Modelling a Satellite Network Communications as a Markov Process

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Abstract. *We model a single orbit satellite constellation with six satellites as a continuous time Markov chain (CTMC). Earth-fixed constellations and constellations with handoffs are considered. The models developed are based on Zaim et al.(2002)´s models.*

Markov processes have been used to model satellite constellations [Usaha and Barria 2002], [Zaim et al. 2002]. The basic property of a Markov process [Çinlar 1975] is that its future behavior is conditionally independent of its past, provided that its present state is known.

We consider a single orbit satellite constellation with six satellites as shown in Figure 1.

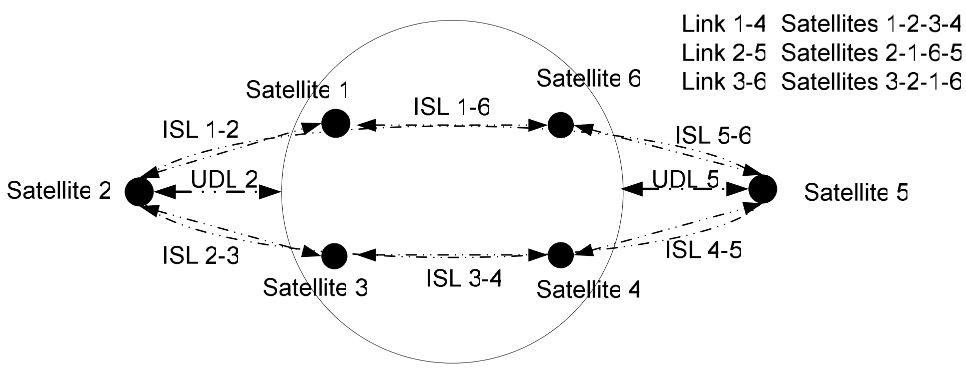

Figure 1. Six satellites in a single orbit

In this system, we assume that call requests arrive at each satellite according to a Poisson process with rate λ, and the call holding times are exponentially distributed with rate μ. Therefore, it is possible to model it as a *continuous time Markov chain* (*CTMC*) following the proposal of [Zaim et al. 2002]. Satellites communicate directly with each other by line of sight using inter-satellite links (ISL). The connection between the earth and the satellite is called "up-and-down link", UDL. Each satellite i has the capacity to support up to C_{UDL} i-i bidirectional calls in the connections of type UDL, and C_{ISL} i-j bidirectional calls in the connections of type ISL with satellite j. In this system, we consider that a call originated at satellite 1 and terminated at satellite 4 is routed through satellites 2 and 3, a call originated at satellite 2 and terminated at satellite 5 is routed through satellites 1 and 6, a call originated at satellite 3 and terminated at satellite 6 is routed through satellites 1 and 2, and, for all the others connections, the shortest-path routing.

Let n_{ij} be a random variable representing the number of active bidirectional calls between satellite i and satellite j, $1 \le i \le j \le 6$, regardless whether the calls originated in satellite i or j. Notice that n_{ii} represents the number of calls between two customers under satellite i, and two bidirectional UDL channels are used. The six-satellite system of Figure 1 can be described by a 21-dimensional *CTMC*. The set of all possible states of this *CTMC* is given by:

$$
\begin{array}{l}
E=\{\ (n_{11}, n_{12}, n_{13}, n_{14}, n_{15}, n_{16}, n_{22}, n_{23}, n_{24}, n_{25}, n_{26}, n_{33}, n_{34}, n_{35}, n_{36}, n_{44}, n_{45}, n_{46}, n_{55}, n_{56}, n_{66},)\ / \\
\qquad n_{ij} \in \aleph \text{ for all } i, j; \\
\qquad 2n_{11} + n_{12} + n_{13} + n_{14} + n_{15} + n_{16} \le C_{UDL}; \\
\qquad n_{12} + 2n_{22} + n_{23} + n_{24} + n_{25} + n_{26} \le C_{UDL}; \\
\qquad n_{13} + n_{23} + 2n_{33} + n_{34} + n_{35} + n_{36} \le C_{UDL}; \\
\qquad n_{14} + n_{24} + n_{34} + 2n_{44} + n_{45} + n_{46} \le C_{UDL}; \\
\qquad n_{15} + n_{25} + n_{35} + n_{45} + 2n_{55} + n_{56} \le C_{UDL}; \\
\qquad n_{16} + n_{26} + n_{36} + n_{46} + n_{56} + 2n_{66} \le C_{UDL}; \\
\qquad n_{12} + n_{13} + n_{14} + n_{25} + n_{26} + n_{36} \le C_{ISL}; \\
\qquad n_{13} + n_{14} + n_{23} + n_{24} + n_{36} \le C_{ISL}; \\
\qquad n_{14} + n_{24} + n_{34} + n_{35} \le C_{ISL}; \\
\qquad n_{35} + n_{45} + n_{46} \le C_{ISL}; \\
\qquad n_{15} + n_{25} + n_{46} + n_{56} \le C_{ISL}; \\
\qquad n_{15} + n_{16} + n_{25} + n_{26} + \mathrm{n}_{36} \le C_{ISL}\ \}.
\end{array}
$$

Let λ_{ij} denotes the arrival rate of calls, and $1/\mu_{ij}$ the mean holding time of calls between satellites i and j. Then, the state transition rates $r(e, \hat{e})$ from the current state $e \in E$ to the next state $\hat{e} \in E$ for this CTMC are given by:

- $r(e, \hat{e}) = \lambda_{ij}$, $\forall\, i, j$, if the transition is due to the arrival of a call between satellites i and j. In this case, $\hat{e}$ is equal to e, except in the position that corresponds to the element n_{ij}, that is increased by one;
- $r(e, \hat{e}) = n_{ij}\, \mu_{ij}$, $\forall\, i, j$, $n_{ij} > 0$, if the transition is due to the termination of a call between satellites i and j. In this case, $\hat{e}$ is equal to e, except in the position that corresponds to the element n_{ij}, that is decreased by one.

If the satellites are not fixed in the sky, the same transition rates presented previously plus additional transition rates to account for the handoffs are considered. These transition rates are:

- $r(e, \hat{e}) = \alpha\, n_{ij}$, where α is a constant that depends on the shape of the earth's surface coverage area of the satellite and the satellite's speed. $\alpha\, n_{ij}$ is the rate at which calls experience a handoff from satellite i to satellite j that follows it In this case, $\hat{e}$ is equal to e, except in the position that corresponds to the element n_{ij}, that is decreased by one, due to the call that was handed off, and in the position of the satellite that receives this call, that is increased by one;
- $r(e, \hat{e}) = 2\alpha\, n_{ii}$, where both customers are served by the same satellite. In this case, e is equal to $\hat{e}$, except in the position that corresponds to the element n_{ii}, that is decreased by one, and in the position of the satellite that receives a call, that is increased by one.

The total arrival rate of calls between satellites i and j will include new calls arriving at a rate λ_{ij} and handoff calls, arriving at an appropriate rate. Similarly, the total departure rate of calls between satellite i and j will include termination at rate $n_{ij}\mu_{ij}$ and handoff calls (at an appropriate rate).

With these models, we can obtain several performance measures. For instance, the system's call blocking probability is given by $\pi_b = \frac{\sum_{i,j} \lambda_{ij} \pi_b(i,j)}{\lambda}$, where $\pi_b(i,j)$ is the call blocking probability in link i-j, and λ is the system total arrival rate, $\lambda = \sum_{i,j} \lambda_{ij}$. $\pi_b(i,j) = \sum_{\forall e \in B_{ij}} \pi_e$ where $B_{ij} = \{e | e \in E \quad e \quad e + 1_{ij} \notin E\}$, π_e is the limiting probability of state e and 1_{ij} is a vector with zeros for all random variables except random variable n_{ij}, which it is 1.

Considering a numerical example of six satellites constellation, with $C_{ISL} = C_{UDL} = 3$, $\mu_{ij} = 2$, $\lambda_{ij} = 0.1$, $1 \le i \le j \le 6$. The blocking probabilities are given in Table 1 for different values of α. We can observe that, in a Geostationary orbit constellation, that is $\alpha = 0$, the system's call blocking probability is smaller than in orbits where the calls experience handoffs ($\alpha > 0$).

Table 1. Call blocking probabilities for 6 satellites, C_{ISL}=C_{UDL}=3, μ_{ij}=2, λ_{ij}=0.1

α	Call Blocking Probability
0.00	0.0371
0.25	0.0702
0.50	0.1023
0.75	0.1327
1.00	0.1609

References

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