BLIND EXTRACTION AND SEPARATION OF CHAOTIC SOURCES – RESULTS AND PERSPECTIVES

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In this work, we discuss different approaches for dealing with the problems of blind source extraction (BSE) – which, when chaotic and stochastic signals are mixed, may correspond to a classical time series denoising task – and blind source separation (BSS).

To explain and illustrate the BSE problem in the context of a denoising problem, let us consider that two sources - one being a chaotic signal $s_c(n)$ and the other being a stochastic signal $s_s(n)$ - are linearly mixed, giving rise to $\mathbf{x}(n) = \mathbf{A}\mathbf{s}(n)$, where $\mathbf{x}(n) = [x_1(n)\ x_2(n)]^T$ is the mixture vector, $\mathbf{A}$ is the 2 x 2 mixing matrix (which is assumed to have full rank in the invertible scenario) and $\mathbf{s}(n) = [s_1(n)\ s_2(n)]^T$ is the source vector. The aim of blind source extraction (BSE) is to extract a source from the mixtures without the need for a reference signal or knowledge of coefficients of the mixing matrix [1]. This task can be achieved by multiplying the mixture vector by an adequately chosen separating vector $\mathbf{w}$, so that the output vector yield, for instance, $\mathbf{y}(n) = \mathbf{w}^T\mathbf{x}(n) = Gs_c(n)$, where G is a scaling factor. Figure 1 shows a scheme that represents the described blind extraction problem.

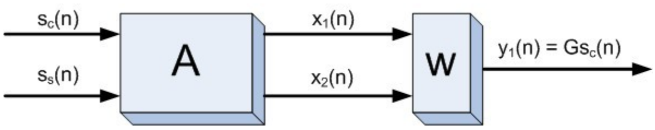

Figure 1: Scheme for blind extraction problem, $s_c(n)$ is the chaotic source, $s_s(n)$ is the stochastic source, $x_1(n)$ and $x_2(n)$ are the observed mixtures, A is the mixing matrix, w is the separating vector and $y_1(n)$ the recovered chaotic signal up to scaling factor G.

Moreover, let us also assume that the mixing matrix is orthogonal (which always can be achieved via a whitening procedure), and, as a consequence, $\mathbf{w}$ can be parameterized in terms of a single variable θ, i.e., $\mathbf{w} = [\cos\theta\ \sin\theta]^T$. Classical ICA approaches look for solutions in θ that ensure, for instance, maximal nongaussianity, which can be evaluated with the aid of the kurtosis of the output components, or, alternatively, maximization of independence between the elements of the output vector $\mathbf{y}(\mathbf{n}) = [y_1(n)\ y_2(n)]^T = \mathbf{W}\mathbf{x}(n)$ (e.g. by minimizing a mutual information measure), being $\mathbf{W}$ the separating matrix given by $[\cos\theta\ \ \sin\theta, -\sin\theta\ \cos\theta]$. It is also important to remark that ICA allows the recovery of the original sources up to scale and permutation ambiguities [1].

However, when it is known that the original sources are, respectively, a deterministic and a stochastic signal, it is possible to obtain the separating vector based on the maximization of the deterministic character of the output, which can be done by employing a recurrence plot. An immediate consequence is that the permutation ambiguity should not exist, since the measure will establish a difference between deterministic and stochastic sources.

In very simple terms, deterministic signals tend to present structures of diagonals in the recurrence plot related to the temporal and spatial correlation characteristics given by the time evolution of motion equations, which is not the case for stochastic signals. The recurrence behavior of a signal can be quantified by recurrence quantification analysis (RQA) using, basically, three measures [2]: the percentage of determinism, the entropy and the length of the maximal diagonal line. These measures (called fit_d, fit_e and fit_l, respectively) were used to adapt a linear separating system in order to extract the "most deterministic" output, which is associated to the chaotic source.

Figure 2 presents the values of the proposed cost function and also of two commonly used ICA contrasts: the kurtosis of $y_1(n)$ and the mutual information between $y_1(n)$ and $y_2(n)$ (which leaves the extraction framework and deal with BSS problem), evaluated with the estimator developed in [3]. It can be noted that the estimators based on the recurrence statistics have global optima at the solutions that lead to perfect inversion (up to a sign ambiguity) of the mixing matrix, a feature shared by methods based on kurtosis and mutual information. These results reveal that the proposal fulfilled the essential requirements to establish a separation method with a performance equivalent to that obtained via classical ICA methods. In fact, for lower signal to noise ratios (SNR), e.g. 2 dB, the contrast between deterministic and stochastic sources provided by RQA measures is stronger than that provided by ICA-based methods, which allows a better extraction performance [4, 5]. When the underdetermined scenario is considered (more sources than mixtures), the approach using RQA measures reveals a better performance than ICA, something that illustrates the advantage of exploiting *a priori* information about the nature of the sources [5].

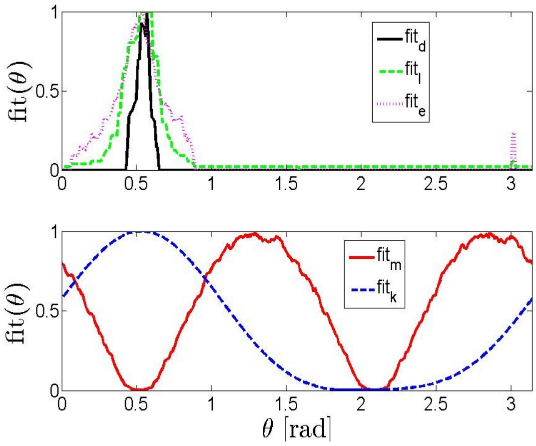

Figure 2: The upper panel shows the percentage of determinism (*fit$_d$*) of $y_1(n)$, longest diagonal *fit$_l$* of $y_1(n)$ and entropy of diagonal lines *fit$_e$* of the recurrence plot for the output vector for different θ values. The stochastic source is a white gaussian signal 10 dB below the chaotic source in power. Lower panel shows the kurtosis of $y_1(n)$ - (*fit$_k$*) - and mutual information between $y_1(n)$ and $y_2(n)$ - (*fit$_m$*) - for different θ values.

It is also possible to extract or separate different chaotic sources using RQA measures. Figure 3 shows the RQA and ICA cost functions for the output vector when a chaotic Lorenz and Rossler time series are mixed by **A**. It can be noted that the first local peak of RQA measures (that occurs at $\theta = 0.52$ rad) leads to the separating vector that recovers the Lorenz time series, and the second peak (global maximum at $\theta = 2.09$ rad) recovers the Rossler series. These results agree with ICA-based methodologies, for which the local and global maxima of kurtosis and the minimum of the mutual information provide the adequate separating vectors for the chaotic time series.

The result presented in Figure 3 is particularly interesting when the problem of multiuser communication systems using chaotic signals is considered, where the separation of several chaotic signals is a central issue, and an issue that, to our best knowledge, has not been addressed before in the literature in the context of the use of RQA. An interesting perspective related to the proposal is to combine ICA and RQA to separate chaotic time series immersed in noise in order to obtain a better separation performance than that reached via methods based only in ICA, which seems restrict to SNRs lower than 5 dB [6]. We feel that this "deterministic component analysis" given by RQA deserves careful analysis.

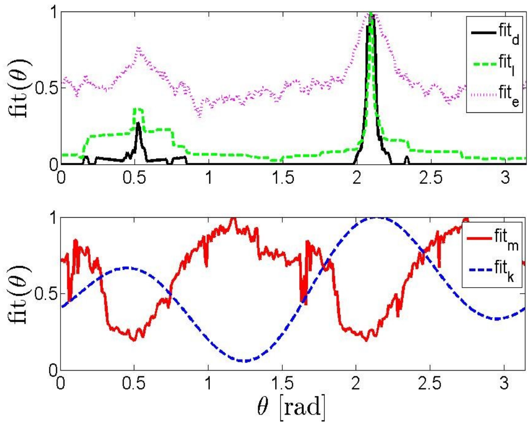

Figure 3: The upper panel shows the percentage determinism (*fit$_d$*) of $y_1(n)$, longest diagonal *fit$_l$* of $y_1(n)$ and entropy of diagonal lines *fit$_e$* of the recurrence plot of the output vector for different θ values considering the mixture of two chaotic sources obtained by Lorenz and Rossler dynamical system. Lower panel shows the kurtosis of $y_1(n)$ - (*fit$_k$*) - and mutual information between $y_1(n)$ and $y_2(n)$ - (*fit$_m$*) - for different θ values.

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