

**ATTITUDE DYNAMICS AND CONTROL OF A LSS DURING THE TRANSLATION OF THE
ROBOT MANIPULATOR AND ASTRONAUT WALK ALONG THE SPACE STRUCTURE**

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Abstract

This paper presents the mathematical model for a large space structure containing a robotic manipulator that translates along the station and is used to execute operations of grasping satellites to be fixed by astronauts. The control efforts are evaluated in two different configurations. One configuration is characterized by a flexible LSS carrying a flexible robotic manipulator. The manipulator is kept in a fixed position on the station. The computational control problem solution for this case includes the jitter effect associated with the various motions along and inside the station and the astronauts walk perturbation on the LSS as input noises. The whole system is excited in rotation and vibration. Then the LQG technique is applied to damp the vibration and at the same time to drive the attitude motion to a nominal operating mode, defined as that of the Earth pointing configuration. The second configuration considers the LSS main bus as a rigid body carrying the flexible manipulator that is allowed to rotate and translate along the station. The manipulator is driven to a certain position while the vibration and the rotation are controlled by the system control. For this case a different control technique is used. This control technique is known as back step technique. This technique allows for the motion of the robotic manipulator to a different final position than zero. Then the vibration is damped so as to allow for the grasping operation. A series of small pulses is used to approach the astronaut walk from one position to another when executing their work in space. The space walk shall not be thought as regular men walking on the ground. The astronaut walk represents the motion of the astronauts from one position to another to develop their space activities, mainly the LSS outside activities.

Introduction

Since the successful Sputnik mission in 1957 vast resources have been driven for space systems development. The results are revolutionary in the communication systems, weather prediction, resource administration and navigation, scientific improvement in understanding of our planet, our solar system, and the universe. The space activities necessary for some missions at the present time differ very much from those of the early days of the space age. The manned spacecraft that started with the in-orbit astronaut walks linked by a type of umbilical cable are today large space stations that include facilities for astronauts in-orbit work by using robot manipulators and autonomous associate technologies. The challenges associated with

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manned space activities gave a great impulse to the development of space robotics. Manned operations require an enormous treaty of risk and are highly expensive because of the associate requirements for room, reliability and life support. In this sense the use of such robotic manipulators and other forms of automation are very attractive. However, completely autonomous systems to operate in orbit are not available so far. The in-orbit work still requires manned interaction from the ground or space sites. This fact has many technological implications that go from mathematical modeling and control law design, to on-board based software and state of the art technological sensors and actuator selection to allow for monitoring, tracking, docking, and controlling the space vehicle attitude. In addition to this, large space structures face structural flexibility problems. Because of the high cost of putting massive objects in space, the guideline is to optimize the weight of the space structure. On the other hand the weight minimization is constrained by the flexibility since very flexible systems are difficult to control. In other words it is necessary to look for the lightest structure with the highest possible stiffness. Further the space zero-g environment requires extra care when the robot manipulators operate. Differently from on-ground based manipulators the space manipulators operate on a mobile base¹. This fact requires taking into account the relative motion between the main bus and the object that is grasped and moved along the main bus structure. It is worth noting that not only the satellites can be grasped and moved along the space station but also other objects. Also the astronauts move to execute extravehicular activity (EVA) and intravehicular activity (IVA). These motions disturb the center of mass of the system and may cause jitter. All these affect the control effort to keep the station in a stable attitude configuration as well as in a level of vibration that do not risk the space operation and make the astronaut work safely and as comfortable as possible.

This work deals with the study of the control efforts in two different configurations. One refers to the whole station and the robotic manipulator moving and vibrating around a nominal operating mode. This mode is defined as the earth-pointing attitude of the station. Small elastic displacements are assumed. In other word the elastic displacement oscillates about zero (non deformed structure) and the pitch angle is slightly misaligned with respect to the local vertical. The motion is constrained to be in a low earth circular orbit. In this configuration the LQG² technique is used to damp the elastic vibration and the rigid body motion represented by the pitch, roll, and yaw attitude angles and the manipulator arms rotation angles.

The mathematical model is obtained by a combination of finite element technique and the Lagrangian formulation for quasi and generalized coordinates. The second configuration is defined by the robotic manipulator translating on the station while the attitude control keeps the whole system stable in attitude. A torque motor activates the manipulator base from one position to another. Then the manipulator base is locked and the vibration is suppressed. After the manipulator locking and the vibration suppressing the manipulator arm is driven from zero to a 30-degrees position. The manipulator rotational and vibrational motions are suppressed by using the controller. For the configuration two the assumed modes method was used to obtain the equations of motion as ordinary differential equations.

LSS Equations of Motion

The mathematical model for the first case under study has been developed in reference 3. Two different approaches have been used to model the LSS. The first approach combines the finite element technique with the Lagrangian formulation for quasi and for generalized coordinates to obtain the system equations. In this formulation the system center of mass has been considered as fixed. For this model the Linear Quadratic Gaussian regulator, LQG has been designed to control the attitude and the vibration of the station. The main steps of the problem formulation (for this case) by using the finite elements in conjunction with the Lagrangian formulation are:

- Divide the main bus into finite elements;
- Approach the elements by tubular beam elements and use appropriate beam element functions
- Derive the kinetic energy for the i^{th} element
- Derive the elastic potential energy for the finite element
- Assemble the mass and the stiffness matrix according to the finite element technique
- Derive the expression for the gravity-gradient torque
- Write the Lagrangian function given by the difference between kinetic energy and the potential elastic and gravitational energy
- Use the Lagrange's formula for quasi and generalized coordinates to obtain the equations of the dynamics
- Linearize the equations about the gravity-gradient stable configuration and the nondeformed structure (elastic displacement)
- Write the system state equations
- Design an appropriate control law (in this case the optimal control law represented by the LQG technique)

- Implement the control by using the MatLab[®]/Simulink computer environment
- Analyze the results.

The non linear equations for this case are given by⁴

$$\frac{d}{dt} \left\{ \frac{\partial T}{\partial \dot{\omega}} \right\} + [\tilde{\omega}] \left\{ \frac{\partial T}{\partial \omega} \right\} = \{M_o + f_o\} \quad \text{for} \quad \text{quasi coordinates}$$

$$\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{q}_i} \right) - \frac{\partial L}{\partial q_i} = Q_{q_i} \quad \text{for generalized coordinates}$$

In these formulas L is the Lagrangian function. $\{\omega\}$ refers to the station angular velocity vector and the q_i refers to the LSS flexible and the robot manipulator flexible and rotational degrees-of-freedom. $[\tilde{\omega}]$ is a skew symmetric matrix in the components of $\{\omega\}$. T is the kinetic energy. M_o is the external torque vector, associated with the gravity-gradient torque and f_o is the control torque vector. Q_{q_i} represents the generalized forces. The above formulas have been used to derive the equations of the dynamics for both models, the one formulated by using the finite element methods

hereafter called FEM Model, and the other by using the assumed modes method. The details of the LSS physical model and the mathematical FEM model derivation are shown in reference 3. Figure 1 shows the in-orbit configuration of the LSS

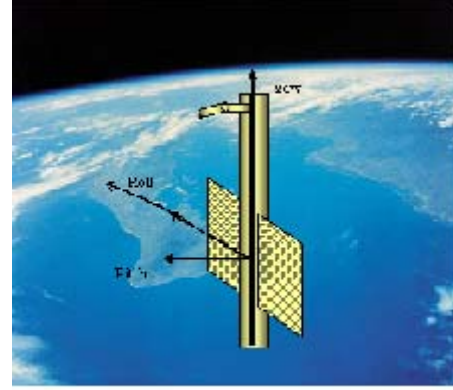


Fig. 1 - LSS in-orbit configuration

Figures 2 and 3 show details of the structure that have been used to guide the derivation of the equations of the dynamics.

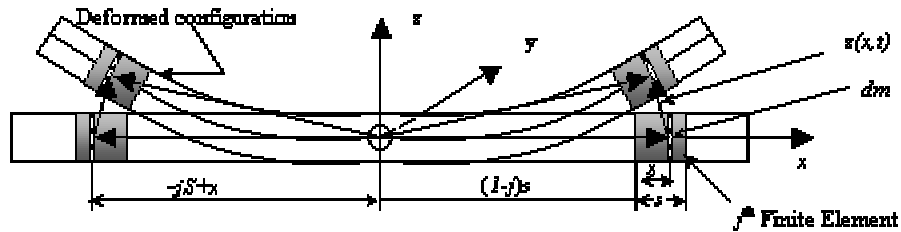


Fig. 2 - j^{th} finite elements, left and right side of the main bus

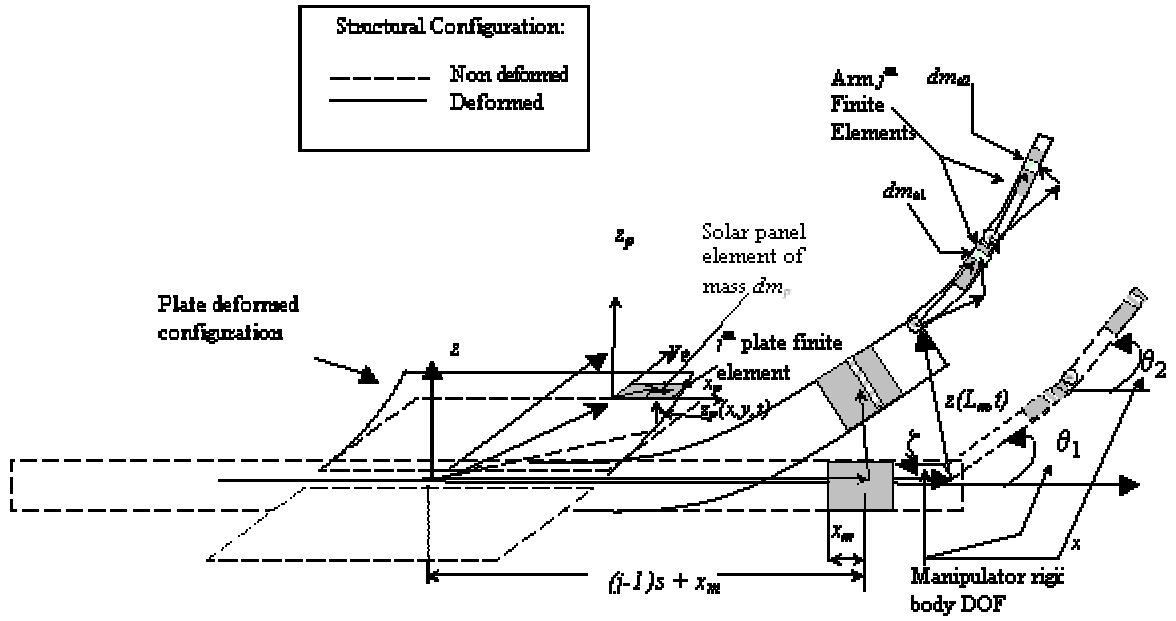


Fig. 3 – Space station components – Finite elements and elemental mass (dm_p , dm_{a1} , and dm_{a2}) positions

The equations of the dynamics for the FEM model can be written as

$$\begin{aligned} \begin{Bmatrix} \ddot{\psi} \\ \ddot{e} \end{Bmatrix} = - \begin{bmatrix} [J] & [E] \\ [E]^T & [m] \end{bmatrix}^{-1} \left(\begin{bmatrix} [G] & [\tilde{Q}][E] \\ [0] & [0] \end{bmatrix} \begin{Bmatrix} \dot{\psi} \\ \dot{e} \end{Bmatrix} + \right. \\ \left. \begin{bmatrix} [C] & [0] \\ [0] & [K] \end{bmatrix} \begin{Bmatrix} \psi \\ e \end{Bmatrix} - \begin{Bmatrix} \{f_o\} \\ \{H\}\{f\} \end{Bmatrix} \right) \end{aligned} \quad (1)$$

Here the rate of change in time of the $\{\omega\}$ is written in terms of the Euler angles (roll, yaw, and pitch, depicted by ψ). The vector $\{e\}$ includes all the elastic displacement and the robot manipulator arms rigid body motion. $[E]$ is coefficient matrix of the vector $\{e\}$. $[K]$ is the structural stiffness matrix. $[m]$ is the elastic structural elastic displacement coefficient matrix.

$$[J] = \begin{bmatrix} j_x & 0 & 0 \\ 0 & j_y & 0 \\ 0 & 0 & j_z \end{bmatrix}$$

$$[G] = \begin{bmatrix} 0 & (I_z - I_y - I_x)\Omega_o & 0 \\ (I_x - I_z + I_y)\Omega_o & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$[C] = \begin{bmatrix} (I_z - I_y)\Omega_o^2 & 0 & 0 \\ 0 & -4(I_x - I_z)\Omega_o^2 & 0 \\ 0 & 0 & 3(I_y - I_x)\Omega_o^2 \end{bmatrix}$$

$$[\tilde{Q}] = \begin{bmatrix} 0 & -\Omega_o & 0 \\ \Omega_o & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\{\psi\} = \begin{Bmatrix} \psi_x \\ \psi_y \\ \psi_z \end{Bmatrix}$$

Here Ω_o stands for the orbital velocity and is constant because we have assumed the orbit circular. The off-diagonal elements of $[G]$ are associated with gravity-gradient torque components. $[J]$ is the LSS inertia matrix whose components are the principal moments of inertia around the roll, yaw, and pitch axes, respectively. The LQG control problem formulation requires the development of the state matrix or plant matrix. In order to obtain this matrix let us define the state vector as

$$\{X_1\} = \begin{Bmatrix} \Psi \\ e \end{Bmatrix} \text{ and } \{\dot{X}_1\} = \{X_2\} = \begin{Bmatrix} \dot{\Psi} \\ \dot{e} \end{Bmatrix} \text{ so that}$$

we can write the state equation as

$$\begin{Bmatrix} \dot{X}_1 \\ \dot{X}_2 \end{Bmatrix} = - \begin{bmatrix} [0] \\ [\bar{K}] \end{bmatrix} - [\bar{M}]^{-1} [\bar{G}] \begin{Bmatrix} X_1 \\ X_2 \end{Bmatrix} + \begin{bmatrix} [0] \\ [0] \end{bmatrix} - [\bar{M}]^{-1} \begin{bmatrix} [0] \\ \bar{u} \end{bmatrix} \quad (2)$$

where

$$[\bar{M}] = - \begin{bmatrix} [J] & [E] \\ [E]^T & [m] \end{bmatrix} \quad [\bar{K}] = \begin{bmatrix} [C] & [0] \\ [0] & [K] \end{bmatrix}$$

$$[\bar{G}] = \begin{bmatrix} [G] & [\tilde{Q}]^T [E] \\ [0] & [0] \end{bmatrix} \quad \{\bar{u}\} = \begin{Bmatrix} \{f_o\} \\ \{[H]f\} \end{Bmatrix}$$

The system equations can be rearranged to write the state equation in the classical form:

$$\{\dot{x}\} = [A]\{x\} + [B]\{u\} \quad (3)$$

LQG Control Law Design

The control law technique used to control the FEM Model is based on the Linear-Quadratic-Gaussian, LQG, optimal control design. The LQG is a modern state-space technique for designing optimal dynamic regulators. It allows for trade-off between regulation performance and control effort, and takes into account process disturbances and measurement noise. The LQG regulator consists of an optimal state-feedback gain and a Kalman filter state estimator. This technique requires a state-space model of the plant as given by Eq.(3) with the addition of the noise effect as shown in Eq.(4). So the problem formulation can be stated as

$$\begin{aligned} \{\dot{x}\} &= [A]\{x\} + [B]\{u\} + [G]\{w\} \\ \{y_v\} &= [C]\{x\} + [D]\{u\} + [H]\{w\} + \{v\} \end{aligned} \quad (4)$$

where $\{y_v\}$ is the measurement equation with known inputs and processes the noise $\{w\}$ and $\{v\}$. The regulation index is given by a quadratic performance criterion of the form

$$J(u) = \int_0^\infty \left(\{x\}^T [Q] \{x\} + 2\{x\}^T [N] \{u\} + \{u\}^T [R] \{u\} \right) dt \quad (5)$$

where $[Q]$, $[N]$, and $[R]$ are weighting matrices that defines the trade-off between regulation performance and control effort, i.e., how fast the state space component $x(t)$ goes to zero. In this work the correlation matrix $[N]$ is not taken into account.

The Kalman filter can be state as:

Given the inputs, the process and measurements white noises $\{w\}$ and $\{v\}$ satisfying

$$\begin{aligned} E(\{w\}) &= E(\{v\}) = 0 \\ E(\{w\}\{w\}^T) &= [Q] \\ E(\{w\}\{v\}^T) &= [N] \\ E(\{v\}\{v\}^T) &= [R] \end{aligned}$$

construct the state estimate $\{x\}$ that minimizes the steady-state error covariance

$$P = \lim_{t \rightarrow \infty} E(\{x - \hat{x}\}\{x - \hat{x}\}^T)$$

The optimal solution is the Kalman filter with equations

$$\{\hat{x}\} = [A]\{\hat{x}\} + [B]\{u\} + [L](y_v - [C]\{\hat{x}\} - [D]\{u\})$$

where L is known as the filter gain and is computed by solving an algebraic Riccati equation.

The FEM Model vibrational and rotational control has been implemented by using the MatLab®/Simulink software package. The block diagrams as they were implemented in the MatLab®/Simulink is shown in figures 4 to 7. The reference shown in the first block (left) requires the control system to guide the station to a configuration characterized by zero roll, yaw, and pitch angles as well as the robotic manipulator arm angles. Details of the input noise are shown in figure 5. The Kalman Filter block diagram is shown in figure 6. The block on the top describes

the perturbation caused by the astronauts walk and other source of jitter. The astronaut walk perturbation has been approached by a discrete sequence of pulses and other jitter effects were approached by a sinusoidal function. The details of that input noise are shown in figure 7.

The second model has been derived by using the Lagrangian formulation for generalized coordinates⁴ and the Newton-Kirchhoff formulation for the electromechanical equations. These equations account for the motor torque equations. Differently from the previous model, the elastic displacement has been approached by the assumed modes method⁵ in the form of a product of space dependent functions by time dependent generalized coordinates. This method allows for model simplification by taking into account in the problem formulation only the elastic degrees-of-freedom previously chosen. The assumed modes method does not require choosing a strict space dependent function to represent the elastic motion. The main requirement is that the space dependent function be compatible with the geometrical boundary conditions. No natural boundary conditions are required in the assumed modes method formulation. The physical model for this second case study is shown in figure 8. The manipulator translates along the station ($r_o(t)$) while it rotates ($\theta(t)$) and vibrates ($v(x,t)$). The mathematical model takes into account the first elastic vibration mode, one robotic manipulator represented by one arm with a planar rotational degree-of-freedom and one translational degree-of-freedom along the station. The equations of motion for this case are

$$\ddot{q}_1 + \mu q_1 + (\omega_1^2 - \dot{\theta}^2) q_1 + \alpha \ddot{\theta} + \beta \ddot{r}_o \cos(\theta) = 0 \quad (6)$$

$$L_m \frac{di_{a1}}{dt} + R_a i_{a1} + \frac{K_b N_g}{R} \dot{r}_o = U_1 \quad (7)$$

$$I_1 \ddot{r}_o + b_m N_g^2 \dot{r}_o - K_t N_g R i_{a1} = 0 \quad (8)$$

$$L_m \frac{di_{a2}}{dt} + R_a i_{a2} + R_a i_{a2} + K_b N_g \dot{\theta} = U_2 \quad (9)$$

$$I_2 \ddot{\theta} + b_m N_g^2 \dot{\theta} - K_t N_g i_{a2} = 0 \quad (10)$$

It is worth noting that to move the manipulator in translation and rotation we consider two independent motors. One is a prismatic motor, to provide the manipulator translation along the LSS and the other is a revolute motor, responsible for the arm rotation. The motor equations are Eqs.(8) and Eq.(9) for the manipulator translation and rotation, respectively.

In those equations i_{ai} ($i=1,2$) stands for the electrical variable (current), U_1 and U_2 stand for the control voltage. The other parameters are given in table 1.

Table 1 – Parameter values used in the simulation

Parameter	Description	Value (I.S.)
ω_1	Arm elastic fundamental frequency	4.7846 Rad/Sec
μ	Structural damping	0.025 Sec ⁻¹
α	$\int_0^{\ell_{arm}} y_1 \phi(y_1) dy_1$	0.82103 m
β	$\int_0^{\ell_{arm}} \phi(y_1) dy_1$	0.7829 Rad
L_m	Electrical Inductance	0.003100 Ω Sec
b_m	Motor viscous damping coefficient	0.004629 Kg/Sec
K_b	EMF constant	0.052814 A Ω Sec
K_t	Torque constant	0.052814 Kg/A Sec ²
N_g	Transmission rate	1
R_a	Electrical resistance	1.914 Ω
R	Motor ratio	0.1 m
I_i ($i=1,2$)	Motor Inertias	7.2×10^{-5} 3.6×10^{-5} A

In order to write the state $\{x\}$ we can define

$$\{x\} = \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_8 \\ x_9 \end{Bmatrix} = \begin{Bmatrix} q_1 \\ \dot{q}_1 \\ i_{a1} \\ r_o \\ \dot{r}_o \\ i_{a2} \\ \theta \\ \dot{\theta} \end{Bmatrix} \quad \text{and} \quad \{U\} = \begin{Bmatrix} U_1 \\ U_2 \end{Bmatrix}$$

This system equation can be written in the state form so as to yield the plant

$$\{\dot{x}\} = [\bar{A}]\{x\} + [\bar{B}]\{U\} + \{\zeta(x)\}$$

where \bar{B} is an 8×2 matrix with $\bar{B}_{3,1} = \bar{B}_{6,2} = 1/L_m$. The other matrix elements are zeros.

$\{\zeta\}$ is a 1×8 vector with components equal to zero, except by $\zeta_{2,1}$, that is given by

$$x_8^2 x_1 - \frac{\beta K_t N_g}{R I_1} x_3 \cos(x_7) + \frac{\beta b_m N_g^2}{I_1} x_5 \cos(x_7)$$

\bar{A} is the plant matrix given by

$$\begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -\omega_1^2 & -\mu & 0 & 0 & 0 & -\frac{\alpha K_t N_g}{I_2} & 0 & \frac{\alpha b_m N_g^2}{I_2} \\ 0 & 0 & -\frac{R_a}{L_m} & 0 & -\frac{K_b N_g}{R L_m} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & \frac{K_t N_g R}{I_1} & 0 & -\frac{b_m N_g^2}{I_1} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\frac{R_a}{L_m} & 0 & \frac{K_b N_g}{L_m} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & \frac{K_t N_g}{I_2} & 0 & -\frac{b_m N_g^2}{I_2} \end{bmatrix}$$

The Backstepping⁶ technique is used to implement the nonlinear control for this second case. It is a stabilization technique, which applies to nonlinear systems with a particular structure in which the actual control input “trickles down” through a series of integrators to the level of a fundamental subsystem. The computer implementation of this technique can be seen in the Simulink block depicting the Backstepping technique, shown in figure 9.

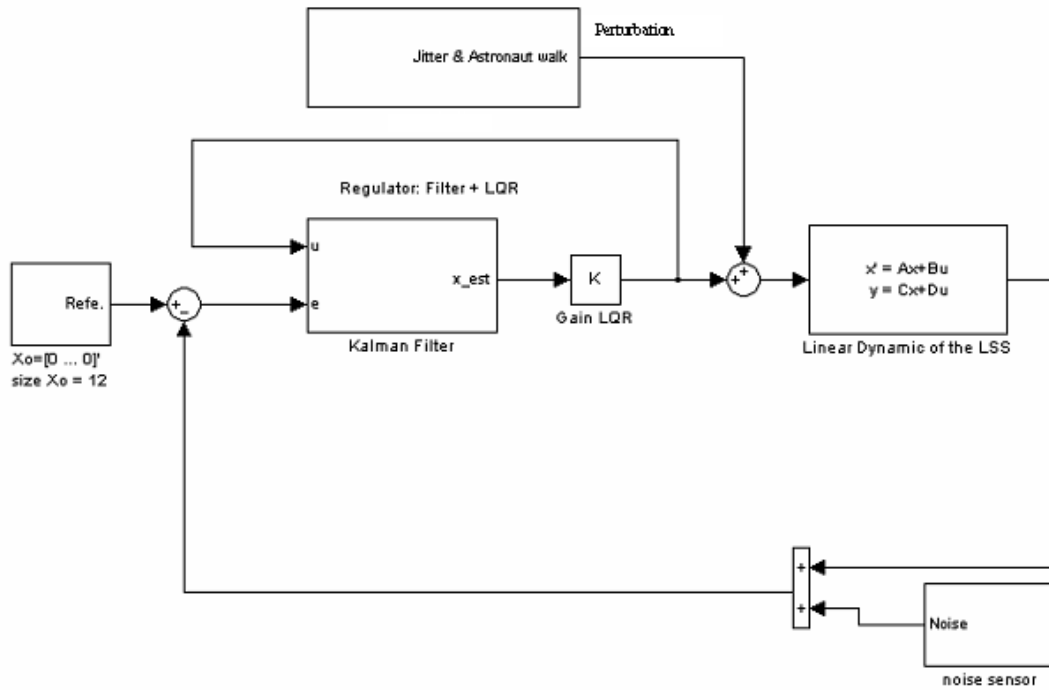


Fig 4 - Simulink block built to implement the LSS attitude dynamics and control

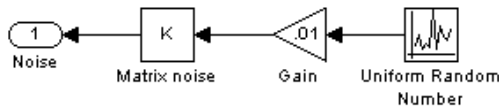


Fig. 5 – Noise input details

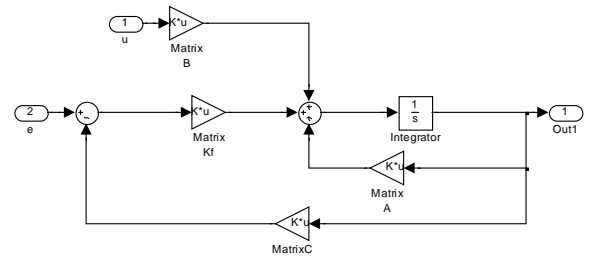


Fig. 6 – Kalman filter block diagram

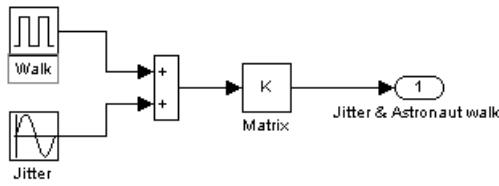


Fig. 7 – Jitter and astronaut walk representation

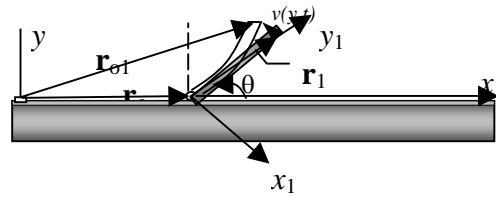


Fig. 8- LSS physical model for the second case study

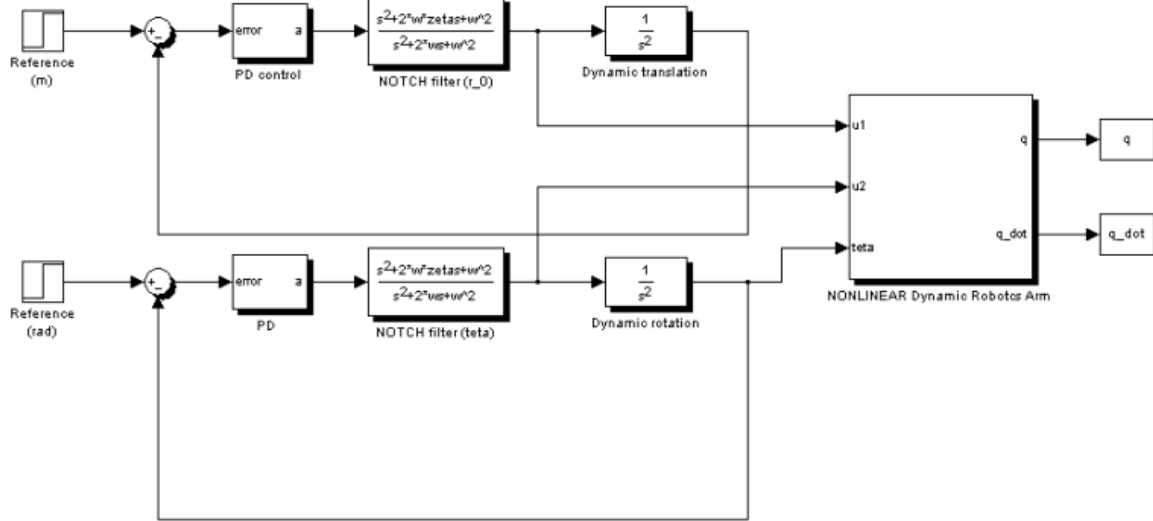


Fig. 9 – Backstepping Simulink blocks

Computer Simulations and Results

The simulations were carried out to check the damping of the attitude angles to the reference position set to zero in yaw, roll and pitch. The same objective has guided the simulation regarding the robot arm. The control should take the robotic manipulator arms from an initial angular position of about 5 degrees to zero. Initial position in the elastic displacement has been set for the simulations and the control should drive them back

to about zero. The structure's parameters used to build the structural model are:

- Material density = 1769 Kg/m^3
- Young's Modulus = $7.3084e10 \text{ N/m}^2$
- Main Bus length = 100 m
- Manipulator arms length = 10 m
- Solar panel dimensions = $10 \times 10 \times 0.003 \text{ m}$
- Average main bus diameter = 5 m
- Manipulator arm diameter = 0.01 m

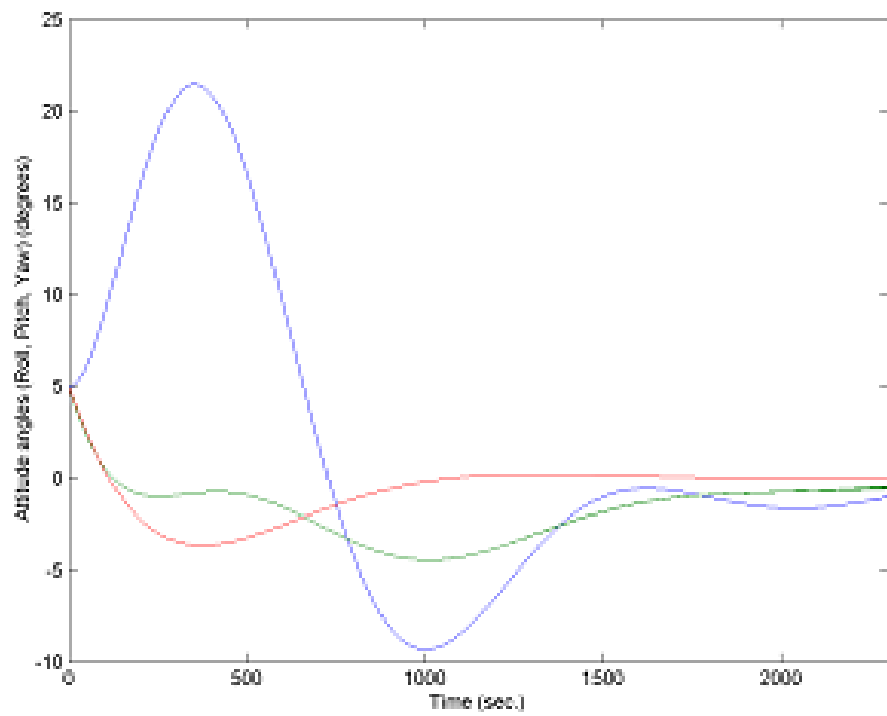


Fig. 10 – Pitch (green), roll (red), and yaw (blue) time history

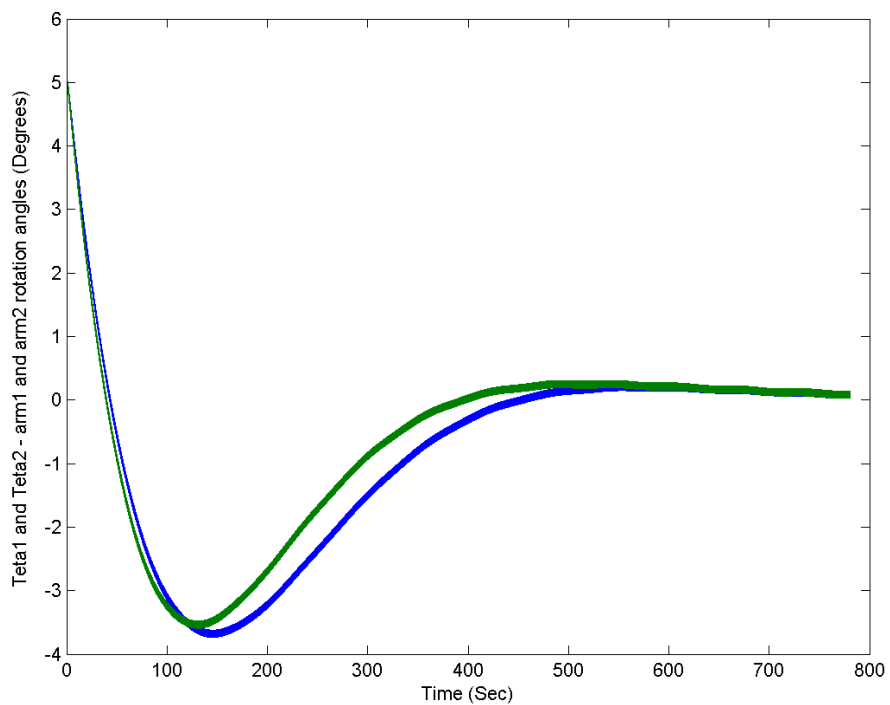


Fig. 11 – Manipulator arms 1 (green) and 2 (blue) time history

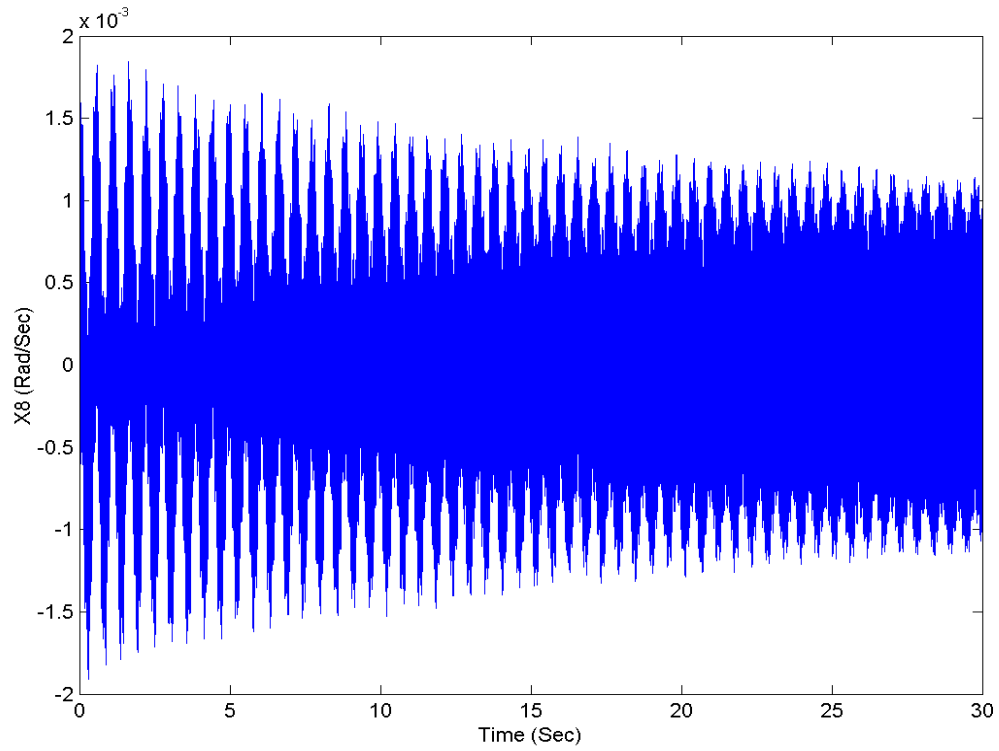


Fig. 12 – RMS tip displacement damping

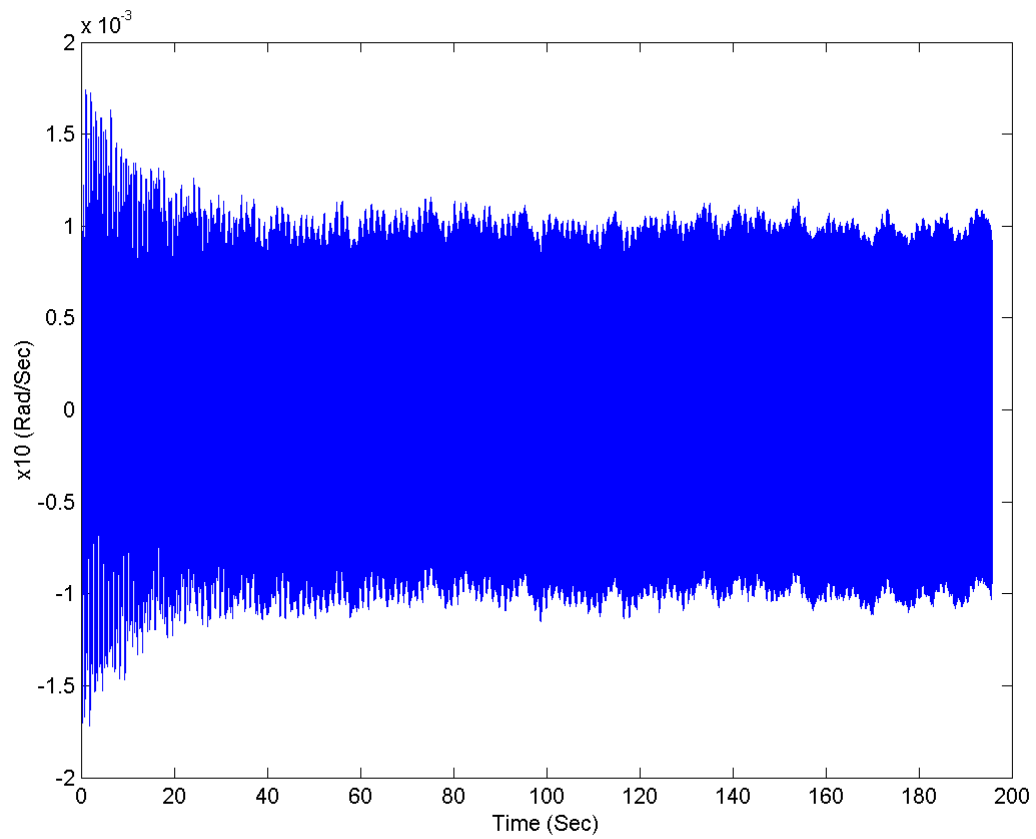


Fig. 13 – Manipulator first link tip vibration damping

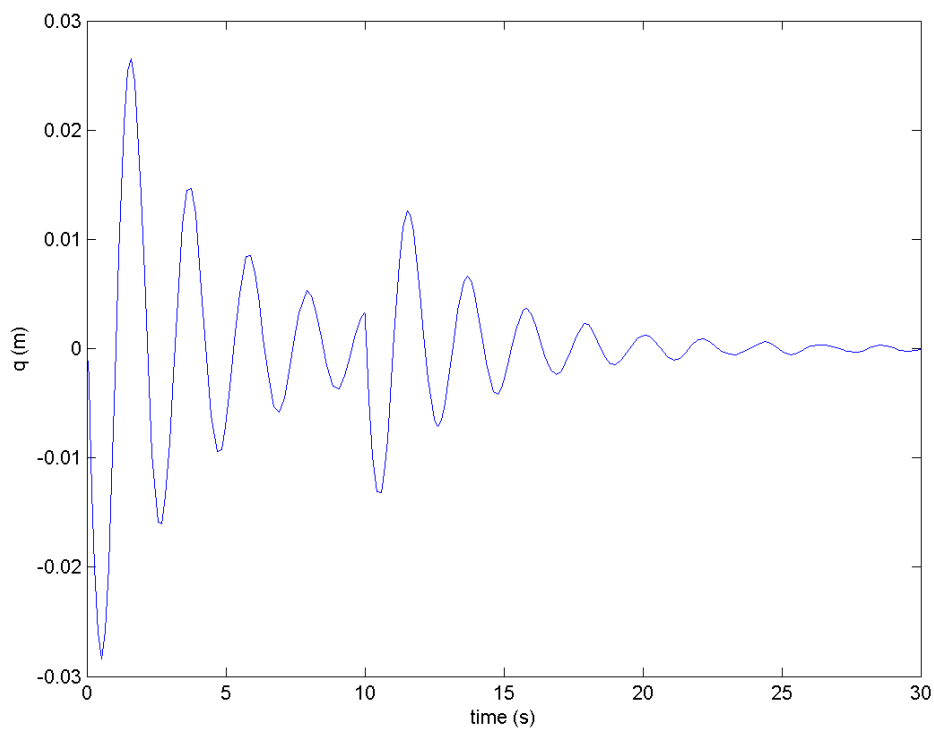


Fig. 14 – Robot manipulator tip vibration for the case 2.

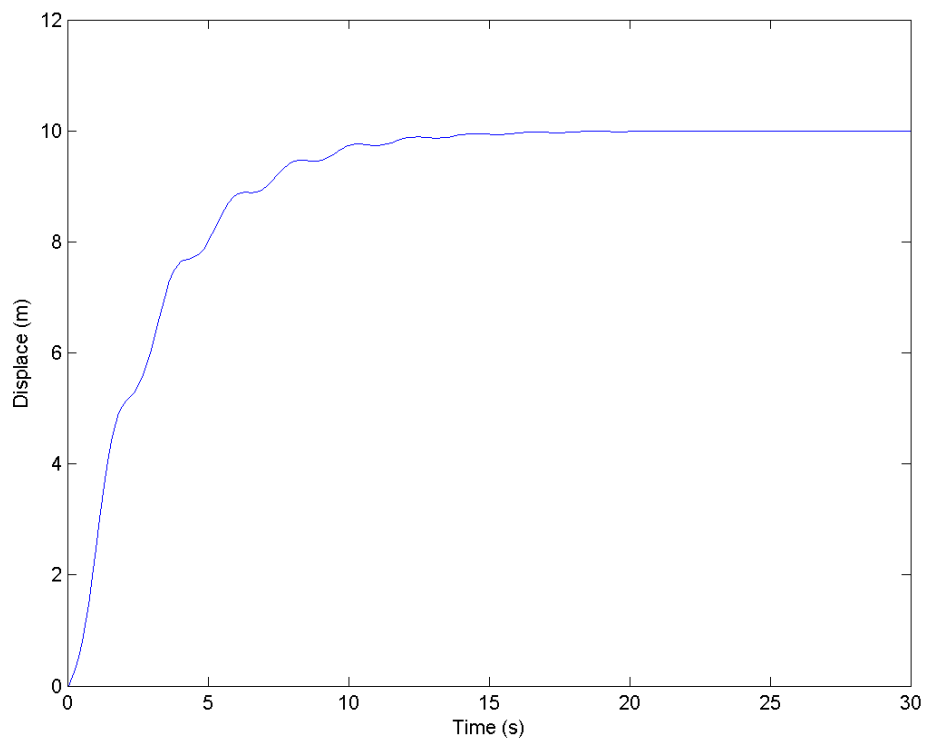


Fig. 15 Robotic manipulator translation along the station

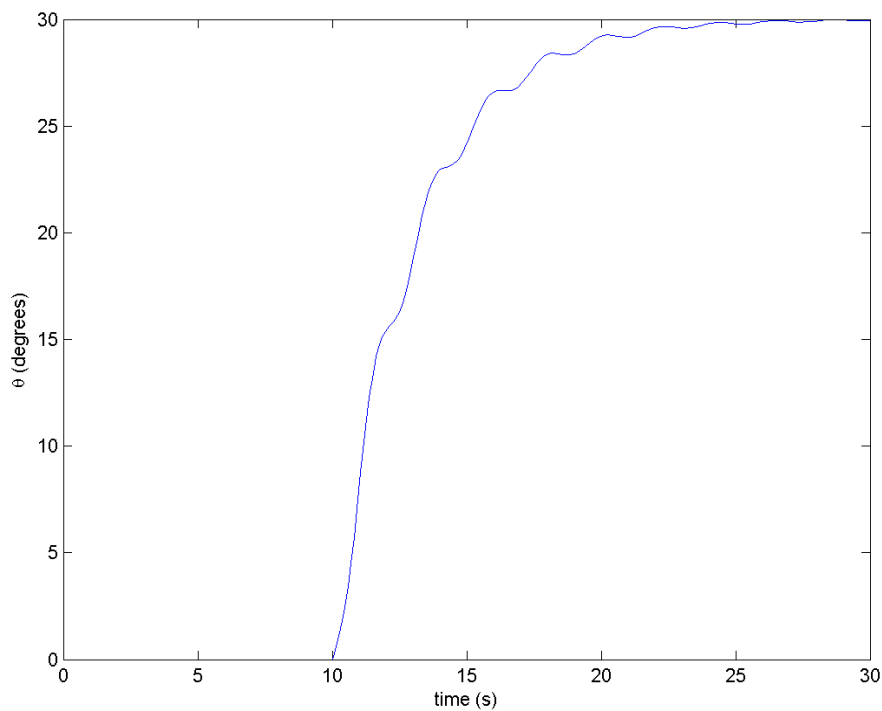


Fig. 16 – Robot arm angular displacement from zero to 30-degrees position

Figure 10 shows the yaw, roll, and pitch angles control. Figure 11 shows the robot arm driving to zero. Note that for the first case study the reference is zero. This means that the attitude angles as well as the robot arm angles are all driven to zero. Figures 12 and 13 are related to the elastic displacements of the main bus tip and the second manipulator arm tip, respectively. Figures 14 to 16 are related to the second case study. Figure 14 traces the elastic vibration of the arm. Figure 15 illustrates the arm translation from zero to a 10-meter position. Figure 16 illustrates the arm rotation. Note that this operation happens after the manipulator is locked in the 10 meters position. The dynamic behavior is over damped.

Conclusion

The LQG control technique has been discussed and implemented aiming at the LSS attitude and structural vibration control. In a second step the LSS has been modeled by a long tubular structure carrying a one-link robot manipulator with a translational degree-of-freedom, in addition to the rotational and elastic degrees-of-freedom. The main features of the Backstepping technique were presented. The technique has been implemented for the second case studied in the paper.

The MatLab[®]/Simulink software packages have been used to execute the simulations. Some of the

software block diagrams has been shown to explain the LQG and the Backstepping technique.

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