some brief results. Full details of the calculations will be presented elsewhere.

1-INTRODUCTION

The semi major axes of Phobos and Triton are decreasing secularly because these two satellites are clearly involved in the well known effects of planetary tidal dissipation. Due to the variation of the semi major axis, the frequencies of the node and perihelium, vary continuously and they can attain some resonant combinations involving the mean motion of the Sun. In a previous work, the evolution of this scenario was studied taking a very simplified model (Yokoyama 2002). In that case, we considered a circular orbit for the host planet and the secular perturbations due to other planets were neglected. In this simplified model it was shown that, in the future, at $a = 2.149R_M$ Phobos can be captured in an interesting $2:1$ resonance which will increase the inclination of the satellite to very high values. The integration time was not long enough and therefore escapes from libration were not studied. This is an important question and deserves to be investigated. To date, the phenomenon of escapes from mean motion resonances and possible existence of a "universal eccentricity (or inclination) "was carefully studied by Beaugé and Ferraz-Mello (1993) and Gomes (1995,1997,1998). However, differently than orbital resonance which involves explicitly two mean longitudes, in the present problem (Phobos), the critical argument of our resonance is basically due to the mean longitude of the Sun and the node of the satellite , that is: $\Omega + 2\lambda_{\odot} - 3\Omega_{\odot}$. In section 2 we discuss some situations related to escape from this resonance and passage through others. Section 3 is devoted to study the elliptic case, that is, if the eccentricity of Mars is considered, then the hole played by a pair of resonances (double resonance which occur at $a \approx 2.619$) is decisive for the $\Omega + 2\lambda_\odot - 3\Omega_\odot$ resonance which will be encountered subsequently. In the section 4 a more complete model is considered: planetary perturbations acting on Mars and the equations that govern the motion of Mars' equator are now included. The main qualitative results are preserved, however, like in the simplified model, in all cases, we see that the problem is very sensitive with respect to the initial conditions. In this work, the results are presented very briefly and the full details with complete description will be given elsewhere.

2-ESCAPE AND DIFFERENT MODES OF LIBRATION

In order to discuss the main point of escape and some transitions between different modes of libration, we still keep the simplified model as given in Yokoyama (2002). To this end, let's consider a satellite perturbed by the action of the Sun and of the oblateness of the host planet. The coordinate system whose reference plane is the equator, is fixed in the center of the planet (Mars). Since we are interested only in the effects of long period, we consider the averaged part of R_{J_2} and $R_\odot$ (oblateness disturbing function and solar disturbing function, respectively). Therefore the principal frequencies of a close satellite besides solar mean motion ($n_\odot$) are:

$$\begin{aligned}
\dot{g} &\approx \frac{3nJ_2R_M^2}{4a^2(1-e^2)^2}\left(5cos^2I - 1\right) \\
\dot{\Omega} &\approx -\frac{3nJ_2R_M^2 cosI}{2a^2(1-e^2)^2}
\end{aligned}$$

where the elements of the satellite: a, e, I, n, g, Ω are respectively: semi major, eccentricity, inclination, mean motion, argument of the pericenter and longitude of the node. J_2 is the oblateness coefficient, k^2 is the gravitational constant and R_M is the equatorial radius of the planet.

A typical resonant combination in this problem occurs when $k_1\dot{g}+k_2\dot{\Omega}+k_3n_\odot$ is nearly zero (k_i are integers). Current Phobos' semi major axis is about $2.76R_M$. Therefore in the future ($a \leq 2.76$), we will have the following resonances:

- $2\dot{\Omega} + 2n_\odot$, with $a \approx 2.6196$
- $2\dot{g} + 2\dot{\Omega} - 2n_\odot$, with $a \approx 2.6193$
- $\dot{\Omega} + 2n_\odot$, with $a \approx 2.1490$

Like in Yokoyama (2002), first let's start with $\dot{\Omega} + 2n_\odot$ resonance. In general, taking the current initial conditions, ($e \approx 0.015$, $I \approx 1^0$) and considering $a \leq 2.15$ only $\dot{\Omega}+2n_\odot$ resonance is possible . All of the remaining resonant combinations which appear in $R_\odot$ cannot be satisfied mainly if I is small ($\approx 1^0$). For decreasing semi major axis, the $\dot{\Omega} + 2n_\odot$ resonance is favorable to capture. In fact, integrating Lagrange's variational equations (Yoder 1982) we have Fig.1A,B. It shows a capture and also an escape at $I \approx 33^0$, when $a \approx 2.04R_M$. The initial inclination is $I = 1^0$.

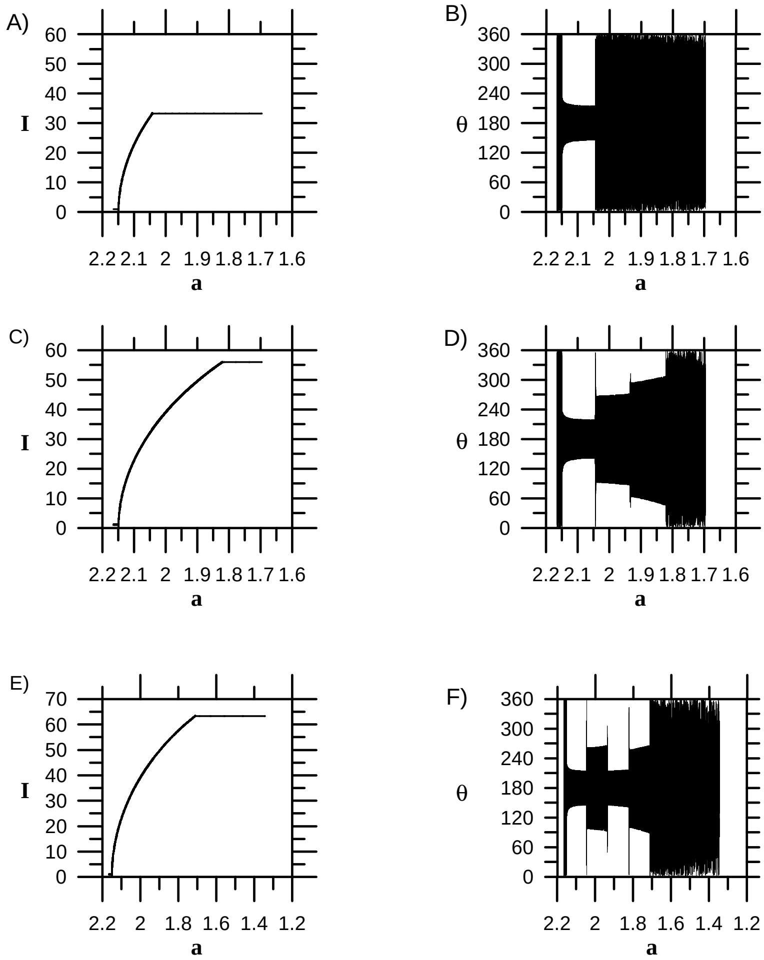

Figure 1: Left panels: variation of Inclination (I) versus semi major axis (a). Right panels: variation of resonant angle ($\Theta = \Omega + 2\lambda_\odot - 3\Omega_\odot$) versus semi major axis. Initial conditions: A,B): $a = 2.165R_M$, $e = 0.01$, $I = 1^0$, $\Omega = 90^0$, $g = 0^0$. C, D): $a = 2.165R_M$, $e = 0.01$, $I = 1.1^0$, $\Omega = 90^0$, $g = 0^0$. E,F): $a = 2.165R_M$, $e = 0.03$, $I = 1^0$, $\Omega = 90^0$, $g = 0^0$.

Now let's see Fig.1C,D. The starting point of the integration for this figure is very similar to the previous one: the only difference is in the inclination, which is changed to $I = 1.1^0$. In spite of this very small difference, note the remarkable change in the escape value of the inclination, which occurs at $I \approx 56^0$ when $a \approx 1.82R_M$. But before that, the amplitude of the libration suffers some clear changes at $a \approx 2.048R_M$, $a \approx 1.926R_M$ and $a \approx 1.82R_M$. At these points the inclination is $\approx 32.5^0$, $\approx 46.6^0$, $\approx 56^0$, respectively. In the Fig.1E,F, the initial conditions are the same of those used in Fig.1A,B, except the eccentricity which is increased to $e = 0.03$. This time, the changes in the libration angle occur at: $a \approx 2.044R_M$ ($I \approx 33^0$), $a \approx 1.93R_M$ ($I \approx 46.38^0$), $a \approx 1.82R_M$ ($I \approx 56^0$), $a \approx 1.71R_M$ ($I \approx 63.43^0$). In all cases, in the limits of these changes, the librating angle suffers a jump. Some of them are very high, specially the first one which appears at $a \approx 2.044$. To explain the reason of these changes in the amplitude of libration, in all these figures, we simply calculate the position of a specific resonant combinations $k_1\dot{g} + k_2\dot{\Omega} + k_3 n_\odot$ for all k_i. For the sake of brevity let's consider only Fig.1F.

In the beginning , since I is small, ($I \approx 1^0$), only $2n_\odot + \dot{\Omega}$ resonance is possible. Then the capture occurs at $a \approx 2.149 R_M$ as predicted. However as the inclination starts to increase, others resonant combinations can be satisfied. The first one occurs at $a = 2.044 R_M$, $I = 33.02^0$. At this point the following equations (resonances) can be satisfied:

$$\begin{aligned} \dot{\Omega} + \dot{g} - n_\odot &= 0 \\ \dot{\Omega} + 2n_\odot &= 0 \end{aligned} \tag{1}$$

Similarly at $a = 1.933 R_M$, $I = 46.38^0$ we have:

$$\begin{aligned} \dot{\Omega} + \dot{g} &= 0 \\ \dot{\Omega} + 2n_\odot &= 0 \end{aligned} \tag{2}$$

and at $a = 1.82 R_M$, $I = 56.064^0$ we have:

$$\begin{aligned} \dot{\Omega} + \dot{g} + n_\odot &= 0 \\ \dot{\Omega} + 2n_\odot &= 0 \end{aligned} \tag{3}$$

and finally at $a = 1.71 R_M$, $I = 63.43^0$ when the escape occurs:

$$\begin{aligned} 2\dot{g} + 2n_\odot + \dot{\Omega} &= 0 \\ 2\dot{g} - 2n_\odot - \dot{\Omega} &= 0 \\ 2n_\odot + \dot{\Omega} &= 0 \\ \dot{g} &= 0 \end{aligned} \tag{4}$$

At these points, when two or more resonances occur simultaneously, the problem is essentially a non autonomous system with two degree of freedom. The adiabatic invariant theory should break down in the region where this two resonances are of the same importance. Therefore, no wonder that jumps or changes in the amplitude of the libration appear during the passage through these regions. It is worth mentioning that, in principle, each time that a system face a case of double resonance, the escape can occur. However, numerical experiments indicate that the phenomenon is not predictable. For instance, in Fig.1B, the first encounter already caused the escape, in Fig.1D the third encounter , and in Fig.1F, the system resisted to three passages. Here we have an interesting case where the escape inclination is the classical “critical inclination ”. This means that after the escape, the scenario is changed to the dynamics of eccentricity and pericenter which seems to become more important. These points are left to be discussed elsewhere in a future work.

3-ECCENTRICITY OF MARS

This time we still keep the keplerian orbit for Mars, but we consider its eccentricity ($e_M = 0.0933$). As a consequence, at least 22 new resonant combinations (of first order in Mars 'eccentricity) will appear in the disturbing function. Therefore the probability of an earlier escape is larger than in the circular case (see Fig.2A where escape occurs at $I \approx 33^0$) , but of course, we can still have escapes with $I \approx 53^0$ (Fig.2C).

The main point in this section is the double resonance which occurs at $a \approx 2.619 R_M$. This resonance plays an important role in the future as we are going to show. From the adiabatic invariant theory (Peale 1999, Henrard 1982), studying each one separatedly we conclude that for decreasing semi major axis, $\dot{\Omega} + n_\odot$ is favorable to capture while $\dot{w} + \dot{\Omega} - n_\odot$ is not. Since they appear almost simultaneously they are in a kind of competition. Fig.2F shows a case of complicate behaviour during the passage through $a = 2.619 R_M$, ending with a capture in $\dot{\Omega} + n_\odot$ resonance which causes an increase of the inclination (actually it goes to more than 30^0). However if the precision of the Radau integrator (Everhart 1985)is increased from $LL = 10$ to $LL = 12$, no capture is observed, but the inclination undergoes a significant jump and it stabilizes (after the some short time) around $I \approx 3^0$. Also the eccentricity is excited from $e = 0.015$ to almost $e \approx 0.08$. Our numerical experiments reveal that some of these variations also occur when Mars is in circular orbit, however, they are really much more significant for the elliptic case, when inclination goes to about 4^0 and stabilizes near to 3^0 (Fig.2E). According to our numerical simulations, excitations in eccentricity and inclination reaching these values are possible only when Mars eccentricity is considered. Of course there are some other cases where excitation is weak so that inclination does not undergo any significant increase.

Now, going further toward the future ($a \approx 2.149$), if $\dot{\Omega} + 2n_\odot$ resonance is encountered with this inclination ($\approx 3^0$), numerical integrations show that no capture is possible. This is also predicted from the adiabatic invariant theory, since small area in the $\oint pdq$ is possible only if inclination is small enough. For this reason, the double resonance mentioned before is important since it provides the incoming data that dictates the dynamics of the system when $\dot{\Omega} + n_\odot$ resonance is encountered.

Closing this section, let's remember that the escape value of the inclination is always very sensitive to the initial conditions (section 2). For orbital resonances in the presence of a dissipative force, some interesting formulae relating the variation of the elements, can be derived (Beaugé and Ferraz-Mello 1993, Gomes 1995, 1997, 1998). Under some hypothesis it is possible to derive a formula that suggests a probable value of escape eccentricity or inclination. The so called "universal eccentricity or universal inclination "refers to these specific values. As we mentioned before, our resonance in this work has some clear differences when compared to orbital commensurability. For instance, in the orbital resonance, when an asteroid , or particle is captured, its semi major axis stops to decrease continuously and starts to oscillate around some resonant value. In our case, once Phobos is captured , its semi major still remains decreasing in the same way as it was decreasing before the capture.

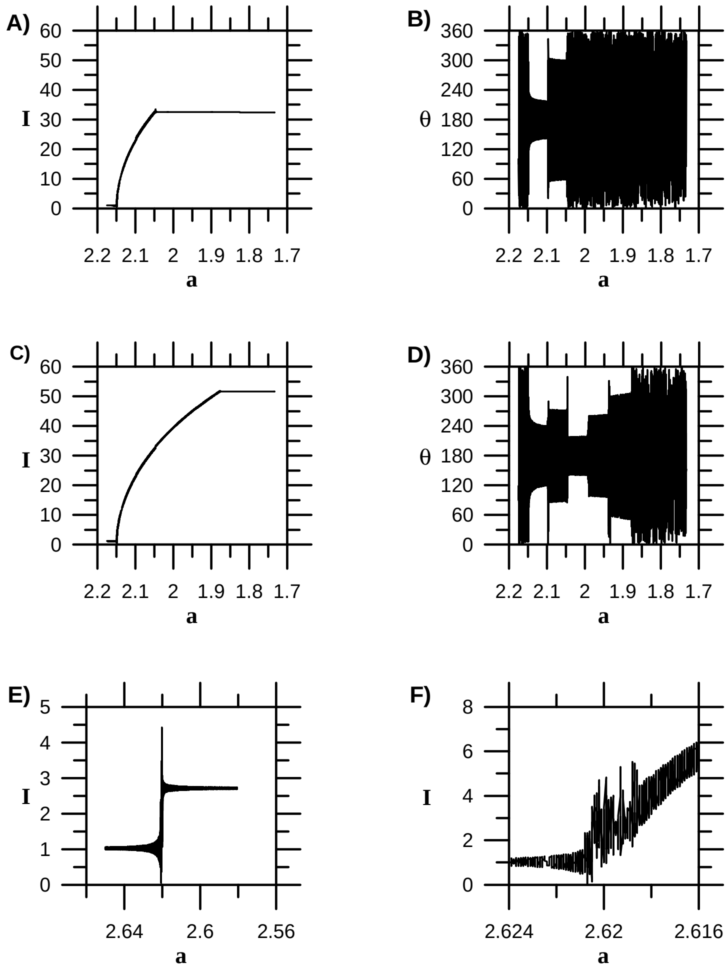

Figure 2: Left panels: variation of Inclination (I) versus semi major axis (a). Panels B,D: variation of resonant angle ($\Theta = \Omega + 2\lambda_{\odot} - 3\Omega_{\odot}$ versus semi major axis. Initial conditions: A,B): $a = 2.175R_M$, $e = 0.01$, $I = 1^0$, $\Omega = 90^0$, $g = 0^0$. C,D): $a = 2.175R_M$, $e = 0.01$, $I = 1.15^0$, $\Omega = 90^0$, $g = 0^0$. Panel E): no capture ($LL = 12$ in Radau integrator). Initial conditions: $a = 2.65R_M$, $e = 0.015$, $I = 1^0$, $\Omega = 90^0$, $g = 0^0$. Panel F): same initial conditions of Panel E) but with capture in $2\Omega + 2n_{\odot}$ where $LL = 10$ in Radau integrator.

This is possible because the variation of the inclination compensates the semi major axis variation such that the relation $\dot{\Omega}+2n_{\odot}$ remains small, locked in the current resonance. Therefore, escapes are not due to a continuous increase of the amplitude of the variation of the semi major axis as is seen in the orbital resonances. In our problem, escapes are mostly due to encounter to additional resonances when proper values of I and a being attained. This seems that no “universal inclination ”is possible in this problem.

4-PLANETARY PERTURBATIONS AND PRECESSION OF MARS EQUATOR

In this section we mention very briefly some additional effects we need to include in this problem. Up to now the orbit of Mars was considered keplerian with fixed equator. Of course it is not our purpose to consider the whole planetary perturbations and integrate from present data until Phobos' semi major axis decreases to $a \approx 2.619R_M$ or $a \approx 2.149R_M$. However it is interesting to have a rough qualitative idea of the main effects when planetary perturbations are included, even considering that the integration starts very near $a \approx 2.149R_M$.

Therefore we considered Bretagnon's secular theory (Bretagnon 1974) and also the variation of Mars equator through the equations $\frac{d\tilde{\Omega}}{dt}$ and $\frac{d\tilde{I}}{dt}$ (Woolard , 1953). Here $\tilde{\Omega}$ is the longitude of the Mars' equator while $\tilde{I}$ is its inclination, both referred to ecliptic of 1850. This time, the inclination of Phobos with respect to this new reference plane, varies very largely, going from 0^0 to more than 90^0. This requires the use of special regularized variables for the inclination. However the most important point to be emphasized is related to the variation of obliquity of the Mars' equator: usually when the obliquity is less than about 19^0 in the vicinity of $a \approx 2.149R_M$ capture does not occur. Also the previous resonant angle (when we were using the equator as the reference plane), now is changed to $2\Omega + 2n_\odot$.

5-CONCLUSION

In this work we integrated Phobos' problem for much longer time. Once a capture occurs, while semi major axis is decreasing and inclination is increasing, several additional resonances will appear and each one can cause interesting jumps and changes in the amplitude of libration. Escapes usually occurs due to the interaction between two or more resonances. In principle it seems that no prediction can be made about escapes. The effect of Mars eccentricity is very important, since during the passage through the double resonance, it can excite Phobos inclination to about 3^0. With this value no capture in the next resonance (at $a \approx 2.149R_M$) is possible. If planetary perturbations are considered, in general, most of the main features are preserved, however numerical experiments show that captures are possible only when Mars' obliquity is larger than $\approx 19^0$.

6-ACKNOWLEDGEMENTS

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