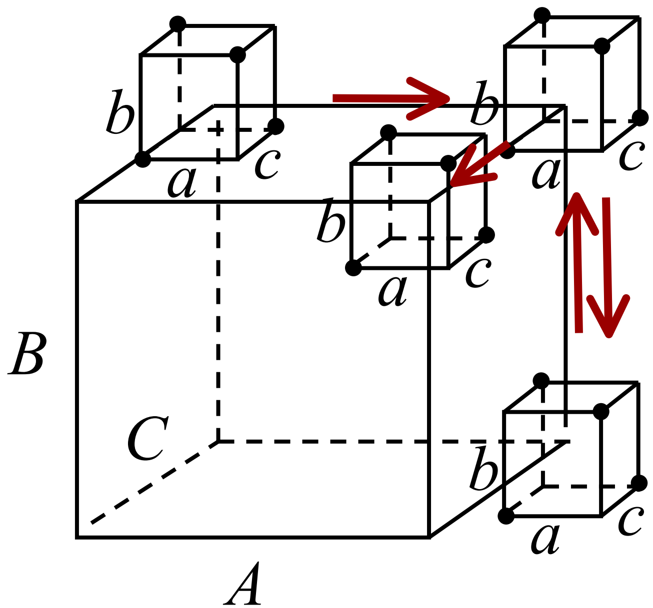
Robust Engineering Design: Making it Efficient, Making it Concurrent

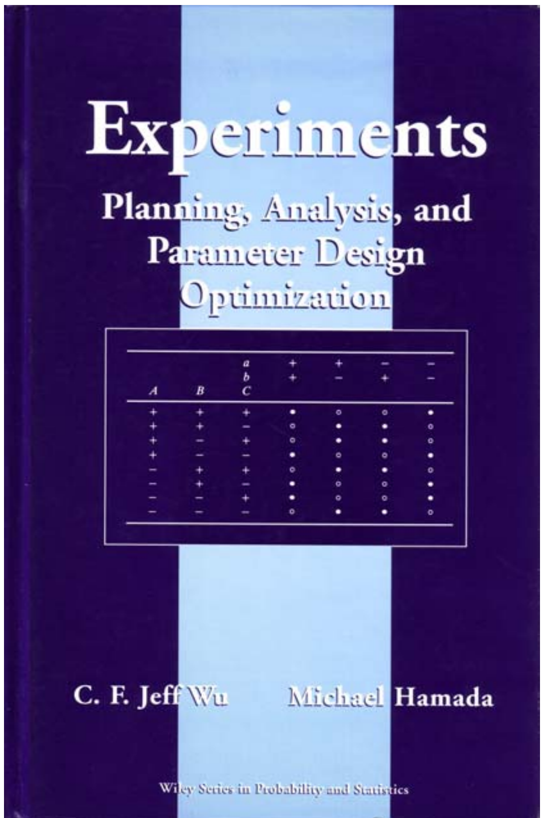


Dan Frey
Associate Professor of Mechanical Engineering and Engineering Systems

Outline

- Introduction and motivation
- Making robust design more efficient
  - Adaptive robust design
  - Characterizing noise
  - Structuring / inducing noise
- Making robust design concurrent

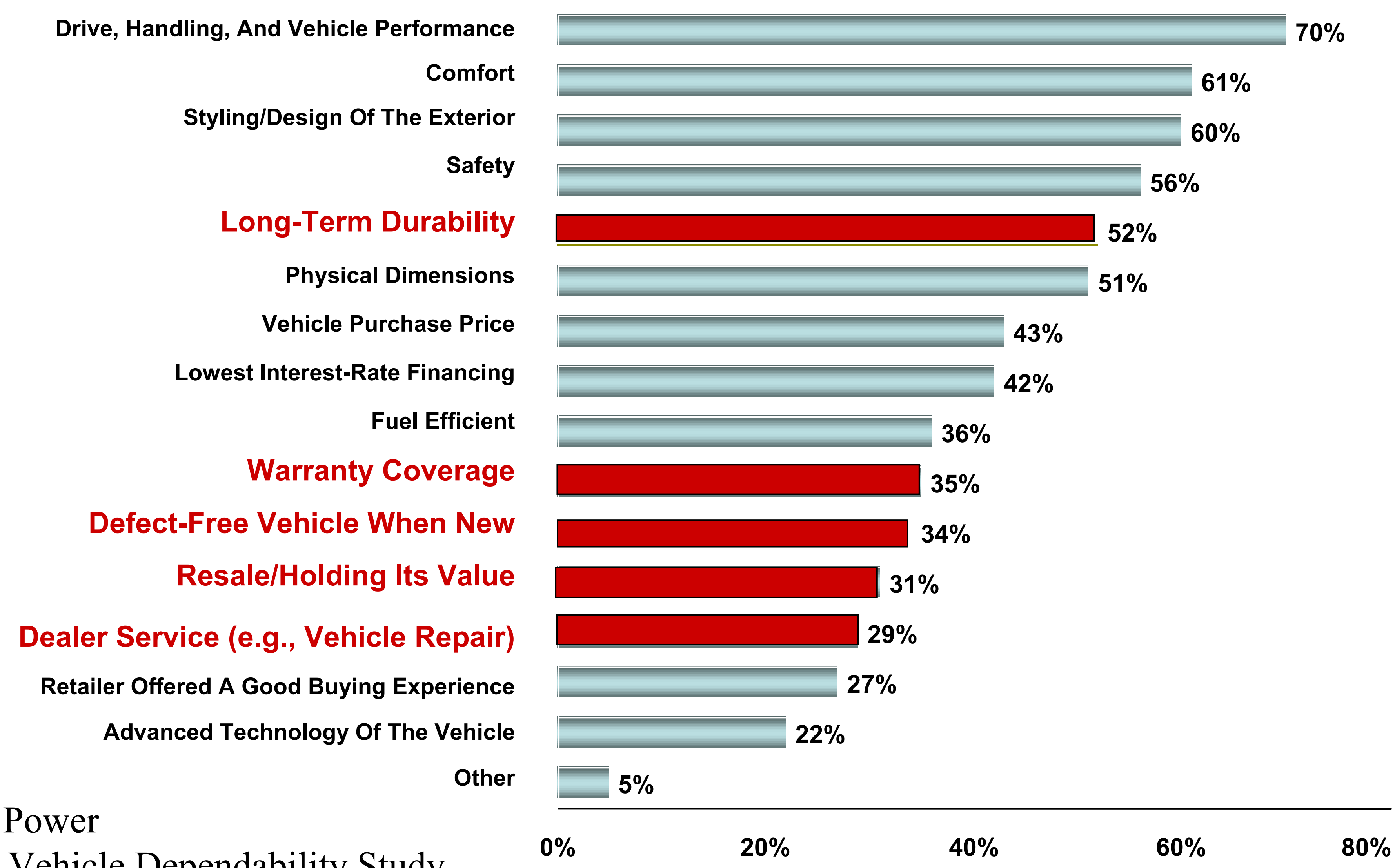
Robust Parameter Design



"Robust Parameter Design … is a statistical / engineering methodology that aims at reducing the performance variation of a system (i.e. a product or process) by choosing the setting of its control factors to make it less sensitive to noise variation."

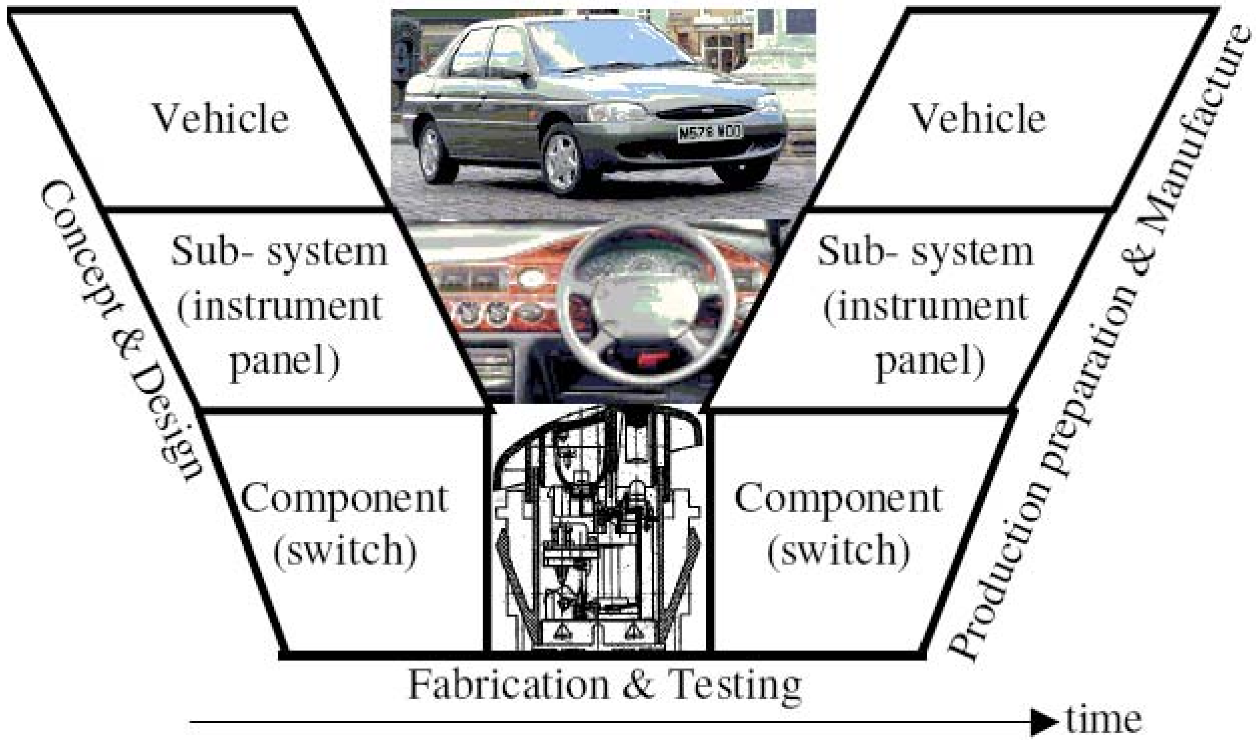
Wu, C. F. J. and M. Hamada, 2000, *Experiments: Planning, Analysis, and Parameter Design Optimization*, John Wiley & Sons, NY.

Robustness and Competitiveness



J. D. Power
2003 Vehicle Dependability Study

Robust Design in Concurrent Engineering



"...the only uncertainty is *when* the failure modes will be found, not *if.*"

Davis, T. P., "Science, Engineering, and Statistics," *Applied Stochastic Models in Business and Industry*, v. 22, pp. 5-6.

Crossed Arrays

Noise Factors 2^{3-1}_{III}

Control Factors

2^{7-4}_{III}

	A	B	C	D	E	F	G					
								a	-1	-1	+1	+1
								b	-1	+1	-1	+1
								c	-1	+1	+1	-1
1												
2												
3												
4												
5												
6												
7												
8	+1	+1	-1	+1	-1	-1	+1					

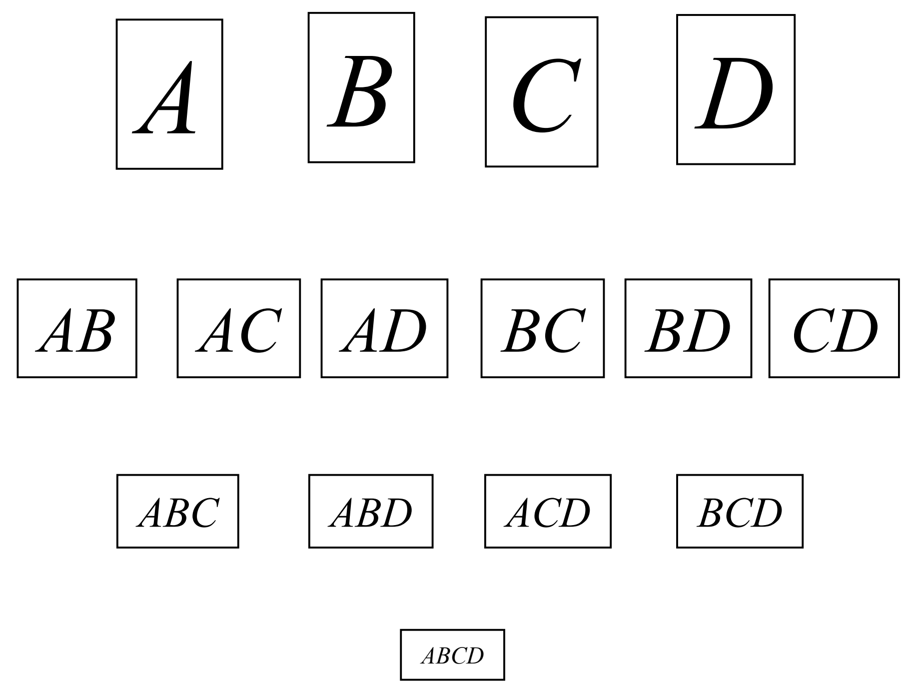
The efforts required for robust design (conducted in this way) scale as <u>the product</u> of the number of control factors and the number of noise factors.

$$2^{7-4}_{III} \times 2^{3-1}_{III}$$

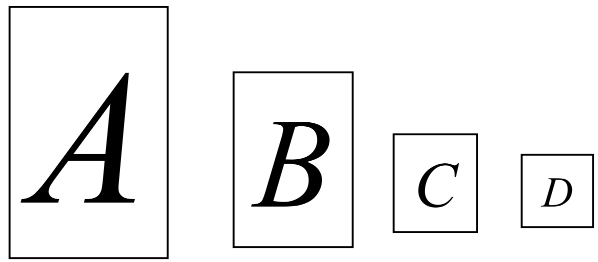
Taguchi, G., 1976, *System of Experimental Design*.

Hierarchy

- Main effects are usually more important than two-factor interactions
- Two-way interactions are usually more important than three-factor interactions
- And so on



Sparsity of Effects



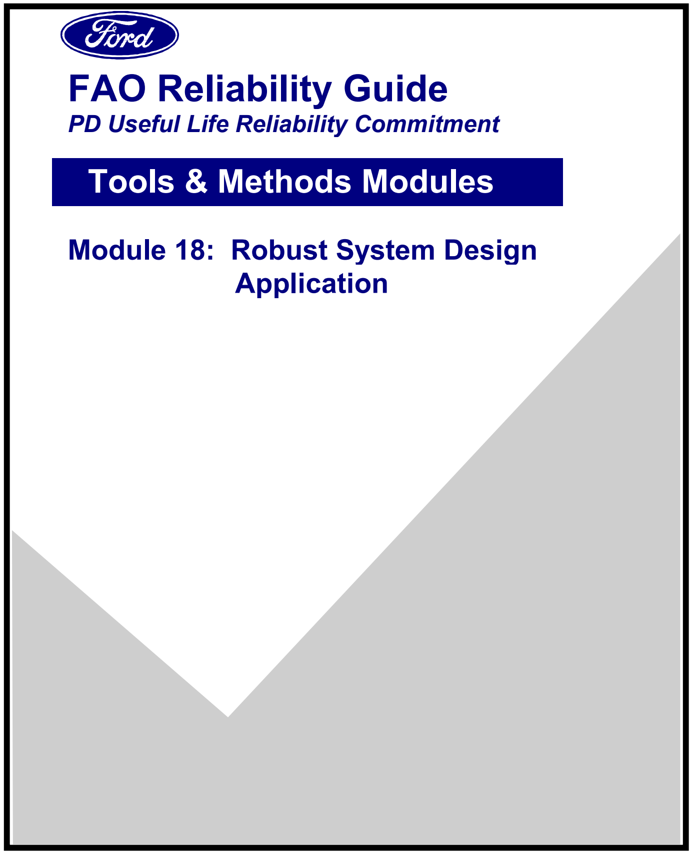
- An experimenter may list a large number of effects for consideration
- A small number of effects usually explain the majority of the variance

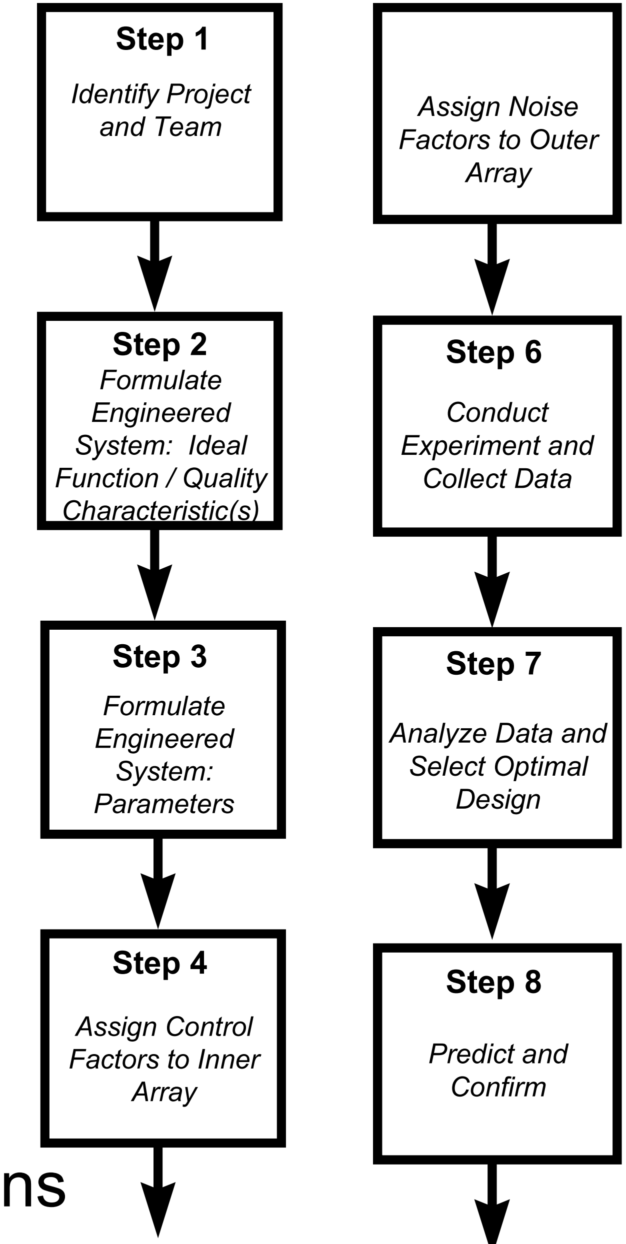
My Observations of Industry

- Fewer than 5% of components and subsystems are subjected to robust design methods
- Of those robust design projects that are planned, more than 50% are not finished
  - Unforseen changes
  - Resource pressure
  - Satisficing

e.g.

“Well, in the third experiment, we found a solution that met all our needs, so we cancelled the rest of the experiments and moved on to other tasks…”

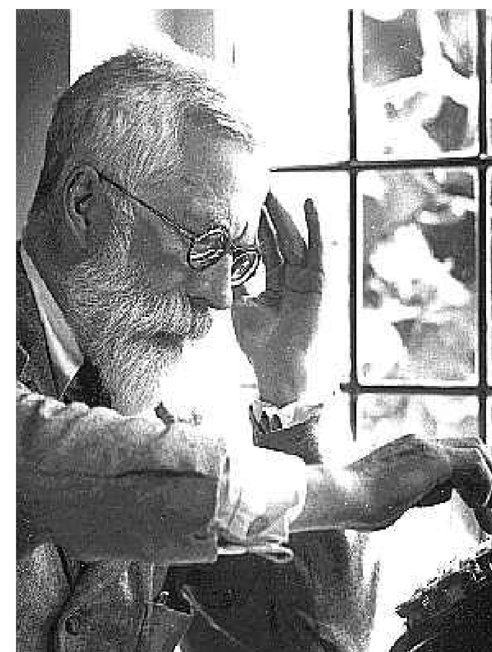




Step 4 Summary:

- Determine control factor levels
- Calculate the DOF
- Determine if there are any interactions
- Select the appropriate orthogonal array

"… The natural remedy for dogmatism has been found in research. … The research worker is ..., therefore, ... a good teacher for the few who wish to use their mind as a workshop, rather than a warehouse."



Fisher, Sir Ronald A.,1935, "Eugenics, academic and practical," *Eugenics Review,* **27,** 95-100, 1935.

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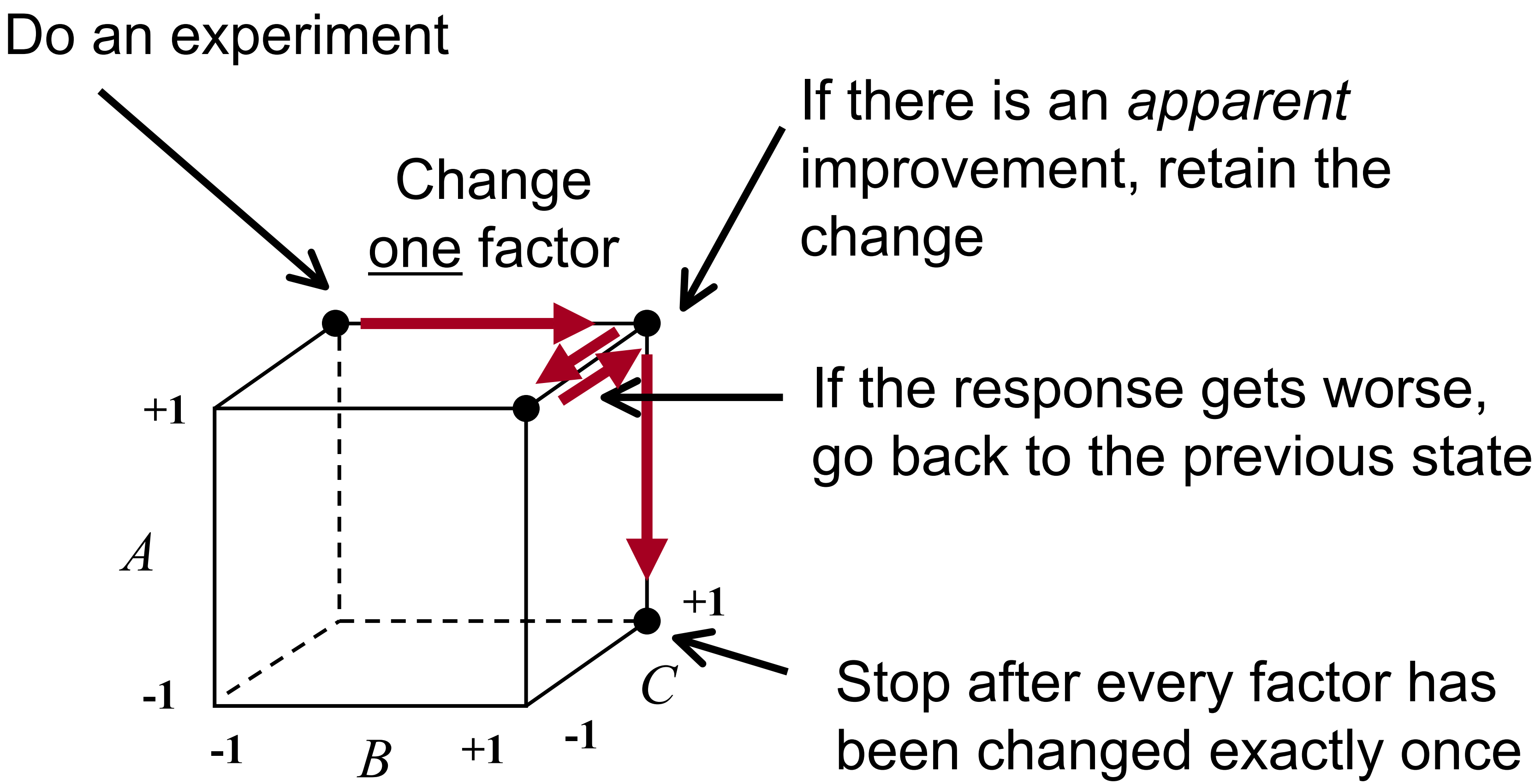
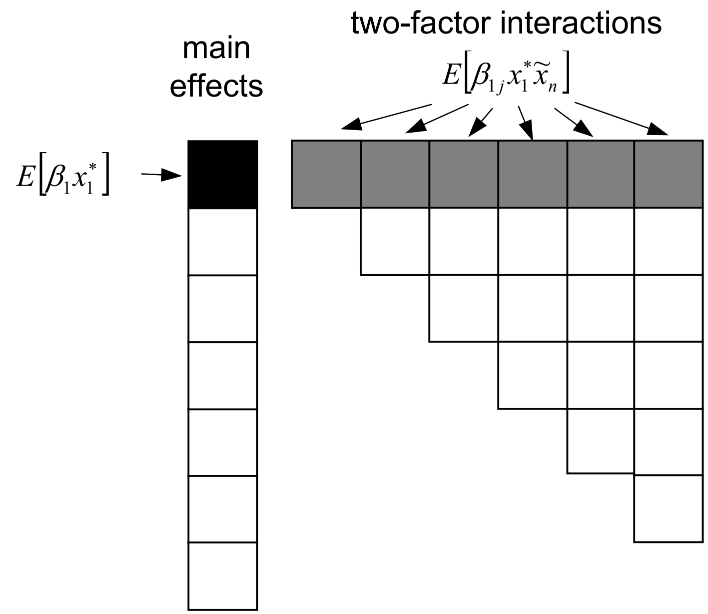
Adaptive One Factor at a Time Experiments



The First Step in *a*OFAT

$$E(y(x_1^*, \tilde{x}_2, \ldots, \tilde{x}_n)) = E\left[\beta_1 x_1^*\right] + (n-1)E\left[\beta_{1j} x_1^* \tilde{x}_j\right]$$



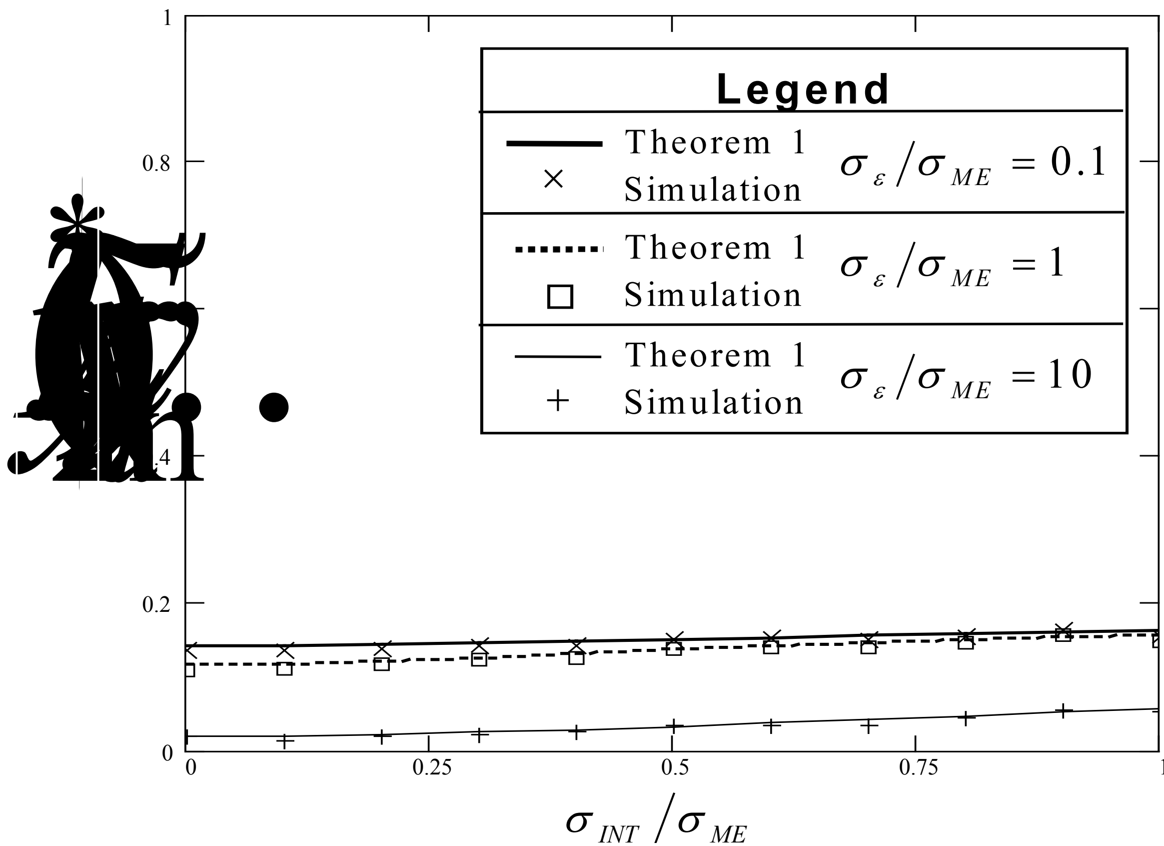
$$E\left[\beta_1 x_1^*\right] = \sqrt{\frac{2}{\pi}} \frac{\sigma_{ME}^2}{\sqrt{\sigma_{ME}^2 + (n-1)\sigma_{INT}^2 + \frac{1}{2}\sigma_\varepsilon^2}}$$

$$E\left[\beta_{1j} x_1^* \tilde{x}_j\right] = \sqrt{\frac{2}{\pi}} \frac{\sigma_{INT}^2}{\sqrt{\sigma_{ME}^2 + (n-1)\sigma_{INT}^2 + \frac{1}{2}\sigma_\varepsilon^2}}$$

$$\Pr\left(\beta_1 x_1^* > 0\right) = \frac{1}{2} + \frac{1}{\pi}\sin^{-1}\frac{\sigma_{ME}}{\sqrt{\sigma_{ME}{}^2 + (n-1)\sigma_{INT}{}^2 + \frac{1}{2}\sigma_\varepsilon{}^2}}$$

Frey, D. D., and H. Wang, 2006, “Adaptive One-Factor-at-a-Time Experimentation and Expected Value of Improvement”, *Technometrics* 48(3):418-31.

Performance after the First Step (n=7)

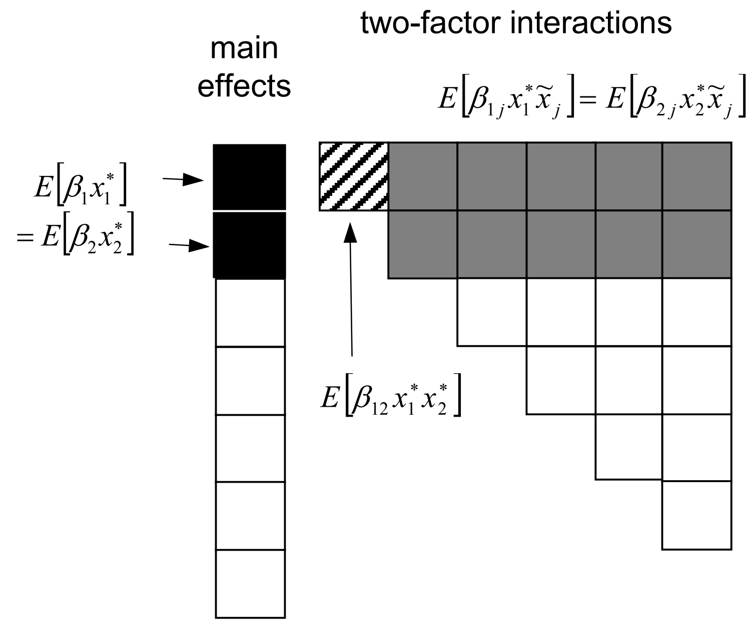


The Second Step in *a*OFAT

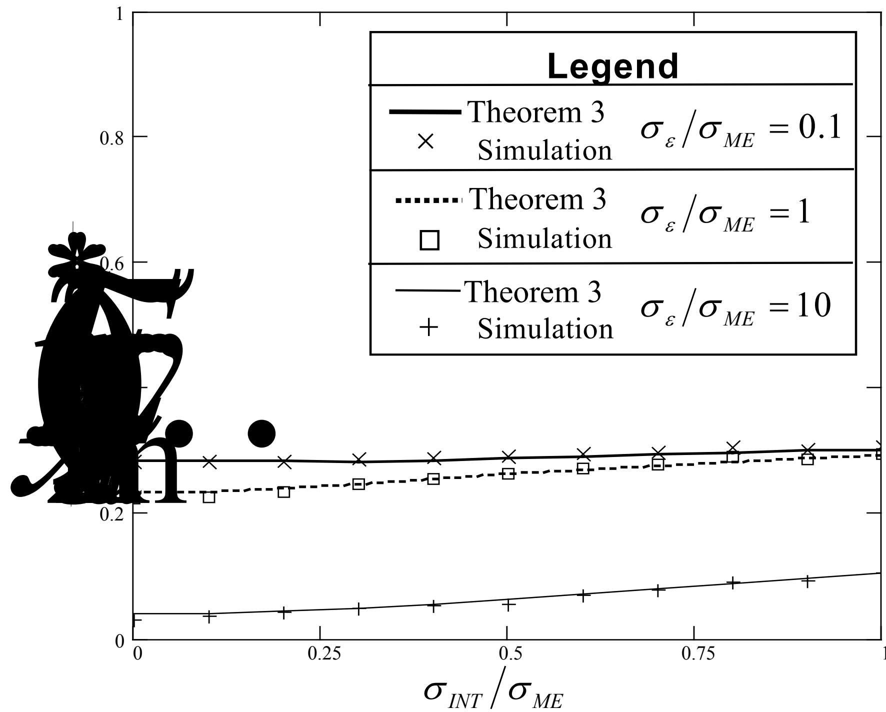
$$E(y(x_1^*, x_2^*, \widetilde{x}_3, \ldots, \widetilde{x}_n)) = 2E\left[\beta_1 x_1^*\right] + 2(n-2)E\left[\beta_{1j} x_1^*\right] + E\left[\beta_{12} x_1^* x_2^*\right]$$

$$E\left[\beta_{12} x_1^* x_2^*\right] = \sqrt{\frac{2}{\pi}}\left[\frac{\sigma_{INT}^2}{\sqrt{\sigma_{ME}^2 + (n-1)\sigma_{INT}^2 + \dfrac{\sigma_\varepsilon^2}{2}}}\right]$$

$$\Pr\left(\beta_{12} x_1^* x_2^* > 0\right) = \frac{1}{2} + \frac{1}{\pi}\tan^{-1}\frac{\sigma_{INT}}{\sqrt{\sigma_{ME}^2 + (n-2)\sigma_{INT}^2 + \frac{1}{2}\sigma_\varepsilon^2}}$$



Performance after the Second Step (n=7)



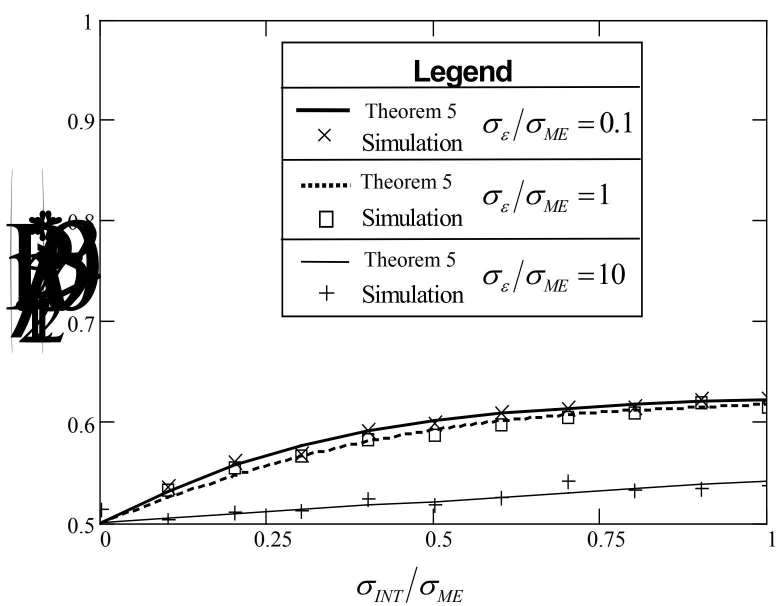
Expected improvement after the second variable is set in adaptive OFAT given a system with seven factors.

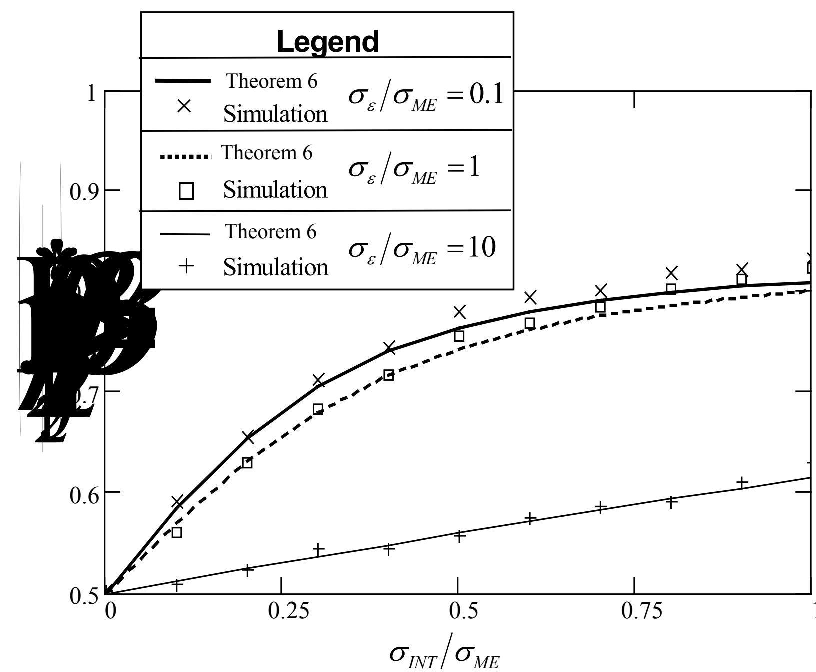
Probability of Exploiting the First Two-Factor Interaction (n=7)

$$\Pr\left(\beta_{12}x_1^*x_2^*>0\right)=\frac{1}{2}+\frac{1}{\pi}\tan^{-1}\frac{\sigma_{INT}}{\sqrt{\sigma_{ME}^2+(n-2)\sigma_{INT}^2+\frac{1}{2}\sigma_\varepsilon^2}}$$

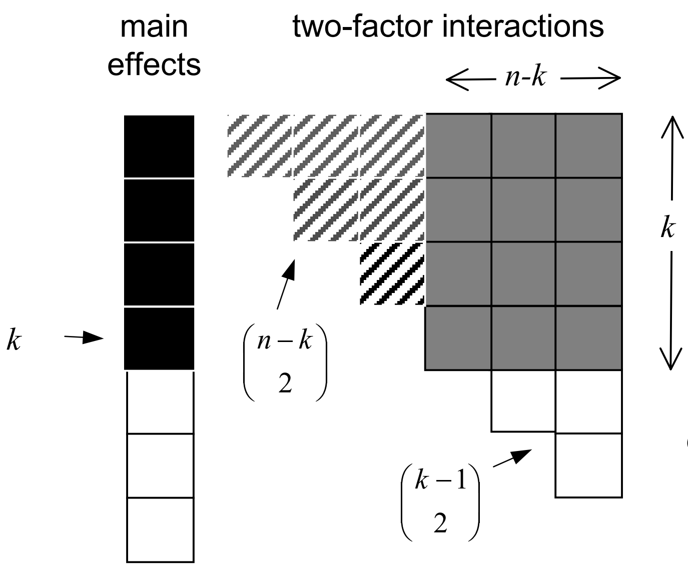
$$\Pr\left(\beta_{12}x_1^*x_2^*>0\middle|\beta_{12}>\beta_{\substack{ij\\ i\neq j}}\right)>$$

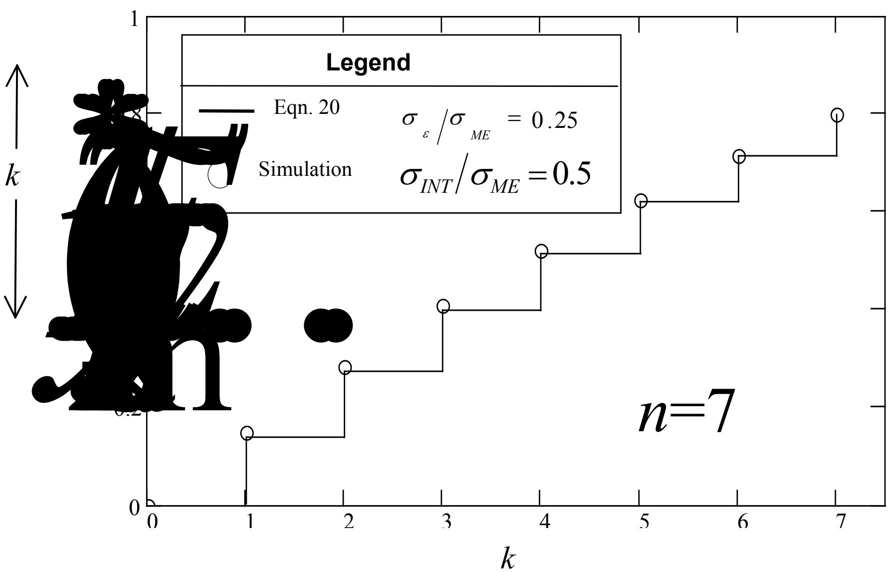
$$\frac{1}{\pi}\binom{n}{2}\int_0^\infty\int_{-x_2}^\infty\frac{\left[erf\left(\frac{1}{\sqrt{2}}\frac{x_1}{\sigma_{INT}}\right)\right]^{\binom{n}{2}-1}e^{\frac{-x_1^2}{2\sigma_{INT}^2}+\frac{-x_2^2}{2\left(\sigma_{ME}^2+(n-2)\sigma_{INT}^2+\frac{1}{2}\sigma_\varepsilon^2\right)}}}{\sigma_{INT}\sqrt{\sigma_{ME}^2+(n-2)\sigma_{INT}^2+\frac{1}{2}\sigma_\varepsilon^2}}dx_2dx_1$$





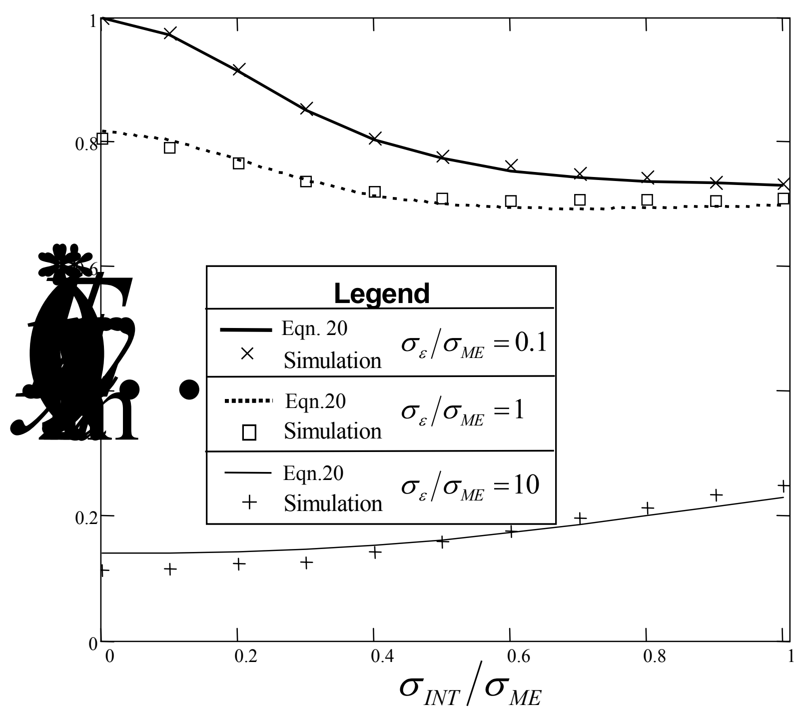
Performance after Multiple Steps



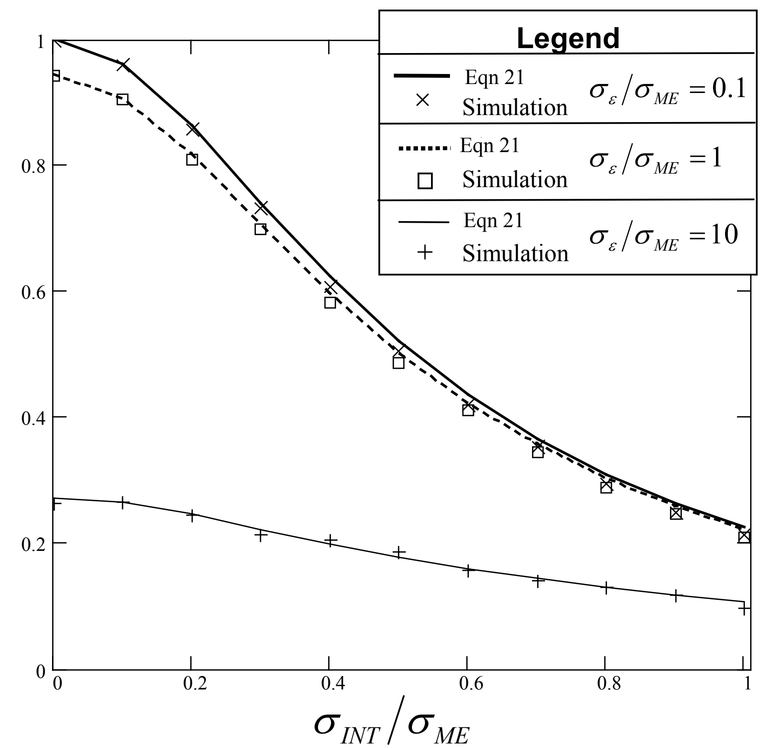


$$\Pr\left(\beta_{ij}x_i^*x_j^*>0\right)\geq\Pr\left(\beta_{12}x_1^*x_2^*>0\right)$$

Final Outcome (n=7)

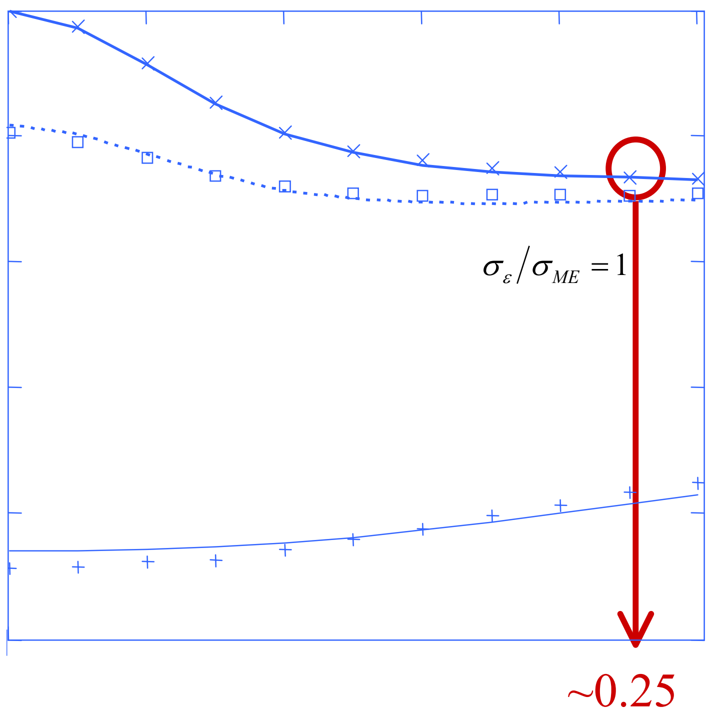


Adaptive OFAT

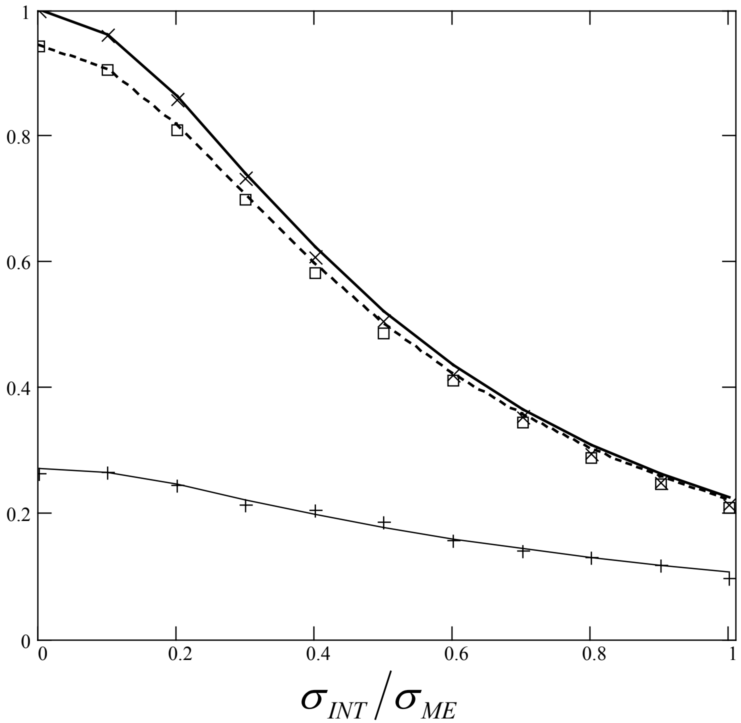


Resolution *III* Design

Final Outcome (n=7)



Adaptive OFAT



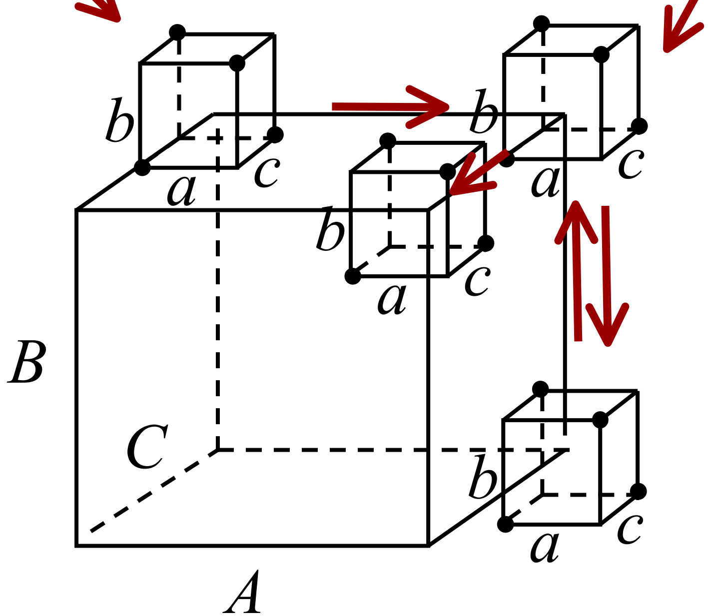
Resolution *III* Design

Adaptive "One Factor at a Time" for Robust Design

Run a resolution III on noise factors

Change <u>one</u> factor

Again, run a resolution III on noise factors. If there is an improvement, in transmitted variance, retain the change

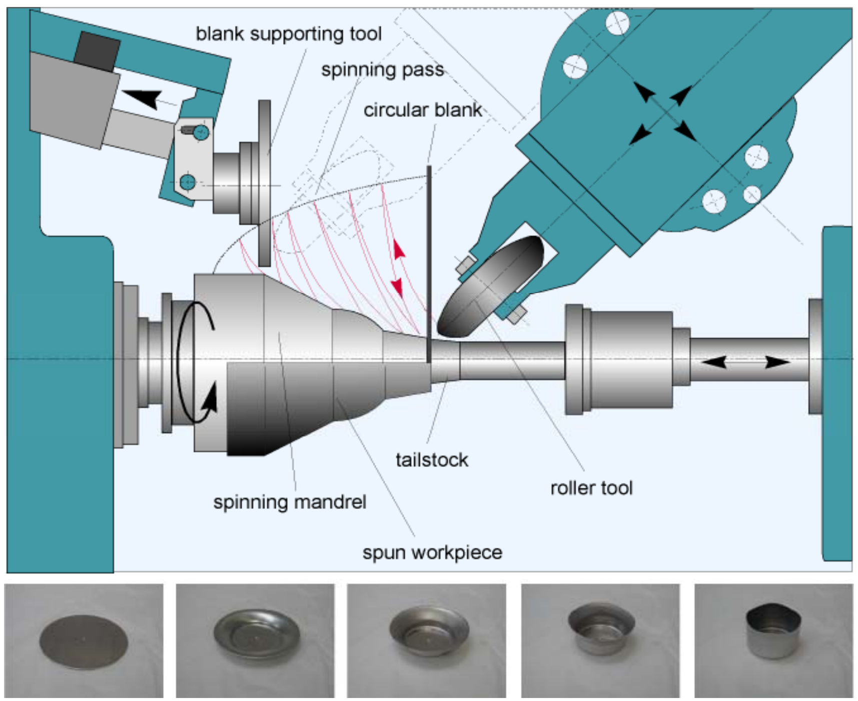


If the response gets worse, go back to the previous state

Stop after you've changed every factor once

Frey, D. D., and N. Sudarsanam, 2006, "An Adaptive One-factor-at-a-time Method for Robust Parameter Design: Comparison with Crossed Arrays via Case Studies," accepted to *ASME Journal of Mechanical Design*.

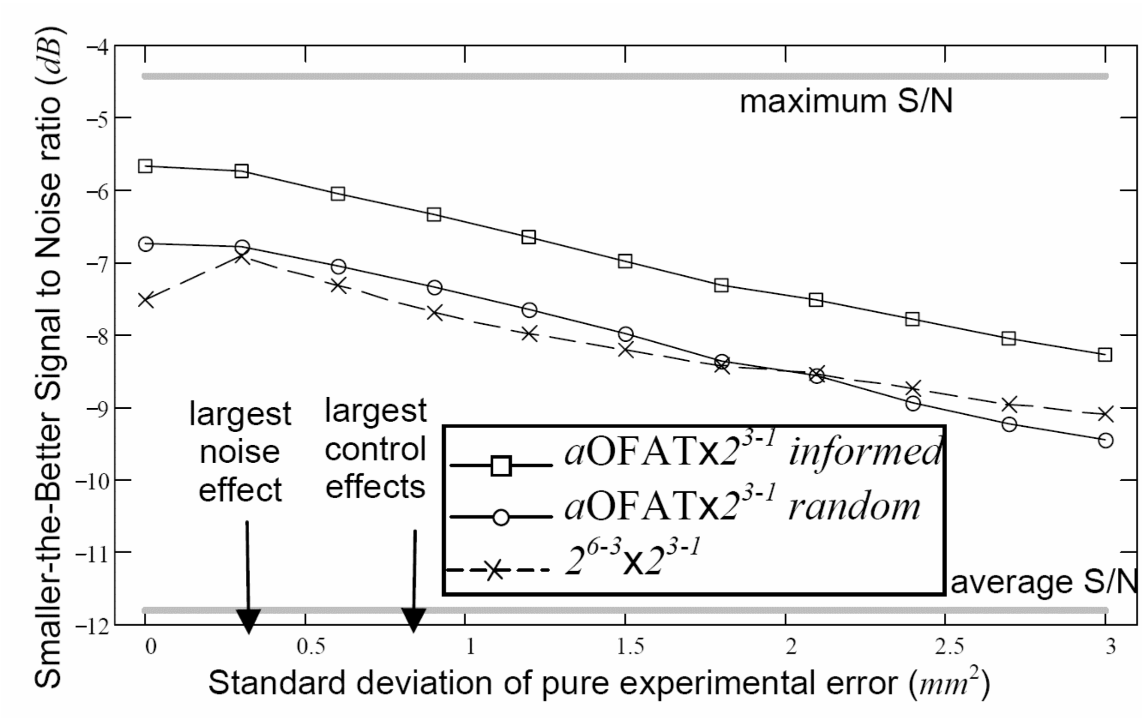
A Manufacturing Case Study



- Sheet metal spinning
- 6 control factors (number of passes of the tool, etc.)
- 3 noise factors (material properties, etc.)
- Goal = more uniform geometry

Kunert, J., et. al., 2004, "An experiment to compare the combined array and the product array for robust parameter design," *J. of Quality Technology* **39**(1)17-34. .

A Manufacturing Case Study



- *a*OFAT worked better if experimental error not too high
- Especially true if an informed starting point was used

Frey, D. D., N. and Sudarsanam, 2006, "An Adaptive One-factor-at-a-time Method for Robust Parameter Design: Comparison with Crossed Arrays via Case Studies," accepted to *ASME Journal of Mechanical Design*.

Results Across Four Case studies

		Method used		
		Fractional array $\times 2_{III}^{k-p}$	$aOFAT \times 2_{III}^{k-p}$	
			informed	*random*
sheet metal spinning	Low ε	51%	75%	56%
	High ε	36%	57%	52%
op amp	Low ε	99%	99%	98%
	High ε	98%	88%	87%
paper airplane	Low ε	43%	81%	68%
	High ε	41%	68%	51%
freight transport	Low ε	94%	100%	100%
	High ε	88%	85%	85%
Mean of four cases	Low ε	74%	91%	84%
	High ε	66%	70%	64%
Range of four cases	Low ε	43% to 99%	75% to 100%	56% to 100%
	High ε	36% to 88%	57% to 88%	51% to 87%

Frey, D. D., N. and Sudarsanam, 2006, “An Adaptive One-factor-at-a-time Method for Robust Parameter Design: Comparison with Crossed Arrays via Case Studies,” accepted to *ASME Journal of Mechanical Design*.

Conclusions : Adaptive Experimentation

- If the goal is maximum improvement rather than maximum precision in estimation
- And experimental error is not too large
- Then adaptive experimentation provides significant advantages over factorial plans
- Mostly because it exploits interactions, especially the largest ones
- Demonstrated to be effective for robust parameter design

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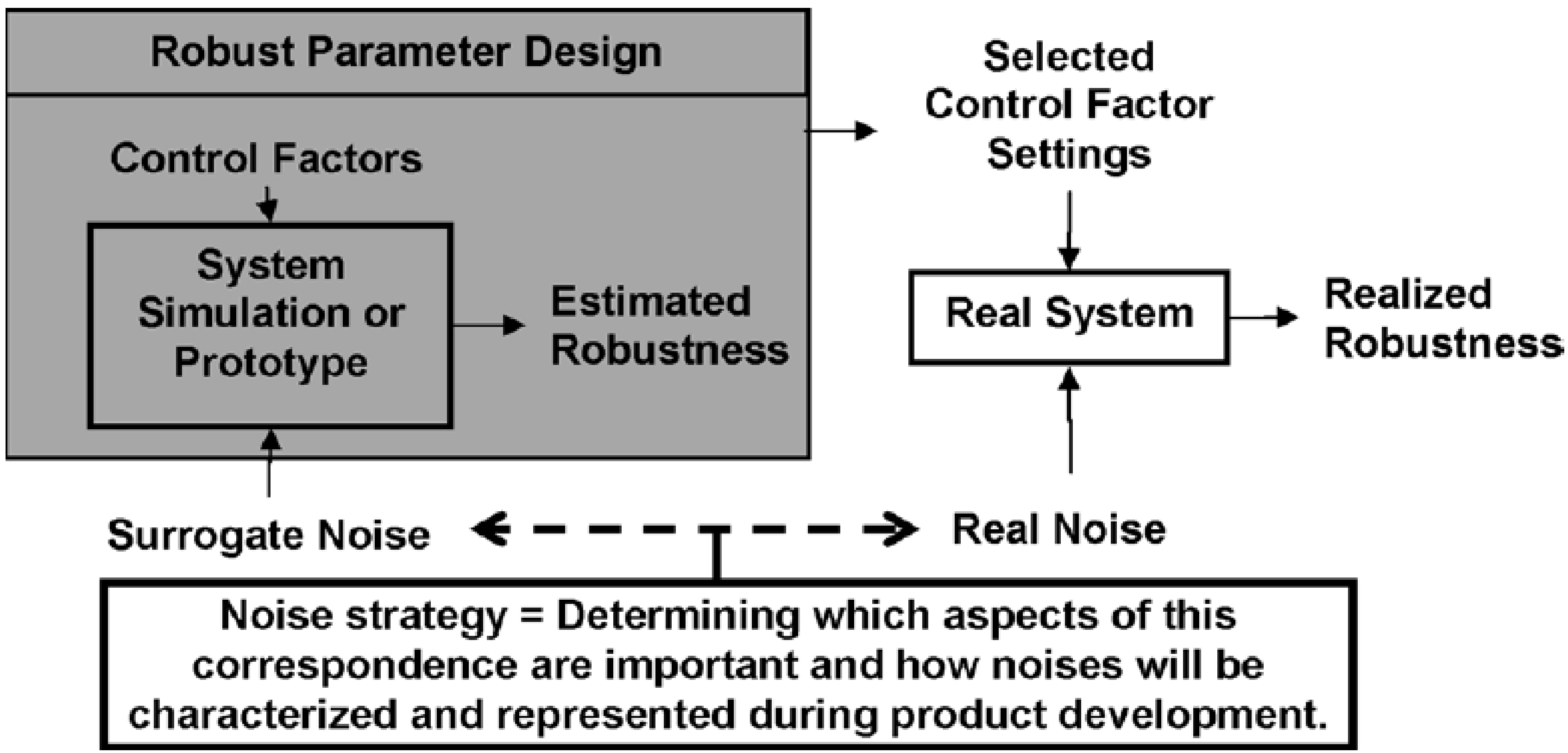
Noise Strategy



Results of a Model-Based Study

Median fraction of the maximum possible improvement attained in hierarchical probability model simulations

		Strong Hierarchy Model (active main effects and two-factor interactions)			Weak Hierarchy Model (active main effects, two-factor, and three-factor interactions)		
		Correlation			Correlation		
		None	Mild	Strong	None	Mild	Strong
Pure Experimental Error							
Large error $\varepsilon \sim \mathrm{NID}(0,10^2)$	Noise						
	Matching	0.97	0.89	0.81	0.94	0.84	0.70
	Amplified	1.00	0.92	0.86	0.90	0.81	0.56
Small error $\varepsilon \sim \mathrm{NID}(0,1^2)$	Noise						
	Matching	1.00	0.96	0.90	1.00	0.89	0.72
	Amplified	1.00	0.96	0.90	0.92	0.82	0.57

Essentially all results are very good when there are no three-factor interactions

Fairly bad results with correlation and three-factor interactions

Singh, J., D. D. Frey, and N. Soderborg, 2006, “Noise Strategy in Robust Design: What Aspects of Noise Factors are Important in Quality Engineering?” *Quality Engineering* **18**:367-377.

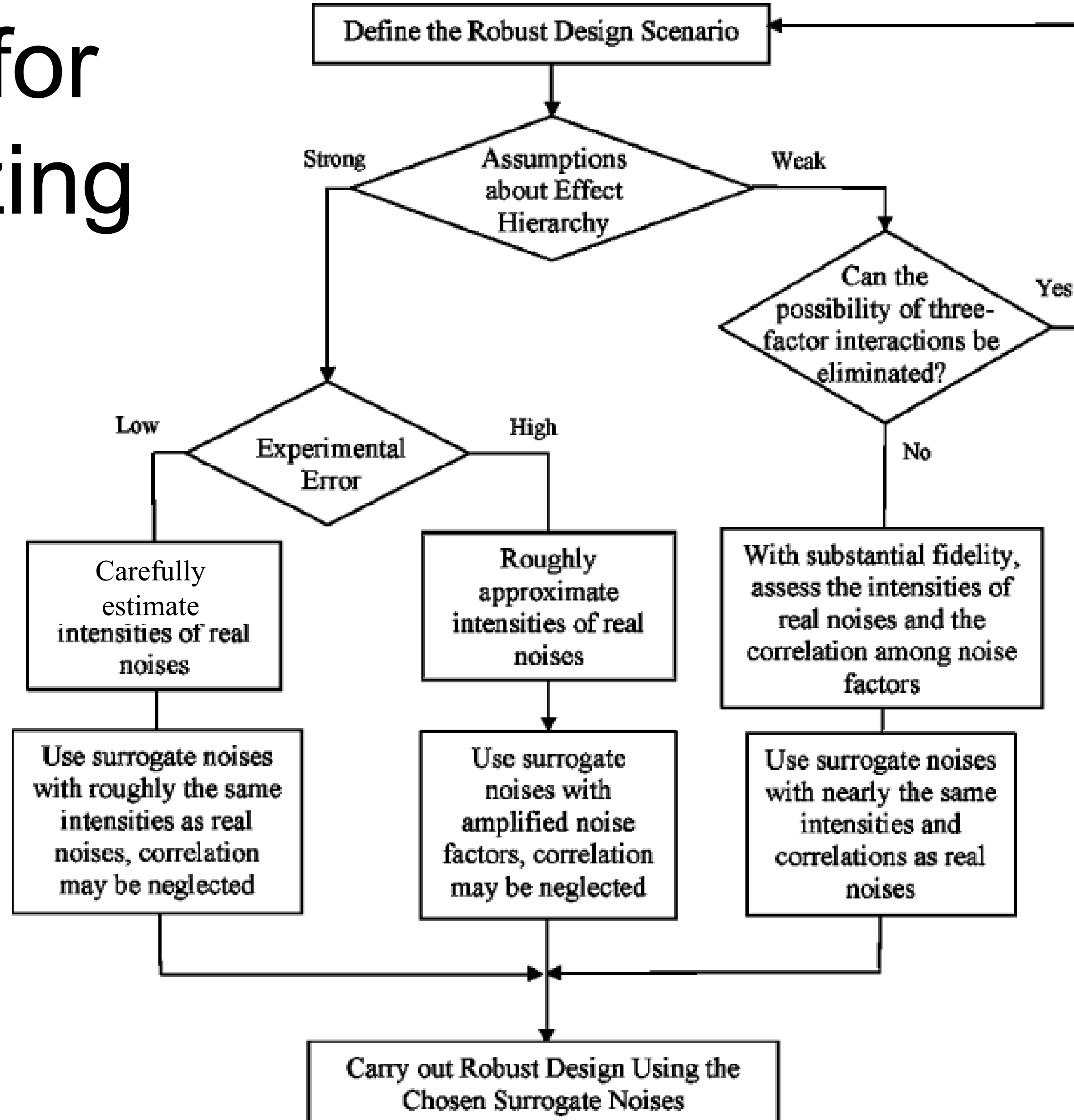
Results of Two Case Studies

Median fraction of the maximum possible improvement attained in case study simulations

		Operational Amplifier (active main effects and two-factor interactions)			Continuous Stirred Tank Reactor (active main effects, two-factor, and three-factor interactions)		
		Correlation			Correlation		
		None	Mild	Strong	None	Mild	Strong
Pure Experimental Error							
Large error	Noise						
	Matching	1.00	0.95	0.89	0.95	0.83	0.69
	Amplified	1.00	0.99	1.00	0.69	0.49	0.30
Small error	Noise						
	Matching	1.00	1.00	1.00	0.99	0.96	0.70
	Amplified	1.00	1.00	1.00	0.71	0.51	0.35

Similar trends. Model-based conclusions verified although cases support even stronger warnings against neglecting correlation when there are three-factor interactions.

A Suggested Procedure for Characterizing Noise



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Compounding Noise

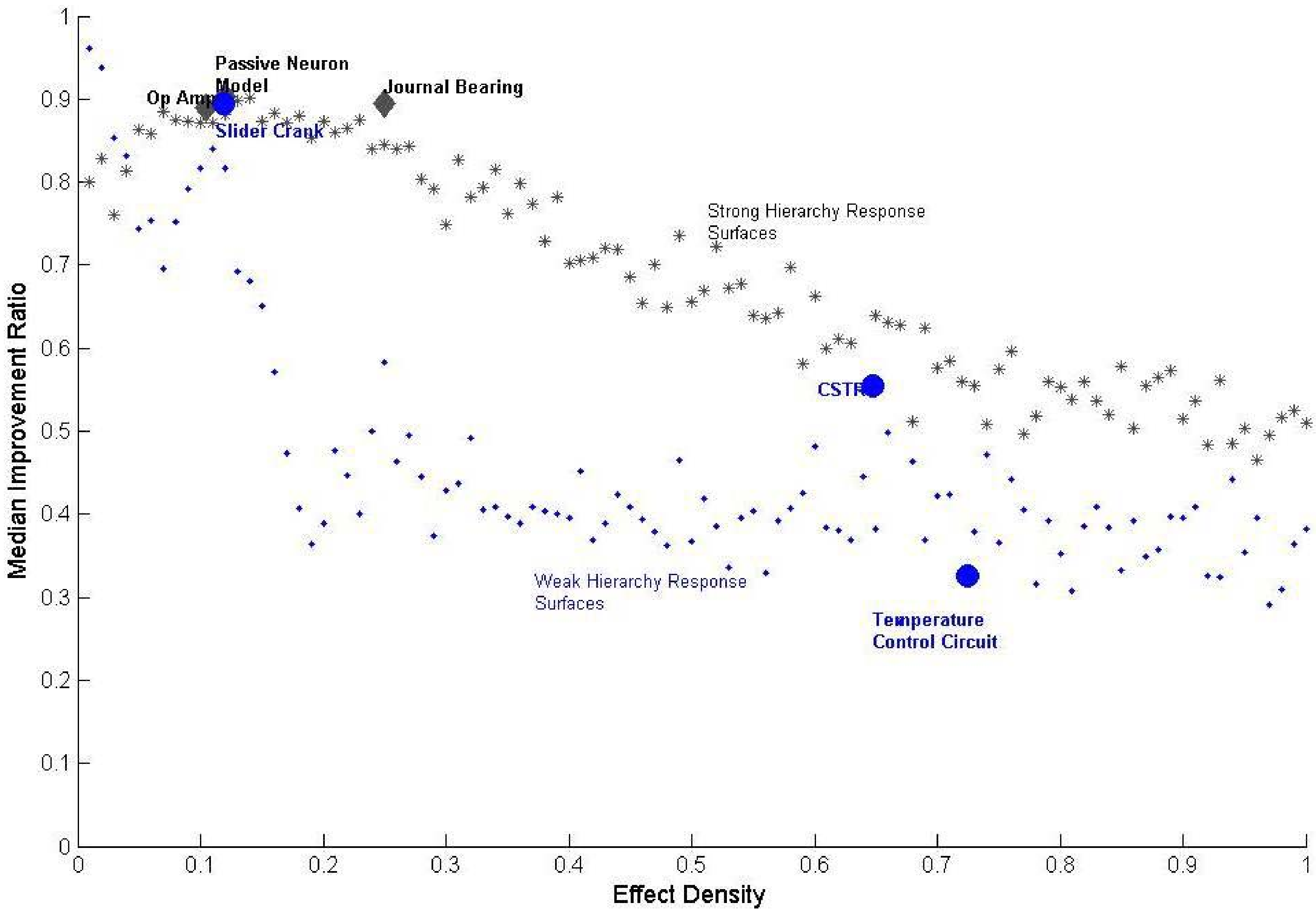
- If the physics are understood qualitatively, worst case combinations may be identified *a priori*

				E1	~~E1~~	E2	~~E2~~
				F1	F2	~~F1~~	~~F2~~
				G1	G2	~~G2~~	~~G1~~
A1	B1	C1	D1	*1*	*2*	*3*	*4*
A1	B2	C2	D2	*5*	*6*	*7*	*8*
A1	B3	C3	D3	*9*	*10*	*11*	*12*
A2	B1	C2	D3	*13*	*14*	*15*	*16*
A2	B2	C3	D1	*17*	*18*	*19*	*20*
A2	B3	C1	D2	*21*	*22*	*23*	*24*
A3	B1	C3	D2	*25*	*26*	*27*	*28*
A3	B2	C1	D3	*29*	*30*	*31*	*32*
A3	B3	C2	D1	*33*	*34*	*35*	*36*

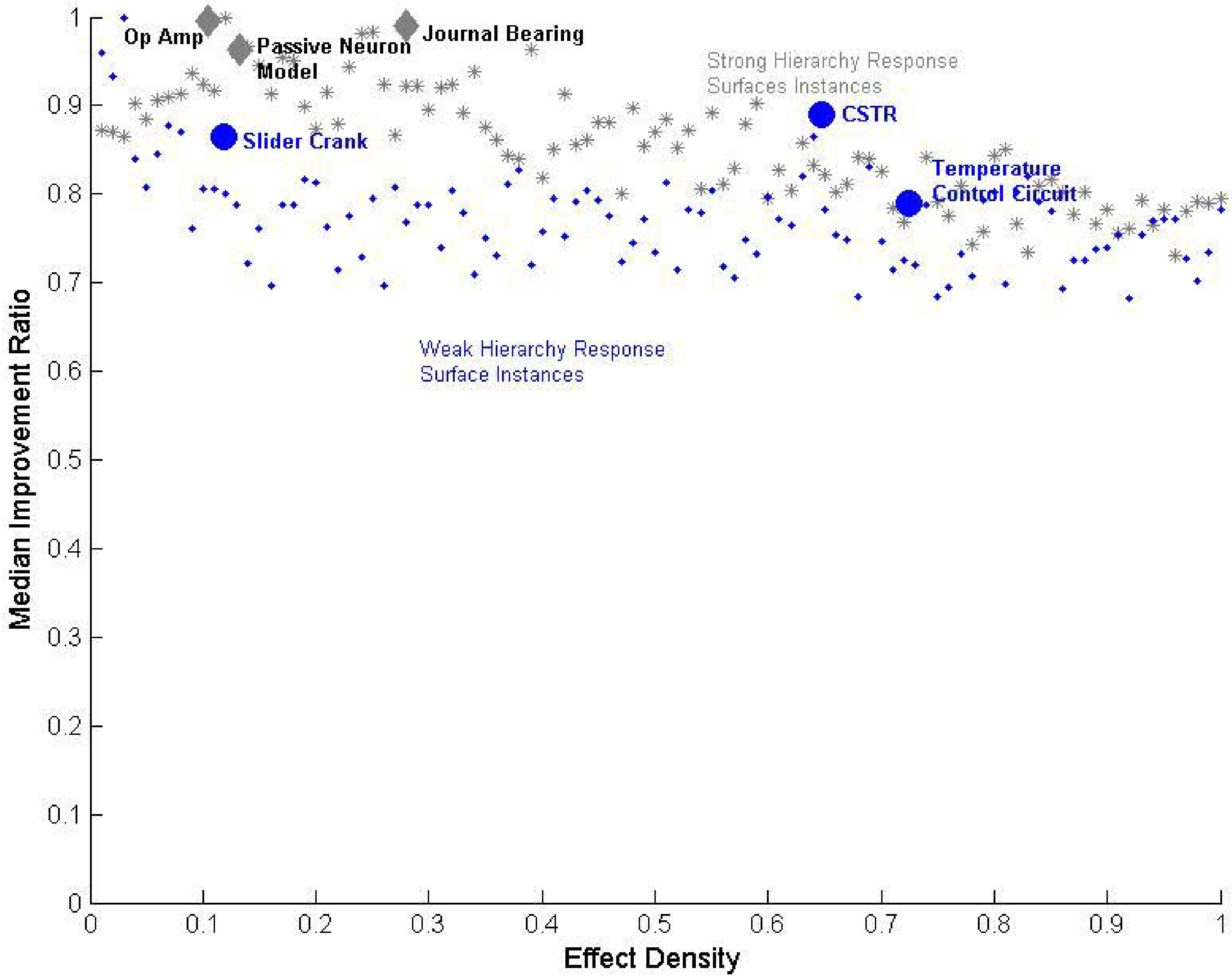
inner x outer = L9xL4= ~~36~~ 18

Taguchi, G., 1976, *System of Experimental Design*.

How Well Does Compounding Work?



An Alternative: Take the Best Few



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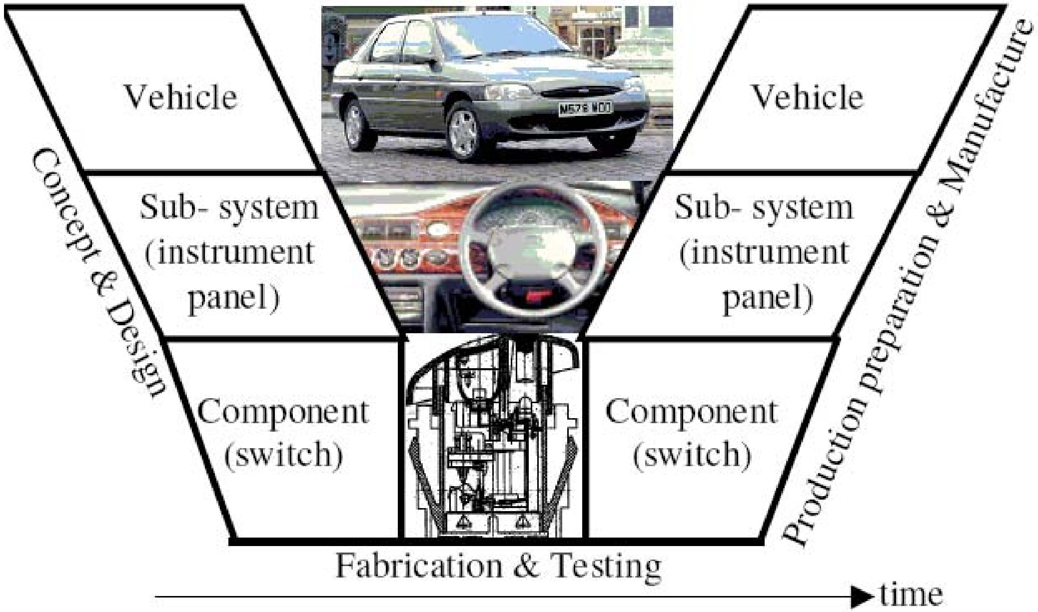
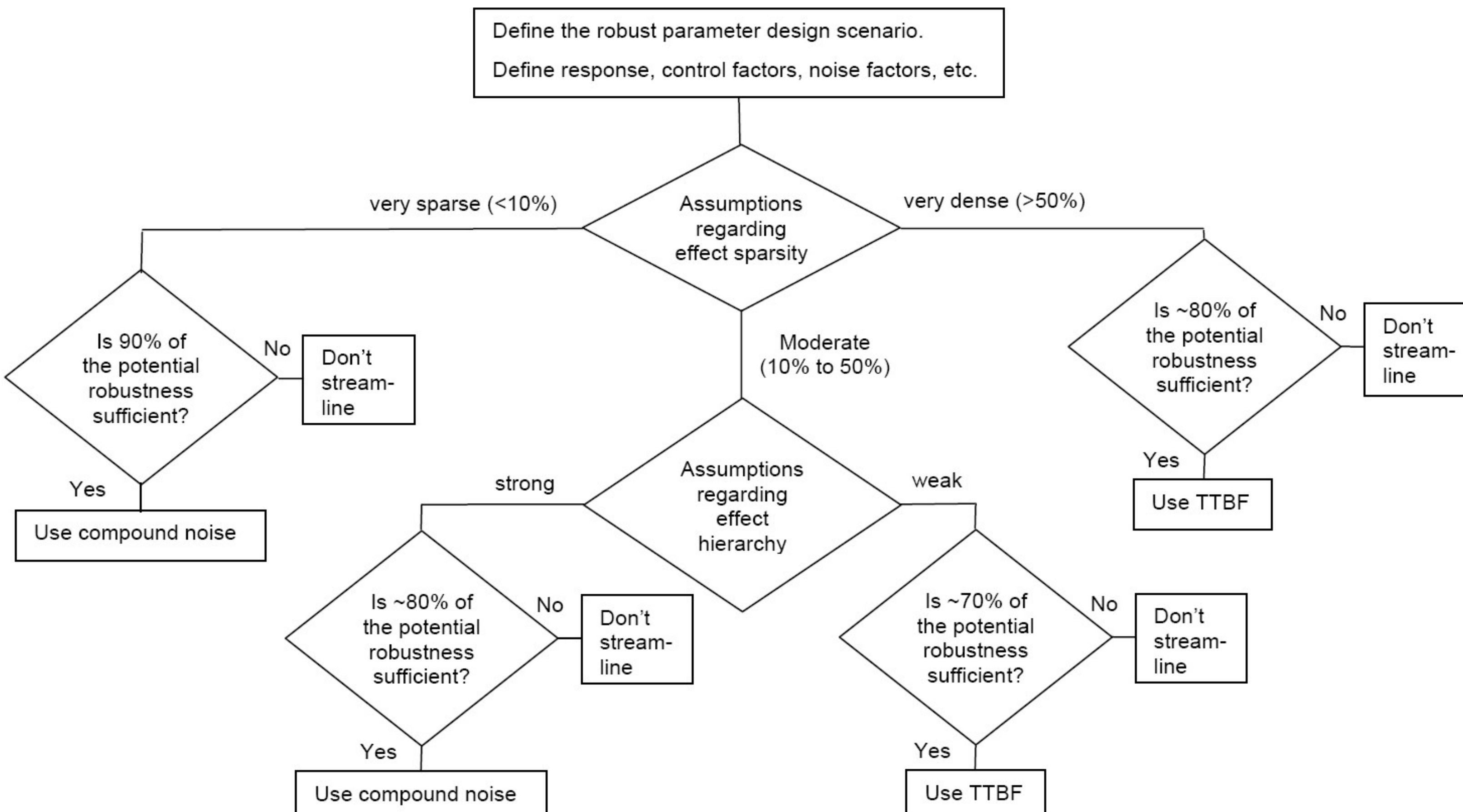
Setting Priorities / Allocating Resources

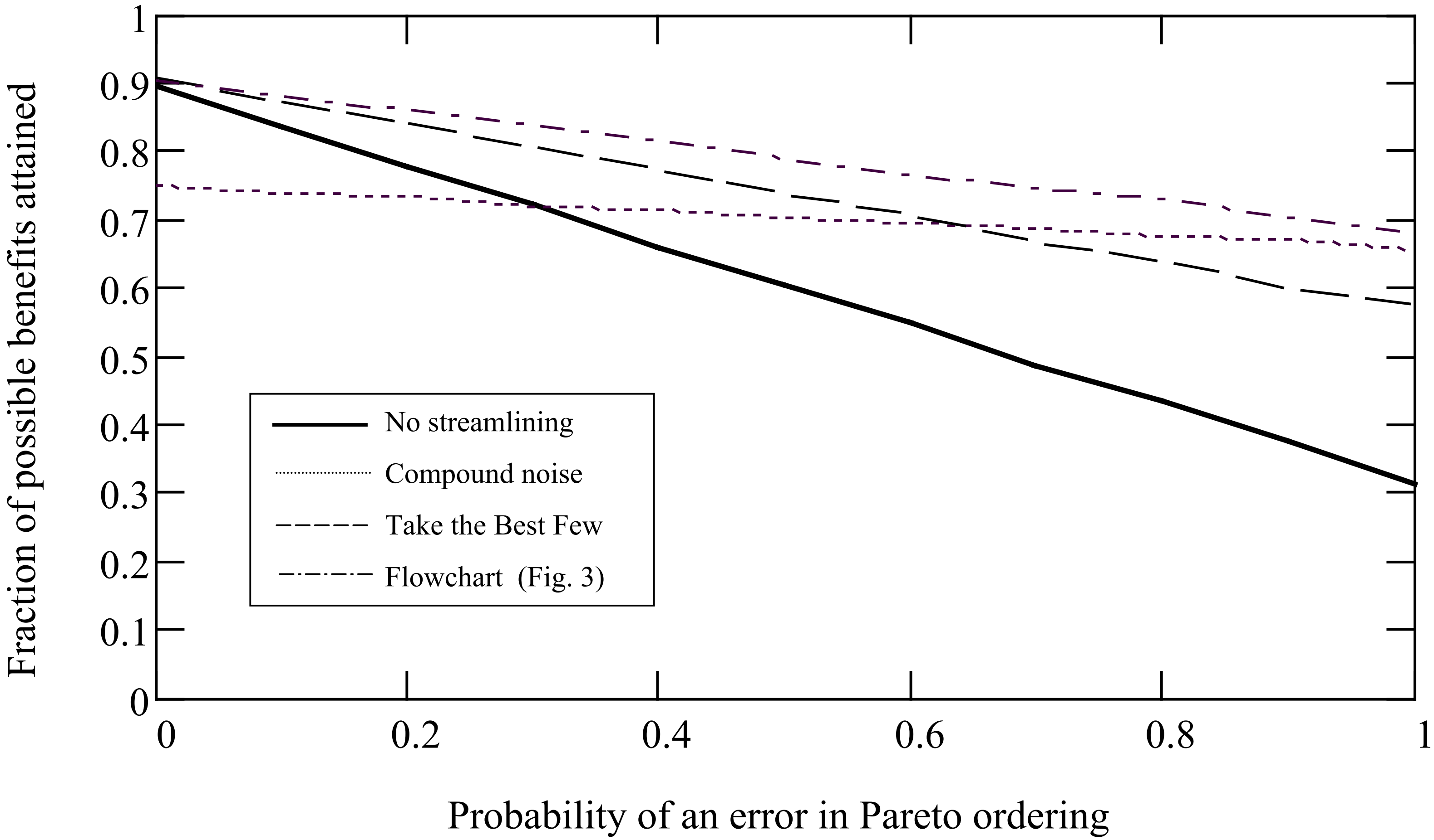
- Imagine a system with 100 components
- Each contributes to variation of the system, but some more than others
- You want to place rsources on the largest contributors
- But do you really know enough?



Combining Two Strategies



Best of Both Worlds?



Conclusions

- Robustness plays a major role in concurrent engineering
- To deploy it widely requires more efficient approaches
  - Adaptive experimentation
  - Characterization / structuring / inducing noise
- Don't concentrate resources if there uncertainty about the location of the biggest problems

Questions?

danfrey@mit.edu

Title and Abstract

Concurrent Robust Parameter Design:
How to Enable it and What it Will Enable

- Robust parameter design methods are used to make systems less sensitive to variations in the environment, manufacturing, and customer usage patterns. Such methods are difficult to run concurrently across complex development projects becuase they are expensive and time-consuming. This paper proposes and evaluates techniques to streamline robust parameter design, that is, to make products robust using fewer experimental runs. The evaluations are conducted using a combination of hierarchical probability models and six case studies. An established technique known as "compound noise" is shown to give good results if factor effects in the system are very sparse (fewer than 1 in 10 possible effects are active), or if the system does not exhibit any interactions involving three or more factors (i.e., weak hierarchy). On the other hand, "compound noise" appears to be much less effective when these assumptions are violated which is the case in two out of the six case studies conducted in this study. An alternative approach we call "Take the Best Few" is shown to give generally better results and to be far less sensitive to assumptions of effect sparsity and hierarchy. "Take the Best Few" scales well with problem size but it does require twice the resources of "compound noise." A procedure is presented for choosing between "compound noise" and "Take the Best Few" across projects within a larger system development effort. Simulations of robust parameter design being streamlined and distributed in this way suggest that this approach gives good results and reduces sensitivity of the design process to the fidelity of a priori judgments of which subsystems most need robustness improvement.

Dan:

Just saw your well-written article on "one-at-a-time" experiments in *Mechanical Engineering*. … I especially liked your observations about abbreviated test programs, which probably constitute half of all the testing done…

Nice job!

Ray Erikson, Principal Engineer

Flight Technology Corporation
Two Collins Road
Wakefield MA 01880-2513