Vertical colunm of beads under external vibrations

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Abstract. *We present results from computational experiments of one-dimensional granular fluid. With characteristic properties lying between those of liquids and solids, this class of material consists of assemblies of solid particles which interact via mechanical forces (contact and friction) and are maintained together by a gravitational field. The system is composed of a vertical column of spherical beads driven by a sinusoidally vibrating plate. Results for N-beads experiments show the fluidization and condensation phenomena, both dependent on the amplitude and frequency of the driving plate. Some scaling properties arising from the dynamics of the beads are also shown.*

Resumo. *Usando a técnica de Evento-Dirigido, apresentamos resultados de simulações computacionais da dinâmica de um fluido granular unidimensional. Com características semelhantes aos sólidos e aos líquidos, esses tipos de materiais são formados por um conjunto de partículas sólidas que interagem via forças de contato mecânicas e de fricção, sendo mantidas juntas por um um campo gravitacional. O sistema compõe-se de uma coluna vertical de esferas submetidas à ação de uma base vibratória. Os resultados das simulações para o caso geral com N esferas apresentam fenômenos de fluidização e condensação, ambos dependentes da amplitude e frequência da base. Leis de escalas, incluindo a posição média do centro de massa e a dilatação da coluna, resultantes da dinâmica das esferas, também são verificadas.*

1. Introdução

Granular materials are present in everyday life playing a critical role in several fields of applications such as pharmaceutical, cosmetic and housing industries. The studies about granular proprieties can improve procedures involving extraction, transport, storage and mixing process . Each process can present desired and undesired phenomena ocasioned by intrisic features of the granular dynamic. Moreover, understanding these proprieties is essential to optimize costs of granular handling such as waste of energy for example. Granular flow systems can present clogging and mixture procedures that may lead to segregation given wrong conditions [Duran 2000]. Initially, due to the large and practical applicability, this field was first restricted to engineering studies. However, as granular systems can also exhibit period doubling followed by chaotic behavior, they have been investigated by physicists and computational mathematicians.

The great challenge to understand the behavior of granular system is to determine the state of matter of the system. Granular matter can behave either like fluid

or like a solid. Therefore new theoretical ideas beyond classical mechanics are needed [Kadanoff 1999].

Solid matter has initial shape called unstressed shape which can be modified by a shear stress. The modified quantity from the rest shape is called defomation and it is proportional to Young modulus. Granular matter can not be considered as solid because they can deform continually under an applied shear stress regardless of how small the applied stress is [Anderson 1990]. In addition, granular matter can flow. Granular matter display similar phenomena as in the ordinary fluids such as freezing, plasticity and hysteresis [Kadanoff 1999]. By contrast, granular matter is composed of discrete and macroscopical units and present substantial change of density histeresis [Du et al. 1995]. The density of the fuid is either null (void), or material density (ρ) breaking the continuum mechanics approximations [Landau and Lifshitz 2004]. Therefore they can not be considered as fluid and can not be modeled by the Navier-Stokes equation.

Moreover, granular matter and molecular gases are composed of discrete units and behave according to Newton's laws of mechanics. However, beyond the diferences in the size of particles, collisions between units dissipate energy leading to emergence of some phenomena exclusively found on granular systems. Granular gases exhibit a tendency to coagulate into cluster without the aid of attractive interactions [Goldhirsch 1993]. This phenomenon are consequence of the dissipative interactions and they do not have correspondence in the elastic world. Collapses occur in groups between near-interacting particles. It happens when there is a high number of collision between particles in short interval of time and the overall state of the system does not present relevant changing. The assembly of particles has a small relative velocity leading to the emergence of particles strings with vanishing relative velocities. Colapse are consequence of local interation [Goldhirsch 1999]. Other important phenomena is clustering. Consider a uniform distribution of particles over a domain. As a result of statistical fluctuation, the density in certain region is increasead locally without change in granular temperature. High density leads to a high number of collisions and dissipation which in turn makes the temperature to rise in the region of high density as a result of energy dissipation. As collisions are inelastic, the granular temperature and consequently the pressure in the dense regime decays faster than neighboring region. Other particles are attracted to the denser region in attempting to equalize global pressure. The number of particles and collisions increases and self-amplify the process [Goldhirsch 2003].

The present work focuses on the condensation and fluidization phenomena verified in one-dimmensional granular system. Explaining the model and the numerical procedure, section 2 presents the simulation of a colunm of beads driven by a vibrating plate. Section 3 gives some results and discussions about fluidization and condensation state as well as some scaling laws related to center of mass. Conclusions are presented in section 5.

2. The Numerical Procedure

The simulation system consists of a vertical container with spheres with radius r equally spaced and driven by a sinosuidally plate of infinitely large mass. Spheres are allowed to move only in the vertical coordinate and therefore they can not exchange postions. Dissipation only occurs during collisions between particles with a coeficient of restitution

$0 < e < 1$. Dissipation may also occur between the first sphere and the plate, according to coeficient of restitution e_w.

Collisions between spheres are instantaneous and head to head since there is no angular momentum transfer between particles. The post collisions velocities obey the following relation [Luding et al. 1994]:

$$\begin{pmatrix} v_i \\ v_{i-1} \end{pmatrix} = \begin{pmatrix} \frac{1-e}{2} & \frac{1+e}{2} \\ \frac{1+e}{2} & \frac{1-e}{2} \end{pmatrix} \begin{pmatrix} u_1 \\ u_{i-1} \end{pmatrix} \tag{1}$$

where u and v are the corresponding velocities before and after collision of the i-th particle. A similar procedure is applied to sphere-base interaction. The plate is governed by sinusoidal function $Asin(\omega t)$ with A as the amplitude and $\omega = 2\pi f$ as the angular frequency. One must obtain the exact instant of collision between the bottom sphere and base. This can be done by solving a system of non-linear equations

$$\begin{cases} y = y_0 + v_0 t^* + \frac{gt^*}{2} \\ X(t) = Asin(\omega t^*) \end{cases} \tag{2}$$

where t^* is collision time. The current model is non-linear and is called Bouncing Ball which is widely studied due to period doubling and chaotic behavior. In order to avoid problems in numerical methods, one must normalize the model grouping parameters according to

$$\tilde{t} \equiv \tau = \omega t, \quad \tilde{y}_n = \frac{y_0}{g/\omega^2}, \quad v_n = \frac{v_0}{g/\omega}, \quad \Gamma = \frac{Aw^2}{g}$$

given the nondimensional variables

$$\tilde{y}(\tau) = y_n - v_n \tau - \frac{\tau^2}{2} \tag{3}$$
$$\tilde{x}(\tau) = \Gamma sen(\tau) \tag{4}$$

where Γ is the control parameter of the system.

Upon using numerical methods, time t^* of next collision is calculated. Once defined the collisional time, one must update the post-collisional velocity of the bottom sphere after collision, described by the followig relation

$$\begin{pmatrix} v_0 \\ v_1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 1+e_p & -e_p \end{pmatrix} \begin{pmatrix} u_0 \\ u_1 \end{pmatrix} \tag{5}$$

where e_w is coeficient of restitution between the sphere and the base.

Event driven molecular dynamics consists of estimating all possible collision between particles according to equation 6. [Porschel and Schwager 2005].

Table 1. Results for $\Gamma = 1.5$ and $\Gamma = 2.0$. In figure a, particles move compactly and periodically in the condensed regime. In figure b, condensed regime vanishes as we increase Γ values.
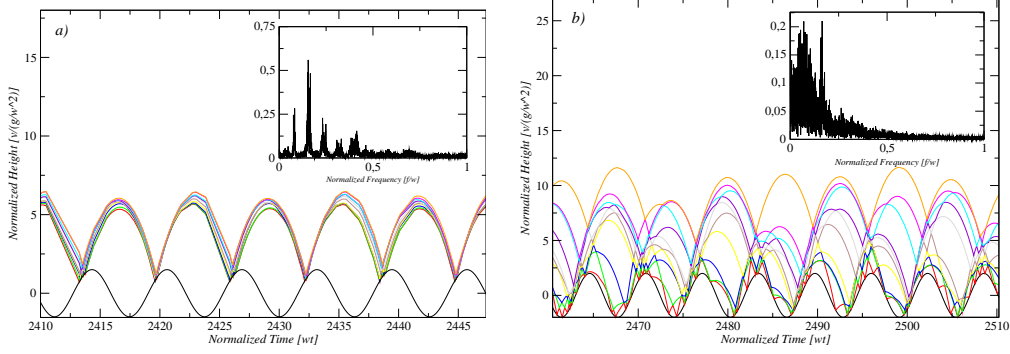
$$t^* = \frac{x_{i,t} - x_{i-1,t}}{v_{i,t} - v_{i-1,t}} \qquad (6)$$

where x and v are the position and the velocity of i-th particle respectively. All values are stored and the smallest Δt determines the ocurrence of the next event. Within this period, we assume all particles describe trajectories following classical laws of mechanics. Positions and velocities of all particles must be updated in each event. The list of time events ought to be modified considering new values.
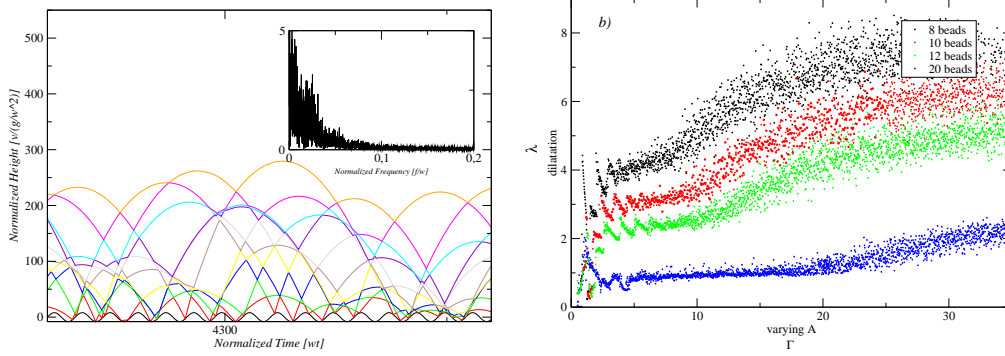
3. Results

We focus our attention on the proprieties and effects of energy dissipation. Simulations used $N = 8, 10, 12$ and 20 spheres. Dissipation ocurrs in particle-particle collision, $e_w = 1$, $e = 0.92$. We use A to adjust the energy input keeping the frequency constant at $20hz$.

First, we adopted a low oscillation with $A = 9, 30888 \times 10^{-4}$ and $\Gamma = 1.5$. Result is shown in figure 1a. Particles oscillate together grouped as a cluster due to the low energy input. We obtained the trajectory of the center of mass of the system and applied the FFT in order to analyse the periodicity of the system. Trajectory of the center of mass is periodic with a characteristic period and also present a subharmonic component in the frequency spectra. If energy is decreased still, a Feigenbaunn scenario is obtained, with a birfurcation cascade displaying period doubling. At this regime, system moves compactly resembling a solid system. We say system is under the condensation regime.

However, if we increase $\Gamma = 2.0$, the condensation regime starts to vanish and particles can propagate separately. The global system still has a compact aspect and some periodic frequencies show up the spectra as we can see in figure 1b.

At higher values, $\Gamma = 8.0$, particles move independently and motion looks erratic. This state of the figure 2a is called fluidization. The center of mass has a chaotic behavior and no characteristic behavior can be determined. As in the previous example, this regime resembles the behavior of a fluid, with sparse distance among particles.

Table 2. **Figure** a **presents results for** $\Gamma = 8.0$ **showing the fluidized state and chaotic moviment of center of mass. Figure** b **shows dilation diagram for** $f = 20hz$**.**

We take the mean dilation as an indicator of the state of the system. This nondimensional quantity is defined by:

$$\lambda = \frac{\langle z_n - z_1 \rangle}{A\omega} \tag{7}$$

Figure 2b shows a dilation diagram as a function of Γ. The result shows distinct regions with condensed and fluidized regimes separeted by a linear transition. The amount of energy of the system is directly proportional to dilation. System with fewer beads has lower dissipitative rates and higher dilation index. The simulation with 20 beads indicates higher dissipation with the corresponding curve below all others. Figure 3a shows a law scale between center of mass and coeficiente of restitution. Energy decays exponentialy as we decrease e. The same can be obtained by the number of beads. Figure 3b shows similar results as we vary the number of components. Defining a new function $X = (1 - e)N$ and multiplying the previous results of figure 4 by the number of particles we can adjust all curves in one single curve with a resulting slope of -1.15.

4. Conclusion

The dual behavior of granular materials can be found even in the simplest case. Fluidization and condensation can be obtained by adjusting the vibration rate of the base. As we increase the input energy dilatation reaches a limit in the fluidization regime. Power laws can be adjusted from the flux of energy of the system as function of dissipation, which is independent on the number of beads and the coeficient of restitution.

Table 3. Scale laws for the dissipative parameter of the model. Both, coeficient of restitution (Figure a) and number of particles (Figure b) are related exponentialy with the mean value of the center of mass.
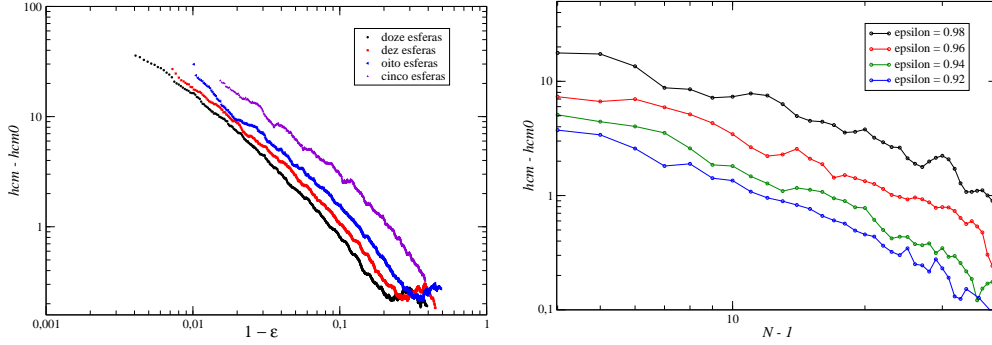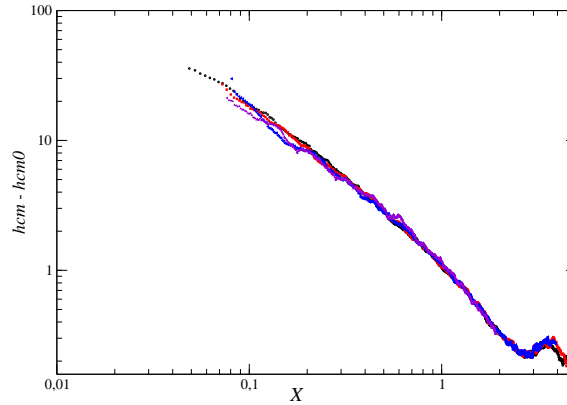

Table 4. Scale law for the mean height of the center of mass in function of X given by equation $X = (1 - e)N$. All curves can be adjusted in one curve if we multiply by a constant factor.

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