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ASYMPTOTIC APPROACH FOR THE NONLINEAR EQUATORIAL LONG WAVE INTERACTIONS

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ABSTRACT

It is common in large-scale tropical dynamics the use of the so called long wave approximation for studies related to the El Niño and/or the Madden-Julian oscillation ([1, 2]). This approximation filters out inertia-gravity waves of all wavelengths while being accurated for Kelvin and long Rossby waves, however is completely inacurate for short Rossby waves ([3]). Furthermore, the completeness of the remaining waves has not been proven, consequently the representation of any phenomenon is questionable. In the present work, asymptotic methods are used to obtain the long wave approximation as a limiting case of the shallow water equation. The difference with previous works is that the transformation of one regime to another is controled by an external parameter ($\delta \in [0,1]$) which is a measure of the anisotropy of the space and time scalings. The $\delta \to 1$ limit correspond to the shallow water equation, whereas $\delta \to 0$ is for the long wave approximation, any value in between correspond to intermediate realizable states. The model equations developed here are valid for both limiting cases and also for the intermediate states. The advantage of this method is that it allows a continuous approach to the long wave equations while keeping the completeness of the solutions of the Shallow water equations. With this approach we have studied nonlinear resonant wave energy exchanges. It was verified in both theoretical an in numerical experiments that the period of the nonlinear slow energy modulation increases as δ decreases, some implications for the Tropical dynamics are discussed.

1. GOVERNING EQUATIONS

We start with the nonlinear shallow water equations in the β plane in its dimensional form (Eq. 1)

$$\partial_t u + \mathbf{v}\cdot\nabla u - \beta y v + g\partial_x H = 0 \tag{1a}$$
$$\partial_t v + \mathbf{v}\cdot\nabla v + \beta y u + g\partial_y H = 0 \tag{1b}$$
$$\partial_t H + \mathbf{v}\cdot\nabla H + H\nabla\cdot\mathbf{v} = 0 \tag{1c}$$

Where $H = \bar{H} + \eta$, $\bar{H}$ is the mean thickness of the fluid layer, η is its dilatation and βy is the equatorial Coriolis parameter. $C \in [10, 50]$m/s is the baroclinic wave speed. The equations are nondimensionalized by taking units of length and time as in 2.

$$L = (C/\beta)^{1/2} \approx [500, 1500]\text{km} \tag{2a}$$
$$T = \frac{1}{(C\beta)^{1/2}} \approx [8, 18]\text{hrs} \tag{2b}$$

the nondimensionalized and anisotropically scaled (δ dependency) variables are obtained by doing:

$$x = (L/\delta)x''; y = Ly''; t = (T/\delta)t'' \tag{3a}$$
$$u = Cu''; v = \delta C v''; H = (C^2/g)H''; \eta = \eta'' \tag{3b}$$

Where $0 < \delta \ll 1$, therefore the nondimensional zonal coordinate x and time t is dilatated whereas v is contracted (or equivalently x'' and t'' are large scale and slow time respectively whereas v'' is of a small magnitude). With substitution $(x, y, t, u, v, \eta)'' \to (x, y, t, u, v, \eta)$ the equations 4 are obtained.

$$\partial_t u + u\partial_x u + v\partial_y u - yv + \partial_x \eta = 0 \tag{4a}$$
$$\delta^2[\partial_t v + u\partial_x v + v\partial_y v] + yu + \partial_y \eta = 0 \tag{4b}$$
$$\partial_t \eta + u\partial_x \eta + v\partial_y \eta + (1+\eta)(\partial_x u + \partial_y v) = 0 \tag{4c}$$

2. SPECTRAL REPRESENTATION

With $\phi = (u, v, \eta)^T$ the spectral representation of the dependent variables and the spectral coefficients are:

$$\phi(x, y, t) = \sum_a \phi_a(x, y) Z_a(t); \tag{5a}$$
$$Z_a(t) = \int \phi_a^+(x, y)\phi(x, y, t)\,dxdy \tag{5b}$$

where $\phi_a(x, y)$ are the eigenfunctions of the linear operator $\mathcal{L}$ defined below

$$(\mathcal{L} - \omega_a I)\phi_a(x, y) = 0 \tag{6}$$

The asymptotic model (equation 4) can be written as:

$$[\partial_t \mathcal{I}(\delta) + \mathcal{L}]\phi + \mathcal{B}(\phi,\phi)\mathcal{I}(\delta) = 0 \qquad (7)$$

Where $\mathcal{I}(\delta) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \delta^2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$;

$$\mathcal{L}(\phi) = \begin{pmatrix} -yv + \partial_x h \\ +yu + \partial_y h \\ \partial_x u + \partial_y v \end{pmatrix}; \mathcal{B}(\phi,\phi) = \begin{pmatrix} \vec{v}\cdot\nabla u \\ \vec{v}\cdot\nabla v \\ \nabla(\eta\vec{v}) \end{pmatrix}. \qquad (8a)$$

3. EVOLUTION EQUATION

Projection of equation (7) onto an arbitrary eigenmode ϕ_a results in:

$$[d_t \mathcal{I} + \omega_a I]Z_a(t) = 1/2 \sum_{bc} \delta_{abc}\sigma_a^{bc}\mathcal{I}Z_b^* Z_c^* \qquad (9)$$

The coupling coefficient (σ_a^{bc}) as a function of δ is:

$$\delta_{abc}\sigma_a^{bc}\mathcal{I} = \int \phi_a^+ \mathcal{B}(\phi_b^*, \phi_c^*)\mathcal{I}dxdy \qquad (10)$$

Which is valid for resonant and non-resonant interactions. Using $\xi_a = -\hat{\imath}v_a/\omega_a$ and $T_a^{bc} = u_a\xi_b\xi_c$ the coupling coefficient can be written as

$$\delta_{abc}\sigma_a^{bc}\mathcal{I} = \hat{\imath}\omega_a\gamma_{abc} + \hat{\imath}(\omega_a + \omega_b + \omega_c)T_a^{bc} \qquad (11)$$

The interaction coefficient (γ_{abc}) is clearly modified by δ

$$\gamma_{abc} = \delta\left[\frac{(\eta_a\vec{v}_b\cdot\vec{v}_c + C.P.) - T_a^{bc}}{\delta} - \delta\{T_b^{ca} + T_c^{ab}\}\right] \qquad (12)$$

Conditions for interaction ($n_a + n_b + n_c =$ odd) and resonance ($\omega_a+\omega_b+\omega_c = 0; k_a+k_b+k_c = 0$). Thus the resonant coupling coefficient shows that there is an effective reduction of the interaction frequency as δ decreases, whereas the interaction strenght is also modified, increasing in absolute values ($\omega_a > \delta\omega_a$ for $0 < \delta < 1; \lim_{\delta\to 0}|\gamma_{abc}| > \lim_{\delta\to 1}|\gamma_{abc}|$)

4. NUMERICS

For the numerical integration a semi-analytic method is used, it assumes that the nonlinear terms are constant within a $2\Delta t$ time interval. If we know the expansion coefficients Z_j,$j = a, b, c, \cdots$ for $[t, t-\Delta t]$, it is possible to obtain analytic solutions at time $t+\Delta t$. The nonlinear terms are defined at the central time t. Zonal and meridional dependency of the wave components, as well as their derivatives are computed analitically. Both, initial amplitudes ($A_j(\omega_{max}) \gg A_{i\neq j}$; defined in eq.13) and phase relationship ($\Sigma\lambda_j = \pi/2$) are choosing as to maximize the energy exchanges (Raupp & SilvaDias [4]).

$$Z_j(0) = A_j e^{\hat{\imath}\lambda_j}; j = \{a, b, c\}; \sum_j \lambda_j = \pi/2 \qquad (13)$$

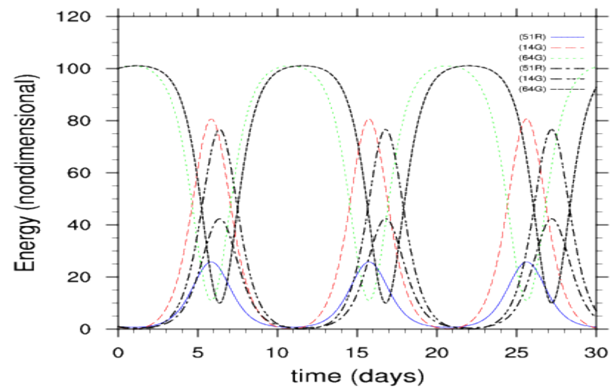

Figure 1 – Slow nonlinear resonant energy exchanges for a triad: formed of one Rossby (R) and two inertio-gravity waves (G). The waves are labeled by zonal wavenumber (k), meridional quantum number (n) and wave type (R or G in this case). Initial energy partion is given by [$A_1^2 = A_2^2 = 1.0; A_3^2 = 100.0$]. The long wave like approach (thick lines) and shallow water approach (thin lines) can be compared and correspond to values of $\delta =$ 0.5 and 1.0 respectively.

5. CONCLUSIONS

Both theoretical framework and numerical experiments confirm that the asymptotic long wave approximation tend to have a slower energy modulation when compared to the shallow water equation. There is a potential for a better understanding of tropical large-scale phenomenon in connection with smaller spatial scales.

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