10-12 dec.

SC: UFSC

**III ENCIT – Itapema, SC (Dezembro 1990)**



# SUPERSONIC COMBUSTORS
## SOME THERMODYNAMICAL ASPECTS FOR A FIXED GEOMETRY

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SUMMARY

*As it is well known each component of a scramjet (supersonic combustion ramjet, fig. 1 i.e., the air inlet (0-1), the connecting duct (1-2), the combustor (2-4) and the exhaust nozzle (4-5), may be analysed in separate and experimental results can be incorporated where needed. This work reviews and discusses some thermodynamical aspects for fixed scramjet combustor geometries aiming at the design of a test facility.*

## INTRODUCTION

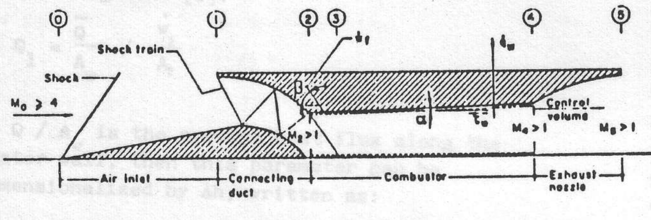


Fig. 1 Schematic of a scramjet engine.
$\tau_w$-wall shear stress, $\dot{q}_w$ - wall heat flux
$\dot{w}_f$ -fuel mass flow rate
$\alpha$-combustor divergence angle, $\beta$-fuel injection angle
$M_j$-mach number at $j^{th}$ station.

The analysis of such problems usually deals with the one dimensional conservation equations plus the appropriate equations of state and a pressure-area relationship, first suggested by L. Crocco, of the form $p_w A^{\varepsilon/\varepsilon-1}$ = constant [1]. This relation, along with the equation of the state and the mass and momentum conservation equations only, yields, for a thermally perfect gas, convenient ratios for pressure, area and temperature [1]. However it might be of interest to notice that those relations must be made compatible with the energy equation [2]. These equations can be simultaneously solved if the equation of state at the exit of the combustor (station 4 in Fig. 1) is known, the wall pressure distribution is given and if working expressions for the wall shear and heat flux are known[3]. The equation of state for station 4 is obtained assuming that thermodynamical equilibrium has been reached there, that the air and fuel properties are known and then making use of the existing computer codes for such calculations [4]. The wall pressure distribution is chosen to obey the above referred Crocco's relation. For the wall shear and heat flux, one assumes that the Reynolds Analogy remains valid for reacting flows. However, as the values for $\varepsilon$ and $p_3/p_2$ (the pressure ratio across the existing shock at section 3 of Fig. 1) are not known a priori, the usual technique is to assume that in the neighborhood of section 4 the flow is one dimensional and that the nozzle flow (between stations 4 and 5 of Fig. 1) is isentropic and then to use the constraints that $dT_t/T_t \to 0$ as $A \to A_4$, were $T_t$ is the total (stagnation) temperature and that $\left.\frac{\partial p}{\partial A}\right|_{\varepsilon=\text{const.}} = \left.\frac{\partial p}{\partial A}\right|_{\text{isentropic}}$ as $A \to A_4$. Notice that as it has been shown elsewhere, this latter constraint is sufficient by itself to yield the relationship between $\varepsilon$ and p3/p2 [2].

This work uses a numerical scheme taken from Operations Research to solve simultaneously this highly non linear system, aiming at the design of a test facility. Thus it obtains the desired solutions for different fixed geometries.

## COMBUSTOR ANALYSIS

As mentioned above the use of one-dimensional conservation equations along with the appropriate equations of state consist in a known technique for the analysis of supersonic combustors. Hence, choosing the control volume shown in Figure 1, one may write [1]:

$$\frac{p_4}{p_2} = \left[\frac{(1+\gamma_2 M_2^2) - (1-\varepsilon)p_3/p_2 - f_1^*}{\varepsilon + \gamma_4 M_4^2}\right]^{\varepsilon} \left[\frac{p_3}{p_2}\right]^{1-\varepsilon} \tag{1}$$

$$\frac{A_4}{A_2} = \left[\frac{(1+\gamma_2 M_2^2) - (1-\varepsilon)p_3/p_2 - f_1^*}{\varepsilon + \gamma_4 M_4^2}\right]^{1-\varepsilon} \left[\frac{p_3}{p_2}\right]^{\varepsilon-1} \tag{2}$$

$$\frac{T_4}{T_2} = \left[\frac{(1+\gamma_2 M_2^2) - (1-\varepsilon)p_3/p_2 - f_1^*}{\varepsilon + \gamma_4 M_4^2}\right]^{2} \frac{\gamma_4 R_4}{\gamma_2 R_2} \; \frac{M_4^2}{M_2^2} \; . \; (1+f)^{-2} \tag{3}$$

where $\gamma$ is the specific heat ratio, $R$ - the specific gas constant, $\dot{w}$ - the mass flow rate, $A$ - the cross-sectional area, $f$ - the fuel to air flow ratio ($f=\dot{w}_f/\dot{w}_2$), the subscripts f, w, 2 and 4 stands for fuel, wall and stations 2 and 4 respectively, and

$$-f^* = [-\int_2^4 \tau_w \cos\alpha \, dA_w + p_f A_f \cos\beta + \rho_f u_f^2 A_f \cos\beta] \; .$$

$$\cdot\ [p_2 A_2]^{-1} \tag{4}$$

Notice that equations (1) to (3) were obtained without the use of the energy equation. However it might be of interest to notice that these relations must be made compatible with the energy equation. Thus, to make equation (3) compatible with the energy equation it is necessary that:

$$\frac{\gamma_2 R_2 (\gamma_4 - 1)}{\gamma_4 R_4 (\gamma_2 - 1)} \left[ 1 + f \frac{h_{tf}}{h_{t2}} - \frac{Q_1}{h_{t2}} \right] =$$

$$= \frac{2 + (\gamma_4 - 1) M_4^2}{2 + (\gamma_2 - 1) M_2^2} \; \frac{\gamma_4 R_4}{\gamma_2 R_2} \; \frac{M_4^2}{M_2^2} \; (1 + f)^{-1} \; *$$

$$* \left[ \frac{(1 + \gamma_2 M_2^2) - (1 - \varepsilon) p_3/p_2 - f_1^*}{\varepsilon + \gamma_4 M_4^2} \right]^2 \tag{5}$$

where, $Q_1 = \int_2^4 \dot{q}_w \, dA_w$ and where the subscript t stands for stagnation conditions.

Defining next [1]:

$$Q_1 = \frac{\bar{Q}}{A_w} \Big/ \frac{\dot{w}_2}{A_2}$$

where $\bar{Q} / A_w$ is the average heat flux along the combustor wall, then this parameter can be nondimensionalized by $\Delta h$, written as:

$$\Delta h = h_{t2} + f h_{tf} + 0.5\, f \eta_c \Delta H_f - \bar{h}_w \tag{6}$$

where $\bar{h}_w$, is the enthalpy of the air at the average wall temperature, $\eta_c$ - the combustion efficiency, $\Delta H_f$ -the fuel lower heating value and the factor 0.5 is used to yield an average for the overall combustor, [1]. Experimental data taken from a variety of fuels and combustor geometries over a wide range of initial conditions allow $Q_1/\Delta h$ to be plotted vs the parameter $f\eta_c \Delta H_f$ [1]. One aproximation to those correlations yields:

$$Q_1/\Delta h = 0.80502\text{x}10^{-3} + 3.450217\text{x}10^{-10}\ \chi + + 1.571077^{-17}\ \chi^2 - 1.057881\text{x}10^{-23}\ \chi^3 \tag{7}$$

where $\chi = f\eta_c \Delta H_f$ and $\Delta H_f$ is in J/kg.

Assuming also that the Reynolds Analogy remains valid in the case of turbulent flow with heat addition [1], then one can write:

$$(\bar{Q} / \bar{A}_w) / (\bar{h}_r - \bar{h}_w) = \bar{\tau}_w / \bar{u} \tag{8}$$

where $\bar{\tau}_w = (1/A_w) \int_2^4 \tau_w \cos\alpha \, dA_w$,

and $\bar{h}_r$ is the mean recovery enthalpy and $\bar{u}$ is the average gas velocity in the combustor which can be taken approximately equal to $u_2$ if it is assumed that the effects of the deceleration through the shock waves and the heat addition are compensated by the combustor divergence [3]. Still following Billig's approach [1] one may choose $\bar{h}_r = C_1 \bar{h}_t$ where $C_1 = 0.93$ and $h_t = h_{t2} + f h_{tf} + C_2 f \eta_c \Delta H_f$ where for the hydrogen fuel, $C_2 = 0.9$ and $\Delta H_f = 1.4329 * 10^8$ jKg$^{-1}$.

Assuming next that $\left(\frac{\partial p}{\partial A}\right)_{\varepsilon = \text{const}} = \left(\frac{\partial p}{\partial A}\right)_{\text{isentropic}}$ as A → A one can obtain the equation

$$\frac{\varepsilon}{\varepsilon + \gamma_4 (1 - \varepsilon)} = \frac{(p_2/p_3)[\gamma_4^{-1} + (\gamma_2/\gamma_4) M_2^2] - (1 - \varepsilon)/\gamma_4}{(A_4/A_2)^{1/(1-\varepsilon)}} + $$

$$+ \frac{f_1^* \gamma_4^{-1} p_2/p_3}{(A_4/A_2)^{1/(1-\varepsilon)}} - \frac{\varepsilon}{\gamma_2} \tag{9}$$

as mentioned earlier.

This leads to the solution for a given heat flux in the combustor which corresponds also to a unique value for $p_2/p_3$.

## SOLUTION PROCEDURE

The highly non linear characteristics of the scramjet balance equations (equations (1) to (9)) make it very difficult, if not impossible, the use of standard solution methods. This led the authors to employ an Operation Research Technique to solve the above equations. Thus it was used the Hooke and Jeeves optimization algorithm to minimize the goal function defined as the sum of the absolute values of each equation in the system [5]. If there is a solution of the system, then the minimum of the goal function is zero; reciprocally, if there is a minimum and it is zero, then each term of the goal function must also be zero and therefore the whole system is solved.

## NUMERICAL RESULTS

As it was already mentioned, this work intends to help the setting up of some design parameters for a ground test facility used to evaluate combustors operating in a supersonic regime.

In order to achieve a situation similar to those met in real flight, one has to simulate the airflow conditions at the combustor entrance, that is, after it went through an oblique shock train. To bring the air flow to the desired conditions (i.e., to a Mach Number between 3 and 5, to a temperature around 1300 K and to a pressure of 0.5 atm), the facility posesses a sue heater besides a supersonic nozzle and a settling chamber. Notice, however, that the air entering the combustor is in the so-called "vitiated conditions" (as one injects fuel to "burn" the enriched air in the sue heater) i.e. it has the following composition: 12% $H_2O$, 69% $N_2$, 19% $O_2$, by weight.

Choosing gaseous Hydrogen at 298.15K and using the data listed below in Table 1 one may use the Hooke routine to solve simultaneously equations (1) to (9). The parameters to be optimized were $p_4$, $M_4$, $T_4/T_2$, $p_3/p_2$ and $A_4/A_2$. The angle $\alpha$ was taken varying from 1º to 5º thus allowing the combustor configuration to be chosen (or tested). The results are shown in Figure 2. They were obtained for a ratio $\rho_f u_f/\rho_2 u_2 = 0.35$ [1] and by using the NASA code SP 273 [4] to obtain $R_4 = R_4(p_4)$ and $\gamma_4 = \gamma_4(p_4)$

Table 1. Parameters used to obtain results shown in Fig. 2.

$\eta_c$ = 0.8
$\alpha$ = 1, 2, 3, 4 and $5^o$
$\beta$ = $90^o$
$\gamma_2$ = 1.3862
$p_2$ = 39215.7 $Nm^{-2}$
$T_2$ = 356K
$M_2$ = 3.633 (which corresponds to a flight regime of $M_o$=10.0, Zo≃ 30000m (flight altitude) as suggested by Billig [1], for proper inlet conditions)
$\rho_2$ = 0.38161 $kgm^{-3}$
$u_2$ = 1401.07 $msec^{-1}$
$T_f$ = 298.15K
$\rho_f$ = 0.06193 $kgm^{-3}$
$p_f$ = $p_2$
$T_{air}$ = 298.15K (temperature outside the scramjet walls)
$Cp_{air}$ = 1119.5 $J(Kg.K)^{-1}$
$\Delta H_f$ = $1.4329 \times 10^8$ $J.Kg^{-1}$

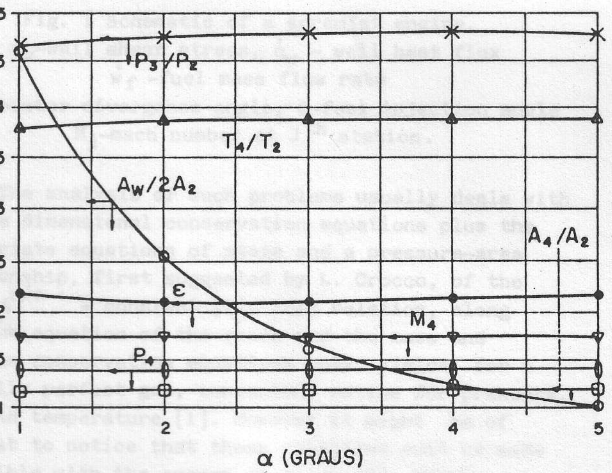


Fig. 2 Design date for supersonic combustor test facility.

□ $A_4/A_2$
0 $P_4$ [atm]
▽ $M_4$
△ $T_4/T_2$
x $P_3/P_2$
o $A_w/2A_2$
● ε

## CONCLUSIONS

An analysis of the scramjet engine cycle and optimization process was done with the purpose of collecting data for the design of a test facility which will be built in the authors' research laboratory. The numerical scheme used to solve the problem has been discussed in detail elsewhere [6,7].

## REFERENCES

[1] Waltrup, P.J., Billig, F.S. and Stockbridge, R.D., "A Procedure for Optimizing the Design of Scramjet Engines", J. of Spacecraft and Rockets, V. 16, May-June 1979, pp. 163-171.

[2] Alves, C.F.E., Sinay, L.R. e Bastos-Netto, D. "Dinamica de Combustores Supersônicos", Anais do ENCIT 88, pp 033-036 (1988).

[3] Billig, S.F., "Combustion Process in Supersonic Flow", J. of Propulsion and Power, V. 4 May-June 1988, pp. 209-216.

[4] Gordon, S. and McBride, B.J., "Computer Program for Calculations of Complex Chemical Equilibrium Compositions, Rocket Performance Incident and Reflected Schocks and Chapman Jouguet Detonations", NASA SP-273 (1971).

[5] Hooke, R. and Jeeves, T.A., "A Direct Unconstrained Search Strategy", J. Assoc. Comp. Mach., Vol. 8, pp 202-229, 1961. For computer program see Kuester, J.L. and Mize, J.H., Optimization Techniques With Fortran, pp 355-366, McGraw-Hill Book Company, New York, 1973.

[6] Bastos-Netto, D., Sinay, L.R. and Alves, C.F.E., "Some Thermodynamic Aspects in the Optimization of Supersonic Combustors", XVII International Symposium on Space Technology and Science (17th. ISTS), May 22-25, 1990, Tokyo, Japan.

[7] Sinay, L.R., Bastos-Netto, D. and Alves, C.F.E. "Numerically Solving Eigenvalue Equations as Minimization Problems" - Paper accepted for 13th CNMAC (Águas de Lindóia, SP, 25-29 Nov., 1990).