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SUPPRESSING FERMI ACCELERATION IN TWO DIMENSIONAL OVAL-LIKE DRIVEN BILLIARDS

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Abstract: We consider a dissipative oval-like shaped billiard with a periodically moving boundary. The dissipation considered is proportional to a power of the velocity V of the particle. The three specific types of power laws used are: (i) $F \propto -V$; (ii) $F \propto -V^2$ and (iii) $F \propto -V^\delta$ with $1 < \delta < 2$. In the course of the dynamics of the particle, if a large initial velocity is considered, case (i) shows that the decay of the particle's velocity is a linear function of the number of collisions with the boundary. For case (ii), an exponential decay is observed, and for $1 < \delta < 2$, an power-like decay is observed. Scaling laws were used to characterize a phase transition from limited to unlimited energy gain for cases (ii) and (iii). The critical exponents obtained for the phase transition in the case (ii) are the same as those obtained for the dissipative bouncer model. Therefore near this phase transition, these two rather different models belong to the same class of universality. For all types of dissipation, the results obtained allow us to conclude that suppression of the unlimited energy growth is indeed observed.

keywords: Stadium–like Billiard, Chaos, Scalling.

1. INTRODUCTION

The famous phenomenon of Fermi acceleration (FA) [1] was studied in various billiard-like models [2–5] where the boundary moves in time, i.e. it is time-dependent. The introduction of time dependence into the boundary yields the particle to change energy upon collision. If the collision is head-on/tail-on then the particle gains/loses energy. A key question addressed in studies on FA [1] is whether a particle's energy can grow to infinity. The answer to this question is not trivial and depends on the geometry of the boundary and the kind of time perturbation. The Loskutov-Ryabov-Akinshin (LRA) conjecture [6] claims that if the dynamics of the particle is chaotic while the boundary is static, thus this is a sufficient condition to observe FA when a time perturbation to the boundary is introduced. It has been shown recently [7] that a time-dependent elliptic billiard can also generate FA, therefore the answer could also be "*yes*" even for (some) integrable billiards.

In this paper, we are concerned with the mechanisms of suppressing FA. We therefore study the dynamics of an ensemble of non interacting particles in a time-dependent oval-like shaped billiard. It is assumed that the particles experience a drag force which is proportional to a power of the particle's velocity. The reflection law is not modified and the particles still keep moving along straight lines, as in standard billiards. However, the velocity of each particle is not a constant anymore and decreases as the particle moves. Depending on the kind of damping force considered, the particles might eventually have all their energy dissipated, leading them to reach the state of rest, thus stopping the dynamics. Our main goal in this paper is to investigate the process of competition between the FA and dissipation of the particle energy via a drag force. We specifically address the question whether FA is observed under the presence of the drag force proportional to a power of the particle's velocity. Our numerical results show convincingly that FA is suppressed, even in the regime of small dissipation. Particularly and depending on the type of damping force, the dissipation leads the particles to reach the state of rest. Recently, inelastic collisions have been considered in a time-dependent version of an oval billiard and the results confirm that FA was also suppressed [8]. We have made a similar investigation in a driven elliptic billiard [9] which led also in suppressing FA. Additionally, a new writing of the LRA conjecture was proposed in Ref. [9]. These results allow us to conjecture that FA is not a structurally stable phenomenon.

2. RESULTS

The three different kinds of dissipation lead to suppression of Fermi acceleration. We concentrate to describe some scaling transition for the model. We then discuss the behavior of the average velocity of the particle when a given initial velocity is very small compared to the maximum component of the moving boundary velocity. The behavior of the average velocity as function of n is shown in Fig. 1(a). One can clearly see that the average velocity starts growing for small n and then, after reaching a critical crossover n_x, it bends towards a regime of saturation, marked by a constant plateau. As the damping coefficient decreases, the average velocity

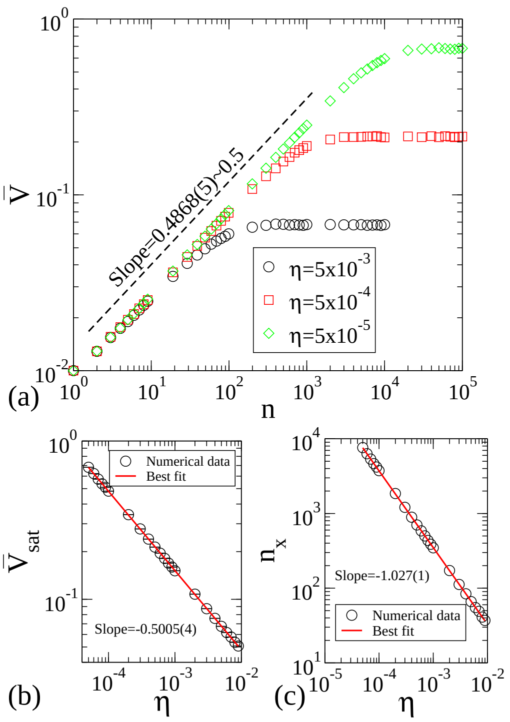

Figure 1 – (Color online) (a) The average velocity as function of n for three different control parameters η, as labeled in the figure. The initial velocity used was $V_0 = 10^{-2}$ and the control parameters were $\epsilon = 0.1$, $a = 0.1$ and $p = 3$. (b) Plot of $\bar{V}_{\rm sat} \times \eta$. A power law fitting yields the slope $\alpha = -0.5005(4)$. (c) Plot of $n_x \times \eta$. The slope obtained is $z = -1.027(1)$.

can reach higher values. Moreover the crossover also rises. We then suppose that

- For small n, say $n \ll n_x$, the average velocity is given by
$$\bar{V} \propto n^{\beta} , \quad (1)$$
where β is a critical exponent;

- For very large n, i.e. $n \gg n_x$, the average velocity is written as
$$\bar{V}_{\rm sat} \propto \eta^{\gamma} , \quad (2)$$
and γ is also a critical exponent;

- Finally, the crossover n_x, which marks the change from the regime of growth to the saturation is given by
$$n_x \propto \eta^{z} , \quad (3)$$
where z is a critical exponent.

Using the formalism shown in [10], it is possible to describe the behavior of $\bar{V}$ using a scaling function. The critical exponents are obtained by numerical fittings as shown in Fig. 1(b) and Fig. 1(c), and the obtained values were $\beta = 0.4868(5) \cong 0.5$, $\gamma = -0.5005(4) \cong -0.5$ and $z = -1.027(1) \cong -1$. Using these three values, we can rescale the axis and obtain a single and universal plot, as shown in Fig. 2.

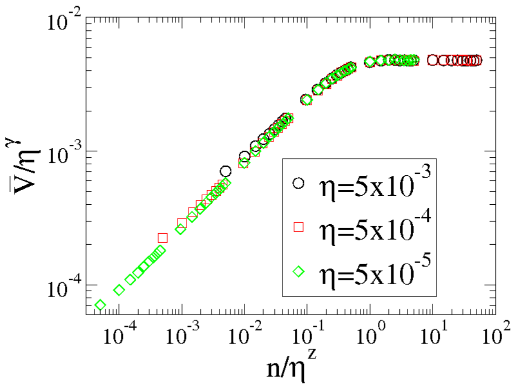

Figure 2 – (Color online) Rescaled axis showing a single and universal plot of three different curves of $\bar{V}$. The control parameters used are labeled in the figure.

3. CONCLUDING REMARKS

Our results confirm that Fermi acceleration is suppressed when a dissipative force of drag-type is introduced in the dynamics. A phase transition from limited to unlimited energy growth was characterized.

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