ORBITAL TRANSFERS BETWEEN HALO ORBITS AND THE PRIMARIES IN THE EARTH-MOON SYSTEM

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ABSTRACT

This paper has the goal of studying the problem of orbital transfers between Halo orbits and the primaries in the Earth–Moon system. Halo orbits are special three-dimensional trajectories that exist around the Lagrangian points of the restricted three-body problem. These orbits are studied in several papers, since they have important applications in astronautics. The first step involved in this research is to perform the determination of the Halo orbits. To do that, an analytic calculation is performed using the Linstedt-Poincaré method. The present paper considers that a maneuver will be performed to transfer the spacecraft from an initial orbit around the Earth to insert the spacecraft in a Halo orbit, and then from the Halo, to the Moon. Transfers between two Halo orbits are also considered. After that, the return trajectories from the Moon to the Earth passing by the Halo are also studied. The control that will be used to achieve that goal is constituted by a series of instantaneous change in the velocity of the spacecraft. A numerical algorithm based in the Lambert Problem is built to calculate the transfer orbits. This maneuver is required when it is desired to use the Halo as a parking orbit to transfer a spacecraft between the Earth and the Moon.

INTRODUCTION

To study the problem of transfer orbits between Halo orbits and the primaries, the restricted three-body problem in three dimensions is used as the mathematical model. It is assumed that the total system (Earth + Sun + spacecraft) satisfies the hypotheses: i) all the bodies are point masses; ii) the Earth and the Sun are in circular orbits around their mutual center of mass. Then, the goal is to study the motion of the spacecraft governed by these two masses. The Halo orbits are trajectories that exist around the Lagrangian points, that are the well-known equilibrium points that appear in the rotating frame of the circular restricted three-body problem (e. g. Szebehely, 1967). They are very important for astronautical applications. Since they are five points of equilibrium in the equations of motion, it means that a particle located at one of those points with zero relative velocity will remain there indefinitely. The collinear points (L_1, L_2 and L_3) are always unstable and the triangular points (L_4 and L_5) are stable in the present case (Earth-Moon system). They are all very good points to locate a space station due to the low cost of the station keeping. The triangular points are especially good for this purpose, since they are stable equilibrium points. In the nomenclature used in this paper, L_1 is the collinear Lagrangian point that exists between the Earth and the Moon. It is located about 58000 km from the Moon. L_2 is the collinear Lagrangian point that exists behind the Moon. It is located about 64500 km from the Moon.

There are many papers in the literature that refers to Halo orbits. In a general form, it is possible to divide them in three groups:

1) Papers that concentrate on the determination of the orbits, like: Farquhar (1972), Farquhar and Kamel (1973), Farquhar *et al* (1977), Breakwell and Brown (1979), Richardson (1980a, 1980b and 1980c), Howell and Breakwell (1984), Popescu (1986), Farquhar (1991), Popescu and Cardos (1995), Felipe *et al* (2000). These papers describe the Halo orbits and show approximate analytical solutions, that can later be used as a starting point to find accurate numerical orbits;

2) Papers that consider the problem of transferring the spacecraft between a parking orbit around the Earth and a Halo orbit. Among these, we can mention, D'Amario and Edelbaum (1974), Stalos *et al* (1993), Howell *et al* (1994), Starchville and Melton (1997);

3) A third line of research studies maneuvers between Halo orbits. Some good samples are: Farquhar *et al.* (1980), Farquhar (1980), Popescu (1985), Simó (1987), Howell and Gordon (1992), Hiday and Howell (1992), Gordon and Howell (1992), Howell and Pernicka (1993). In this category it is also possible to include some papers that consider the Rendezvous between two spacecrafts, like: Jones and Bishop (1993a, 1993b, 1994). A excellent compilation of the results combining all the topics are available in four volumes in Gómez *et al* (2001a, 2001b, 2001c, 2001d).

THE RESTRICTED THREE-DIMENSIONAL THREE-BODY PROBLEM

The model used in this paper is the well-known circular restricted three-body problem. This model assumes that two main bodies (M_1 and M_2) are orbiting their common center of mass in circular Keplerian orbits and a third body (M_3), with negligible mass, is orbiting these two primaries. The motion of M_3 is supposed to be affected by both primaries, but it does not affect their motion (Szebehely, 1967). The canonical system of units are used, and it implies that: i) The unit of distance is the distance between M_1 and M_2; ii) The angular velocity (ω) of the motion of M_1 and M_2 is assumed to be one; iii) The mass of the smaller primary (M_2) is given by $\mu = m_2/(m_1 + m_2)$ (where m_1 and m_2 are the real masses of M_1 and M_2, respectively) and the mass of M_1 is (1-μ), so the total mass of the system is one; iv) The unit of time is defined such that the period of the motion of the primaries is 2π; v) The gravitational constant is one.

There are several systems of reference that can be used to describe the three-dimensional restricted three-body problem (see Szebehely, 1967). In this paper the rotating system is used. In the rotating system of reference, the origin is the center of mass of the two massive primaries. The horizontal axis (x) is the line that connects the two primaries at any time. It rotates with an angular velocity ω in such way that the two massive primaries are always on this axis. The vertical axis (y) is perpendicular to the (x) axis and lies in the plane of movement. In this system, the positions of the primaries are: $x_1 = -\mu$, $x_2 = 1 - \mu$, $y_1 = y_2 = 0$. In this system, the equations of motion for the massless particle are (Szebehely, 1967):

$$\ddot{x} - 2\dot{y} = x - (1-\mu)\frac{x+\mu}{r_1^3} - \mu\frac{x-1+\mu}{r_2^3} \quad (1)$$

$$\ddot{y} + 2\dot{x} = y - (1-\mu)\frac{y}{r_1^3} - \mu\frac{y}{r_2^3} \quad (2)$$

$$\ddot{z} = -(1-\mu)\frac{z}{r_1^3} - \mu\frac{z}{r_2^3} \quad (3)$$

where r_1 and r_2 are the distances from M_1 and M_2, given by

$$r_1^2 = (x+\mu)^2 + y^2 + z^2 \quad (4)$$

$$r_2^2 = (x-1+\mu)^2 + y^2 + z^2 \quad (5)$$

THE HALO ORBITS

The Halo orbits are periodic orbits in the three-dimensional restricted three-body problem in the neighborhood of the collinear Lagrangian points (e. g. Breakwell and Brown, 1979). Fig. (1) shows a schematic view of those orbits and the geometry of the proposed mission. Those transfers are required to make possible orbital maneuvers for space missions. To perform this task, it is necessary to look at the characteristics of the Halo orbits and explore the non-linear system. To determine those orbits, a possible approach is to find an analytical approximation for the family of periodic orbits by using the Linstedt-Poincaré method, which allows us to obtain one solution, in a series form, until a very high order. As a reference orbit it is possible to use the linear solution.

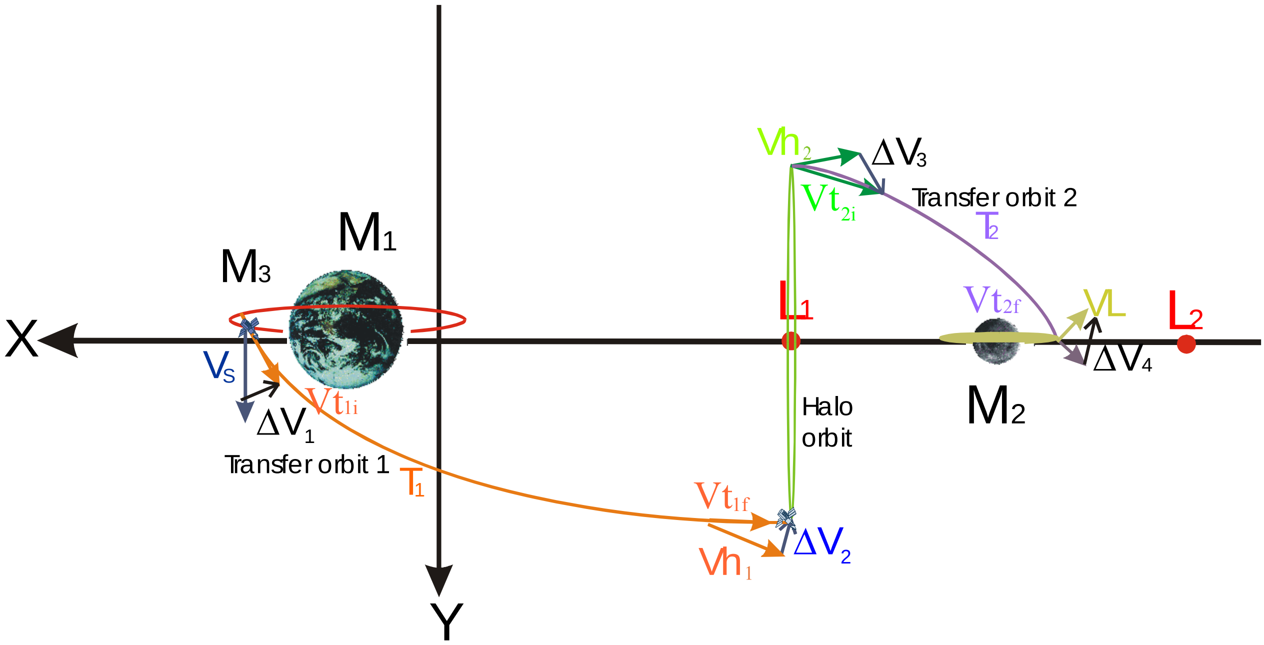

Figure 1. Geometry of the problem (detailed explanation will be show later in this paper).

The basic idea of this method is to use the non-linear part of the differential equations written as a Legendre polynomial (Richardson, 1980b). The solution of the linear approximation can be written as:

$$x = A\exp\{i\omega_o t\}; \quad y = KA\exp\{i\omega_o t\}; \quad z = B\exp\{i\nu_o t\} \tag{6}$$

where ω_0, ν_0 are the fundamental frequencies in the plane and perpendicular to the plane; A, B are the amplitudes (in complex notation) in the x and z axis, respectively, and K is the relation between the amplitudes in the x and y axis. After that, we know that when the non-linear terms of the differential equations are included in the variational equations. The general solution will depend on the linear solution in the plane and in the z axis. In this case the motions are no longer separable. Thas, it is possible to write the general solution as a Fourier series of the type

$$x = \sum_{i,j,k,l} X_{i,j,k,l} A^i B^j \exp\{k\omega + l\nu\}t \tag{7}$$

where $X_{i,j,k,l}$ are complex coefficients. The same is true for the variables y and z. Their determination is made by replacing the proposed solutions in the differential equations, identifying the terms of the same harmonic and power and solving the algebraic equations left. The new frequencies can be determined as a power series in the amplitudes

$$\omega = \sum_{i,j} \omega_{i,j} A^i B^j; \quad \nu = \sum_{i,j} \nu_{i,j} A^i B^j \tag{8}$$

where $\omega_{0,0}$; $\nu_{0,0}$ are the frequencies of the linear system. In this way, it is possible to determine periodic orbits if we request that ω and ν be both equal (i.e. $\omega = \nu$).
Following this determination, it is possible to perform the validation of the solution found by this method, using numerical integration of the restricted three-body problem with the initial condition found by the analytical method.

After finding a set of initial conditions (x, y, z, V_x, V_y, V_z) that allows the spacecraft to enter in an orbit around the Lagrangian point, it is possible to use the Lambert Problem approach in the restricted three-body problem, (which is described below) to find the orbital transfers. It is necessary to choose two points that belong to two different Halo orbits and vary the time of flight involved.

THE LAMBERT'S THREE-BODY PROBLEM

The problem that is considered in the present paper is that of finding trajectories to travel between two fixed points in the rotating frame, that belong to two different Halo orbits. Since those points are in known positions, this problem can be formulated as:

"Find an orbit (in the three-body problem context) that makes a spacecraft leave a given point A and go to another given point B". It is the TPBVP (two point boundary value problem). There are many orbits that satisfy this requirement, and the way used in this paper to find families of solutions is to specify a time of flight for the transfer. Then, the problem becomes the Lambert's three-body problem, that can be formulated as:

"Find an orbit (in the three-body problem context) that makes a spacecraft leave a given point A and go to another given point B, arriving there in a specified time of flight". Then, by varying the specified time of flight it is possible to find a whole family of transfer orbits and study them in terms of the required ΔV. This technique was used before in Prado (1993) and Prado and Broucke (1996).

To solve this problem to follow the steps:

i) Guess a initial velocity, together with the initial prescribed position, the complete initial state is known;
ii) Integrate numerically the equations of motion for a specified transfer time;
iii) Check the final position obtained from the numerical integration with the prescribed final position. If there is an agreement (difference less than a specified error allowed) the solution is found and the process can stop here. If there is no agreement, an increment in the initial guessed velocity is made and the process goes back to step ii).

The method used to find the increment in the guessed variables (initial velocity) is the standard gradient method, as described in Press *et al*, 1989. The routines available in this reference are also used in this research with minor modifications.

After this, we calculated the consumptions from the Earth for the Moon, according to the Figure 1, in the following way:

$$\Delta V_1 = \sqrt{(Vs_x - Vt_{1ix})^2 + (Vs_y - Vt_{1iy})^2 + (Vs_z - Vt_{1iz})^2} \qquad (9)$$

Where:
ΔV_1 = total impulse to insert the spacecraft in the first transfer orbit
$Vs_{x,y,z}$ = x, y, z components of the velocity of the spacecraft when in orbit around the Earth at 7000km, just before the application of the first impulse
$Vt_{1ix,y,z}$ = x, y, z components of the velocity of the spacecraft when it is inserted in the trajectory T_1, just after the application of the first impulse

$$\Delta V_2 = \sqrt{(Vh_{1x} - Vt_{1fx})^2 + (Vh_{1y} - Vt_{1fy})^2 + (Vh_{1z} - Vt_{1fz})^2} \qquad (10)$$

Where:
ΔV_2 = total impulse to insert the spacecraft in the Halo orbit
$Vh_{1x,y,z}$ = x, y, z components of the velocity of the spacecraft when it is insert in Halo orbit, just before the second impulse
$Vt_{1fx,y,z}$ = x, y, z components of the velocity of the spacecraft when it arrives in the Halo orbit just after the application of the second impulse

$$\Delta V_3 = \sqrt{(Vh_{2x} - Vt_{2ix})^2 + (Vh_{2y} - Vt_{2iy})^2 + (Vh_{2z} - Vt_{2iz})^2} \qquad (11)$$

Where:
ΔV_3 = total impulse to insert the spacecraft in the second transfer orbit
$Vh_{2x,y,z}$ = x, y, z components of the velocity of the spacecraft when in Halo orbit in instant it is insert in the trajectory T_2 just before the third impulse
$Vt_{2ix,y,z}$ = x, y, z components of the velocity of the spacecraft in orbit T_1 when it arrives in the in Halo orbit, just after application of the third impulse

$$\Delta V_4 = \sqrt{(VL_x - Vt_{2fx})^2 + (VL_y - Vt_{2fy})^2 + (VL_z - Vt_{2fz})^2} \qquad (12)$$

Where:
ΔV_4 = total impulse to insert the spacecraft around the Moon
$VL_{x,y,z}$ = x, y, z components of the velocity of the spacecraft when in orbit around the Moon at 1960km, just after the application of the fourth impulse
$Vt_{2fx,y,z}$ = x, y, z components of the velocity of the spacecraft when in trajectory T_2 in instant of the insert in lunar orbit, just before the application of the fourth impulse
For the transfers between the Moon and the Earth the calculations are the same ones, considering the theorem of the mirror image (Miele, 1960).

RESULTS

To simulate orbital transfers, the approach used in this paper is to make the spacecraft to leave the Earth's orbit from a point located at 7000 km from the Earth and going to four points of each of the three Halo orbits, analyzing the consumption. Then perform the transfers from the points that belong to the Halo to going to a point of an orbit with 1960 km altitude from the Moon, also analyzing the consumption. The same procedure is used for the return from the Moon to the Earth. Transfers between the Halos are also considered, and for these situations, the points were chosen in such way that the time to travel between them is constant. The fact that the velocity of the spacecraft is not constant for a spacecraft in the Halo orbit, makes those points to be closer to each other in the regions where the velocity is lower. The number of the points in each Halo orbit can be increased to get a more accurate minimum cost maneuver. Then, each point of the initial orbit is combined with each point of the final orbit. For each pair of points, the Lambert problem is solved for a series of times of flight. Then, the maneuver that has the minimal fuel consumption for each pair is listed in a table. After all the simulations are performed, we have a table that gives the minimum fuel consumption maneuver for the whole transfer. Fig. (2) shows several Halo orbits discretized in points. From this collective view it is possible to see the three-dimensional character of the orbits.

Table 2. Position and Velocity, in the rotating frame, for the points at the Halo Orbit

For $\beta = 0.001$ = HALO 1

x	y	z	vx	vy	vz	Point
-0,8550346	0	$1,41181571 \times 10^{-4}$	0	0,13357503945	0	1
-0,8446384	$5,555764 \times 10^{-2}$	$5,5244823 \times 10^{-7}$	$3,5035189 \times 10^{-3}$	$1,4193513 \times 10^{-2}$	$-3,430358 \times 10^{-4}$	2
-0,8236527	0	$-1,6185533 \times 10^{-4}$	0	-0,1261723	0	3
-0,8446384	$-5,555764 \times 10^{-2}$	$5,5244823 \times 10^{-7}$	$-3,5035189 \times 10^{-2}$	$1,4193513 \times 10^{-2}$	$3,430358210^{-4}$	4

For $\beta = 0.05$ = HALO 2

x	y	z	vx	vy	vz	Point
-0,855364	0	$7,0536659 \times 10^{-3}$	0	0,13499743	0	a
-0,8448171	$5,6101007 \times 10^{-2}$	$1,6340096 \times 10^{-5}$	$3,5429034 \times 10^{-2}$	$1,4204071 \times 10^{-2}$	$-1,7149859 \times 10^{-2}$	b
-0,8236527	0	$-7,0536659 \times 10^{-3}$	0	-0,13499743	0	c
-0,8446384	$-5,6101007 \times 10^{-2}$	$1,6340096 \times 10^{-5}$	$-3,5429034 \times 10^{-2}$	$1,4204071 \times 10^{-2}$	$1,7149859 \times 10^{-2}$	d

For $\beta = 0.1$ = HALO 3

x	y	z	vx	vy	vz	Point
-0,856353	0	$1,4075205 \times 10^{-2}$	0	0,13919989	0	e
-0,845349	$5,7701918 \times 10^{-2}$	$-3,4341665 \times 10^{-5}$	$3,659545 \times 10^{-2}$	$-1,422379 \times 10^{-2}$	$-3,428782 \times 10^{-2}$	f
-0,823642	0	$-1,623469 \times 10^{-2}$	0	-0,13044728	0	g
-0,845349	$-5,7701918 \times 10^{-2}$	$3,4341665 \times 10^{-2}$	$-3,659545 \times 10^{-2}$	$1,422379 \times 10^{-2}$	$3,4287826 \times 10^{-2}$	h

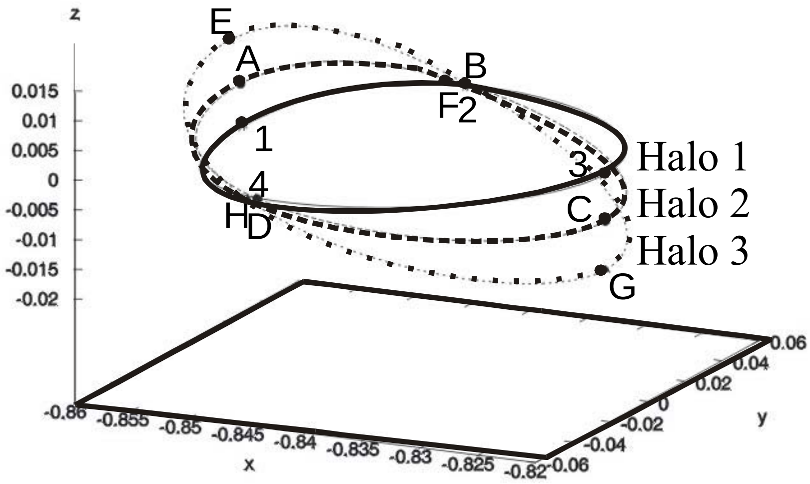

Figure 2. Halo orbits showing the points for the transfers.

For this first study, three different Halo orbits were generated with the Linstedt-Poincaré method. They all belong to the same family around the Lagrangian point L_1. The difference between them can be expressed by a single parameter, usually the parameter B, that represents the linear amplitude in the z axis. In terms of this parameter, the orbits used in this research are: 0.001, 0.05, and 0.100. Table 2 shows the position and velocity, in the rotating frame, for the eight points in which the orbits were divided and for all three orbits. This table also defines the notation used for the points.

Table 3 shows the consumption referent to transfers between the three Halo orbits. Table 4 show the consumption between a point in an orbit around the Earth with 7000 km altitude with the points of the Halos. Table 5 show the consumption between the Halo orbits and a point in an orbit with 1960 km altitude around of the Moon. The shaded lines represent the minimum consumptions between each orbit.

Table 3 – Transfers between Halo orbits

From 1 to:	t	Δv
a	1.03	0.262335939
b	0.63	0.017876695
c	1.24	0.155990324
d	0.82	0.263937248
From 1 to:	t	Δv
e	0.85	0.273686580
f	0.64	0.038318047
g	1.09	0.200032945
h	0.82	0.281566522
From 2 to:	t	Δv
a	2.82	0.263582076
b	0.65	0.082047226
c	0.75	0.018003250
d	1.46	0.028360674
From 2 to:	t	Δv
e	0.82	0.280662052
f	1.32	0.100379785
g	0.74	0.038510893
h	1.41	0.061007929
From 3 to:	t	Δv
a	1.24	0.151801072
b	0.77	0.250639137
c	1.08	0.243324808
d	0.75	0.017946231
From 3 to:	t	Δv
e	1.11	0.193891663
f	0.78	0.267202718
g	0.91	0.253167559
h	0.75	0.038420914
From 4 to:	t	Δv
a	0.62	0.017919696
b	1.25	0.017584640
c	0.77	0.251301416
d	0.75	0.083913889
From 4 to:	t	Δv
e	0.62	0.038531253
f	1.26	0.042885454
g	2.39	1.046032490
h	1.41	0.103938115
From a to:	t	Δv
1	1.03	0.262335934
2	0.62	0.017919671
3	1.24	0.151801055
4	0.82	0.263582007
From b to:	t	Δv
1	0.82	0.263937309
2	0.75	0.083915128
3	0.75	0.017946219
4	1.5	0.029042611

From c to:	T	Δv
1	1.24	0.155990325
2	0.77	0.251301407
3	1.08	0.243324804
4	0.745	0.017946655
From d to:	t	Δv
1	0.63	0.017876647
2	1.25	0.017584569
3	0.77	0.250639145
4	0.65	0.082046492
From e to:	t	Δv
1	0.85	0.273686570
2	0.62	0.038531233
3	1.11	0.193891638
4	0.82	0.280661992
From f to:	t	Δv
1	0.82	0.281566563
2	1.41	0.103938697
3	0.75	0.038420900
4	1.41	0.061007747
From g to:	t	Δv
1	1.09	0.200032932
2	0.77	0.268996540
3	0.91	0.253167554
4	0.74	0.038510888
From h to:	t	Δv
1	0.64	0.038318003
2	1.26	0.042885496
3	0.78	0.267202707
4	1.32	0.100379591
From a to:	t	Δv
e	0.58	0.278320163
f	0.63	0.020512544
g	1.24	0.162766349
h	0.81	0.297647138
From b to:	t	Δv
e	0.81	0.297181476
f	0.113	0.100965826
g	0.75	0.020568875
h	1.43	0.062083624
From c to:	t	Δv
e	1.25	0.150105086
f	0.78	0.284484391
g	0.71	0.257150758
h	0.75	0.020511338
From d to:	t	Δv
e	0.62	0.020549726
f	1.26	0.050864102
g	0.77	0.285457143
h	0.123	0.104397009

Table 4 – Transfers between the Earth and the Halo orbits

From Earth to:	t	Δv
1	0.50	4.708563921
2	0.51	4.680172369
3	3.13	14.050712468
4	0.47	4.863093722
a	0.50	4.711744533
b	0.51	4.679886429
c	0.48	4.834240523
d	0.47	4.864458074
e	0.50	4.721216335
f	0.51	4.679101019
g	0.48	4.852447869
h	0.47	4.868509170

Table 5 – Transfers between the Halo orbits and the Moon

To Moon from:	t	Δv
1	2.61	3.650079659
2	3.06	3.532641386
3	2.45	3.475038607
4	2.63	3.558500329
a	2.59	3.648711924
b	2.43	3.542211996
c	2.39	3.485044319
d	2.67	3.549355841
e	2.56	3.647806028
f	2.45	3.526158372
g	2.35	3.486904404
h	2.63	3.542687700

CONCLUSIONS

The algorithm developed in this paper, that uses the "Lambert Method" for the calculation of orbital maneuvers with minimum consumption of fuel and limit in time for the transfer, was used for a complete transfer between the Earth and the Moon, going from an Halo orbit around the intermediate lagrangian point. The method was shown to be efficient and several transfers trajectories were generated. The same method was applied for the calculation of transfers between two Halo orbits. In all situations it was possible to identify transfers with minimum cost, indicating the initial and final points of each maneuver, as well as the magnitude and direction of the necessary impulses to complete it.

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