An Evaluation of the Extended Logistic, Simple Logistic, and Gompertz Models for Forecasting Short Lifecycle Products and Services

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Introduction

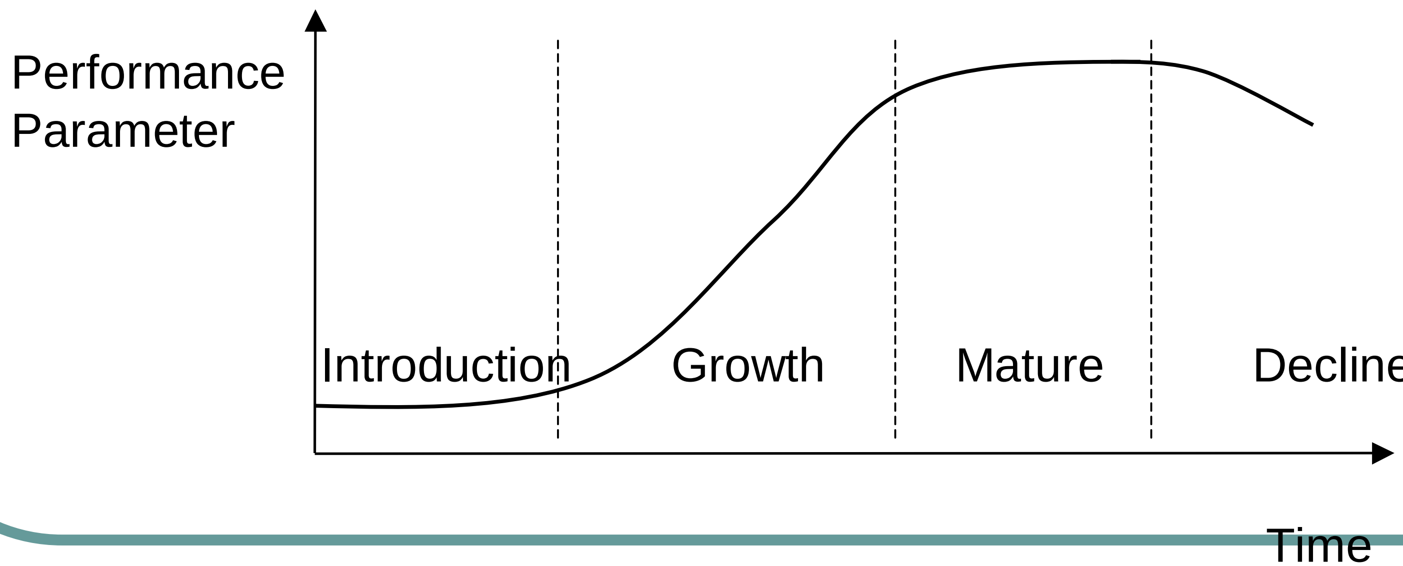
- With the rapid introduction of new technologies, product life cycles (PLC) for electronic goods become shorter and shorter.
- Less data becomes available for analysis
- Using smaller data sets to forecast future trends is important

Research Purpose

- This research studies the forecast accuracy of large and small data sets using the simple logistic, Gompertz, and extended logistic models.
- A time-series dataset describing the Taiwan market growth rates for two types of electronic products and two types of services were used to evaluate model performance.

PLC and S-curve

- Technology product growth follows S-curve
  - Initial growth is often slow
  - Followed by rapid exponential growth
  - Falls off as a limit to market share is approached



Growth Curve Model

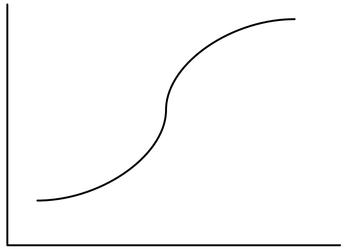
- Growth curves models are widely used in technology forecasting and have been developed to forecast the adoption rate of technology based products.
- Growth curve models are expressed by logistic form, so they are also referred as "S-shaped" curves.
- Simple logistic curve and Gompertz curve are the most frequently referenced.

Technological Forecasting Models

- Simple Logistic Curve Model
- Gompertz Model
- Extended Logistic Model

Simple Logistic Curve Model

$$y_T = \frac{L}{1 + ae^{-bt}}$$



y_t : the variable care to forecast

L : is the upper bound of y_t

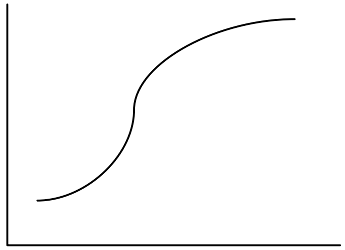
e : natural logarithm

a b : parameter

$$Y_t = \ln\left(y_t / L - y_t\right) = -\ln\left(a\right) + bt$$

Gompertz Model

$$y_t = Le^{-ae^{-bt}}$$



y_t : the variable care to forecast

L : is the upper bound of y_t

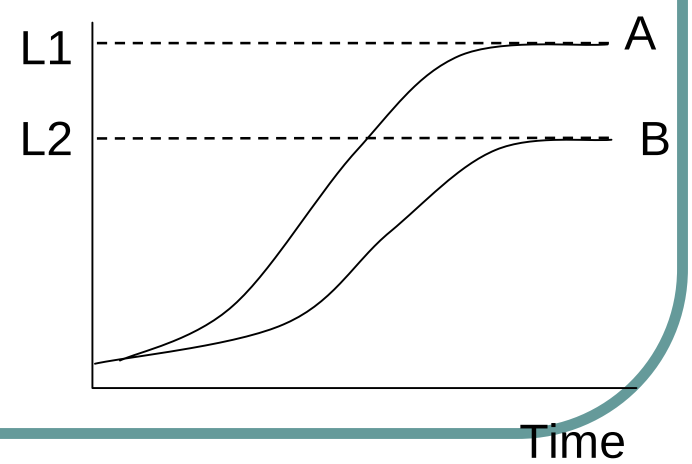
e : natural logarithm

a b : parameter

$$Y_t = \ln(\ln(L/y_t)) = \ln(a) - bt$$

Limitation of the Models

- The upper limit of the curve should be set correctly or the prediction will become inaccurate.
- Setting the upper limit to growth is difficult and ambiguous
  - Necessity product
  - Short life cycle product



Extended Logistic Model

- Meyer and Ausubel (1999)
- The upper limit of the curve is not constant but dynamic over time
- Extending the simple logistics model with a carrying capacity

$$\frac{df}{dt} = by\left(1 - y/k\right) \text{ is extended to } \frac{df}{dt} = by\left(1 - y/k(t)\right)$$

Technological Forecasting Models

- Extended Logistic Model

$$k(t) = 1 - D \times e^{-at}$$

is the time-varying capacity and is the function which is similar to logistic curve

$$y_t = \frac{k(t)}{1 + C \times e^{-bt}} = \frac{1 - D \times e^{-at}}{1 + C \times e^{-bt}}$$

The Measurements of Fit and Forecast Performance

Mean Absolute Deviation (MAD)

$$MAD = \frac{\sum_{i=1}^{n} \left| f_i - \hat{f}_i \right|}{n}$$

Root Mean Square Error (RMSE)

$$RMSE = \sqrt{\frac{\sum_{i=1}^{n} \left(f_i - \hat{f}_i \right)^2}{n}}$$

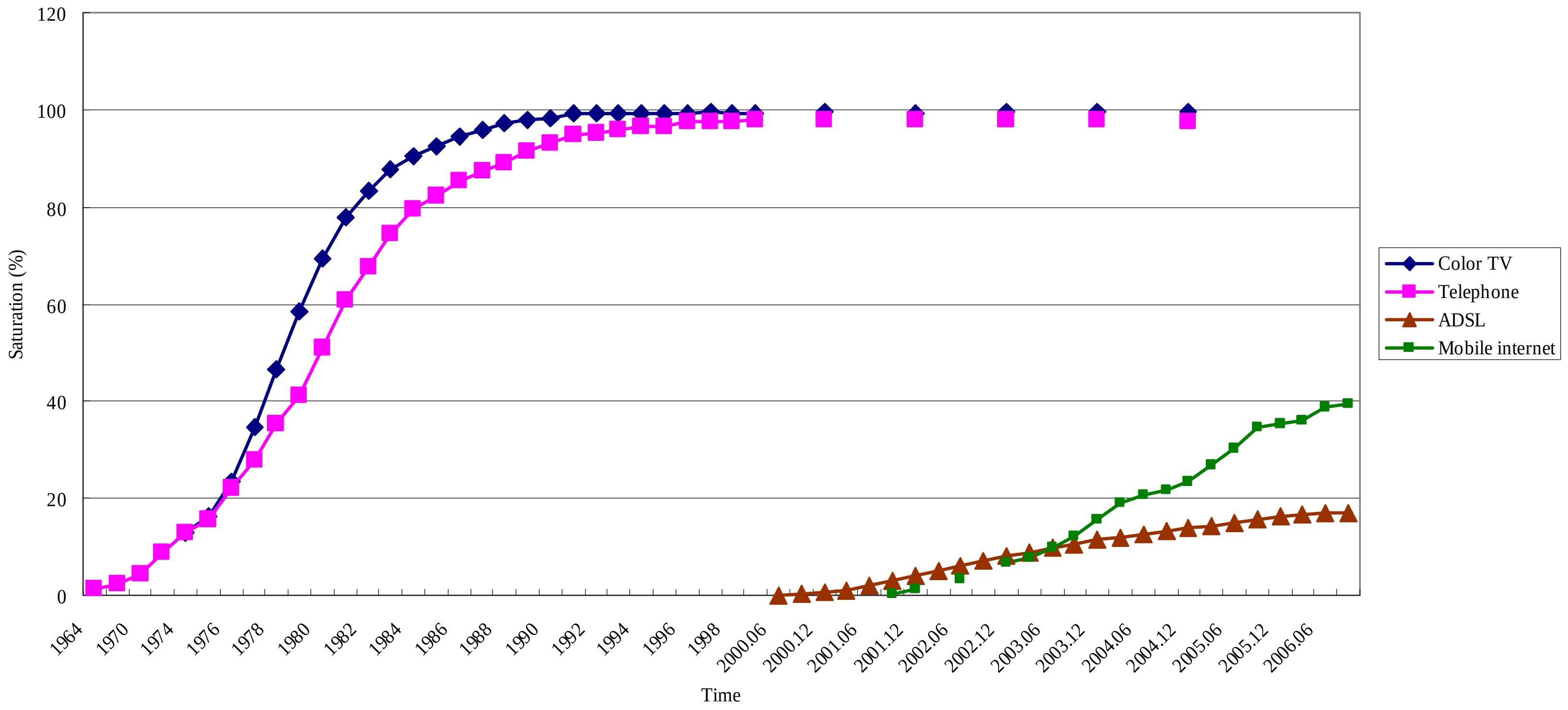
The smaller the MAD and RMSE, the better performance

Data Collection

- The Taiwan market growth rate

Product	Time	Data points	Data length
Color TV	1974 ~ 2004	31	Long data (Long PLC)
Telephone	1964 ~ 2004	35	
ADSL subscription	2000 ~ 2006	26	Short Data (Short PLC)
Mobile internet subscription	2000 ~ 2006	19	

Market Growth for Color TVs, Telephones, ADSL Subscription, and Mobile Internet Subscription



Analysis Steps

- **The first step (Fit performance)**
  - Reserve the last five data points
  - Fit the rest data into the three models
  - Compute the coefficients of the models and the statistics for MAD and RMSE
- **The second step (Forecast performance)**
  - Uses the derived models to forecast the five data points
  - Compare the forecast with the true observations
  - Compute MAD and RMSE for forecast performance

Results

Data length	Model	Statistic	Fit			Forecast		
			Extended logistic	Gompertz	Simple logistic	Extended logistic	Gompertz	Simple logistic
Long data	Color TV	MAD	0.0053	0.0297	0.0361	0.0025	0.0042	0.0045
		RMSE	0.0071	0.0502	0.0551	0.0026	0.0043	0.0046
	Phone	MAD	0.0063	0.0290	0.0323	0.0049	0.0125	0.0171
		RMSE	0.0083	0.0440	0.0453	0.0057	0.0130	0.0174
Short data	ADSL 100%	MAD	0.0036	0.0154	0.0329	0.0140	0.1037	0.2846
		RMSE	0.0043	0.0228	0.0483	0.0147	0.1060	0.2933
	ADSL 50%	MAD	0.0036	0.0140	0.0274	0.0140	0.0671	0.1688
		RMSE	0.0043	0.0170	0.0372	0.0147	0.0688	0.1716
	ADSL 30%	MAD	0.0036	0.0102	0.0212	0.0140	0.0345	0.0833
		RMSE	0.0043	0.0119	0.0264	0.0147	0.0352	0.0839
	Mobile Internet 100%	MAD	0.0057	0.0134	0.0337	0.0117	0.0627	0.2397
		RMSE	0.0073	0.0156	0.0439	0.0152	0.0699	0.2503
	Mobile Internet 50%	MAD	0.0057	0.0059	0.0217	0.0117	0.0146	0.0503
		RMSE	0.0073	0.0076	0.0248	0.0152	0.0166	0.0518

Conclusion

- This study compares the fit and prediction performance of the simple logistic, Gompertz, and the extended logistic models for four electronic products.
- Since the simple logistic and Gompertz curves require the correct setting of upper limits for accurate market growth rate predictions, these two models may not be suitable for short life cycle products with limited data.

Conclusion

- The extended logistic model estimates the time-varying capacity from the data and perform better than the simple logistic and Gompertz model
- Besides, the extended logistic model is also suitable for predicting both long and short lifecycle products.

Limitations

- Since the capacity of extended logistic model is time-varying and is a logistics function of time, it is not suitable for the data with linear curve.
  - Example: Meade & Islam (1995)
    - Using telephone data from Sweden to compare the simple logistic, extended logistic, and the local logistic models
    - the extended logistic model had the worst performance

Meade & Islam (1995)

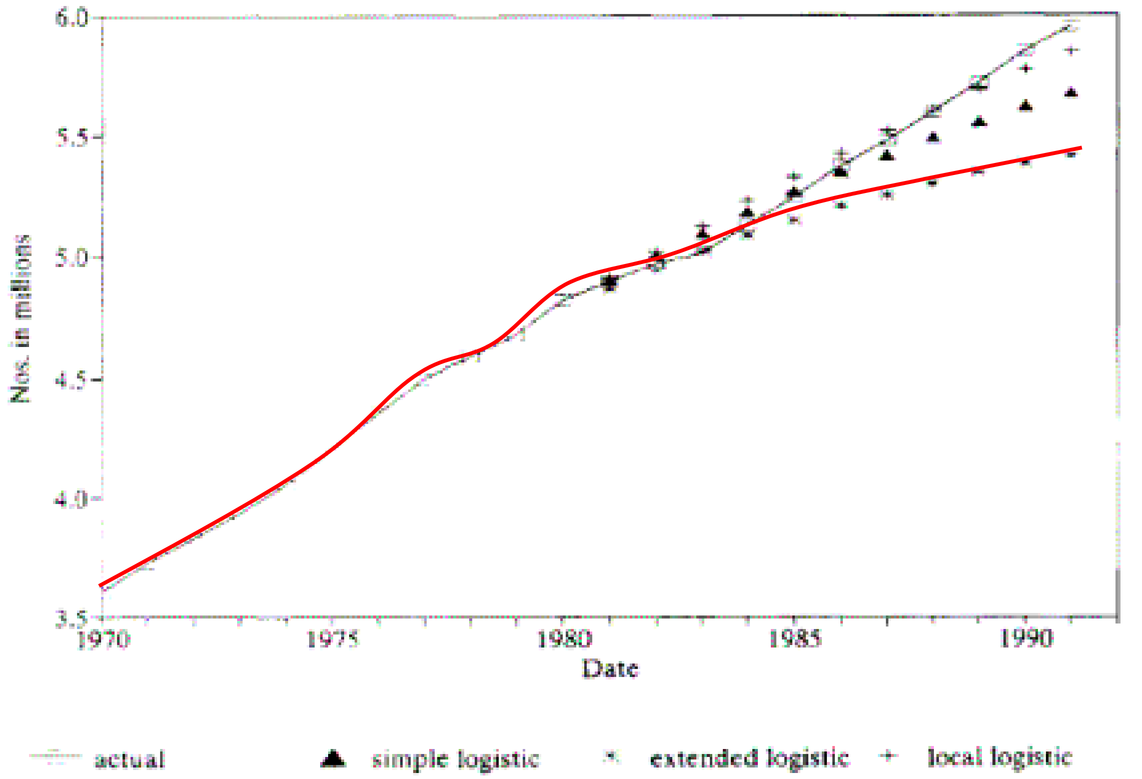


Fig. 1. Three logistic-based forecasts of main telephones in Sweden (forecasts 1981–1991).

Source: Meade N, Islam T. Forecasting with growth curves: An empirical comparison. International Journal of Forecasting 1995;11:199-215.

Thank you.