

# The behavior of a discrete closed-loop control system over a ZOH equivalent benchmark plant in presence of phase compensations \_ investigations about the Tustin rule of mapping

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## ABSTRACT

This work is a part of another and more complete work aimed to project the digital control of an asymmetric satellite with a flexible appendage in that we studied the performance of a closed-loop discrete control of a benchmark plant (DCBP) as a harmonic oscillator with a high sampling period. Instability problems arise immediately and after apply techniques as increase the damping ratio, apply an anti-aliasing filter (fourth-order Buterworth filter) and changing the usual and moderns continuous to discrete mapping methods present in the literature and don't solving the problem, arise the following two questions: would be the instability provoked by the half period delay implanted by the ZOH? May we stabilize the DCBP compensating this delay? This work show how is the performance of the DCBP in presence of phase compensations of the ZHO half period delay. It will be shown also a comparison with the conventional performance of a design with the usual Tustin or bilinear rule and a new propose.

## 1. Introduction

*{ we may apply your introductory part here (“Insights on...”), talking a little bit more about the half period delay of the zero-order hold }*

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## 2. The Zoh half period delay and lead compensation

Using a overdamped single mode ( $\omega_n=0.8244$  rad/sec) DCBP with high sampling period of  $T_s = 1.6$  seconds we may calculate the Zoh half period delay of the following simple manner:

$$\Psi = \frac{180}{\pi} \left( 2\pi \cdot 0.1312 \cdot \frac{T_s}{2} \right) = 37.78 \text{ deg} \quad (1)$$

The phase lead of a PD controller will be given by:

$$\Phi = \tan^{-1} \left( \frac{k_d \cdot \omega}{k_p} \right) \quad (2)$$

where  $k_d$  and  $k_p$  are the derivative and proportional control gains.

None of the cases with Tustin rule was capable to give the stability although we use the high phase leads as shown in Table 1. This fact is due that the phase depends strictly of the ratio between the control gains and, beyond this, the open-loop gain depends strictly and individually of the absolute values of these gains, as we may verify in (3), the transfer function of the closed-loop system of a harmonic oscillator.

$$H(z) = [1 - \cos(\omega_n \cdot T_s)] \frac{z^2 + 2z + 1}{z^3 + [1 - 2 \cdot \cos(\omega_n \cdot T_s) + (k_p + 2k_d/T_s)] [1 - \cos(\omega_n \cdot T_s)] z^2 + \dots} \quad (3)$$

$$\dots \frac{[1 - 2 \cdot \cos(\omega_n \cdot T_s) + (k_p + 2k_d/T_s)] [1 - \cos(\omega_n \cdot T_s)] \left( \frac{k_p \cdot T_s - 2 \cdot k_d}{k_p \cdot T_s + 2 \cdot k_d} + 1 \right)}{z + 1 + \dots}$$

$$\dots \frac{+ (k_p + 2k_d/T_s) [1 - \cos(\omega_n \cdot T_s)] \left( \frac{k_p \cdot T_s - 2 \cdot k_d}{k_p \cdot T_s + 2 \cdot k_d} \right)}{z + 1 + \dots}$$

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Using a modification of the Tustin rule we may note that the instability problem was solved for some phase leads and some values of the  $\xi$  parameter (Table 1).

TABLE 1: COMPARISON BETWEEN THE TWO RULES

$\Phi (^{\circ})$	TUSTIN	NEW RULE
$87^{\circ}.7777$	Unstable	Unstable
$57^{\circ}.0983$	Unstable	Stable ( $\xi=0.13$ )
$51^{\circ}.0370$	Unstable	Stable ( $\xi=0.13$ )

### 3. Conclusions

We conclude that the lead compensation didn't show satisfactory results front the instability problem when we use the usual Tustin rule of mapping. When we use the new rule present in this work we achieved the stability for some phase lead ranges of values and with a  $\xi=0.13$ . This work only shown that the phase lead solution doesn't may solve the instability problem using a usual Tustin rule and although we use a new rule and stabilize the system with it, we don't need to use this proceeding because it changes the control characteristics as rise time, overshoot and settling time.

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