DYNAMICS DAYS SOUTH AMERICA 2010
International Conference on Chaos and Nonlinear Dynamics

INPE – National Institute for Space Research
São José dos Campos – SP – Brazil – July 26-30, 2010

HOMOCLINIC CHAOS IN AXISYMMETRIC BIANCHI-IX COSMOLOGICAL MODELS WITH AN “AD HOC" QUANTUM POTENTIAL

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keywords: Chaos in Hamiltonian Systems

Belinskii, Khalatnikov and Lifishitz [1] started the question of chaotic behaviour of general Bianchi IX models in Relativistic Cosmology. The interest in the chaoticity (or not) of Bianchi IX models has been mainly focused on the Mixmaster case (vacuum Bianchi IX models with three scale factors [2]). The question of the generic behaviour (chaotic or not) of the Mixmaster dynamics remained unsettled mainly due to the absence of an invariant (or topological) characterization of chaos in the model (standard chaotic indicators as Liapunov exponents being coordinate dependent and therefore questionable [3, 4]). For discussions of the issue of chaotic dynamics on these models we refer to the works of [5–9].

In this work we study the dynamics of the axisymmetric Bianchi IX cosmological model. The phase space of such **classical** model is noncompact and the presence of the cosmological constant determines two crucial facts in phase space: first, the existence of a critical point of the saddle-center type; second, two critical points at infinity corresponding to the attractor configuration, one acting as an “atractor" to the dynamics and the other as a “repeller". With respect to the latter point, this system has mathematically the characteristics of a chaotic scattering system with two abosolute outcomes consisting of (i) escape to infinity or (ii) recollapse to the singularity. The presence of this critical point is responsible for a rich and complex dynamics, engendering in phase space topological structures such as homoclinic orbits to a center manifold. The physical singularities are the main point in the whole discussion, that is, when any one of the scale factors crosses zero, meaning a recollapse of the universe. As showed in [10] any homoclinic crossings present in the dynamics of the classical model is not seen by the physical world since the mandatory recurrence is lost because the physical dynamics has to be restricted to $A(t)$ and $B(t)$, the scale factors, greater than zero. Therefore the only separation between recollapse and escape to the attractor at infinity are the unstable and stable manifolds corresponding to the center manifold associated to the Einstein singularity. This establishes the difference between physical and mathematical integrability: in spite of the chaotic dynamics present in the equations the physical meaningful region does not see it (see also [11]).

In the present work we study the dynamics of the Bianchi IX cosmological model as in [10] to which we add a term of quantum potential inspired by the work of Alvarenga et al. [12] whose presence represents exactly the short-range effects due to the quantum behavior of matter in small scales and plays the role of a repulsive force near the singularity. In this work a similar term has been introduced in an “ad hoc" manner. As it will be seen, this potential restricts the dynamics of the model to the positive values of $A(t)$ and $B(t)$ and alters some qualitative and quantitative characteristics of the dynamics of the classical model. We show the common features of a large class of such potentials which depends only on a so-called r-equivalent variable: $(AB^2)^{1/3}$. Picking a particular example, we make a complete analysis of the phase space of the model finding critical points, periodic orbits, stable/unstable manifolds using numerical techniques such as Poincaré section, numerical continuation of orbits and numerical globalization of invariant manifolds. We compare the classical and the quantum models and verify that the addition of this quantum term allows the existence of homoclinic crossings of the stable and unstable manifolds in the physical meaninful region of the phase space (both $A(t)$ and $B(t)$ positive) thus allowing chaotic escape to inflation as well as chaotic bouncing near the singularity due to a new center-center equilibrium point.

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