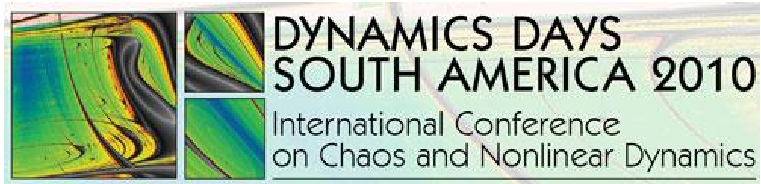

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A NOTE ON THE DYNAMICS OF FERROELECTRIC LIQUID CRISTALS

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We study theoretically the phase diagram of a ferroelectric liquid crystal under an external field. The system is composed of chiral molecules that selforganize in a smectic order. An interesting phase diagram is obtained when a magnetic field is applied parallel to the smectic layers. Musevic et al. [1] and others have studied experimentally this system, using optical and specific heat measurements. On the theoretical side, this system has been extensively studied by means of Landau-Ginzburg expansions of the free energy.

Using this approach, Michelson [2] was the first to propose an expression to study the system. He predicted a very rich behavior on the H-T phase diagram, showing the occurrence of three smectic phases: disordered (sm$A$), homogeneously ordered (sm$C$) one helicoidally ordered (smC^*). The transitions between sm$A$ to sm$C$ and sm$A$ to smC^* are second order; the transition between sm$C$ to smC^* is first order. A novel behavior was found by Michelson: the occurrence of a Lifshitz point, which separates the two second order transition lines and represents a special case of a triple point.

In the Landau theory of smectic liquid crystals under magnetic field, the free energy density $\mathcal{F} = \int \mathcal{L}\, dz$ is written in terms of the projection of the director n onto the layer plane, $n = (n_x, n_y)$, the z axis being taken normal to layer, with $\mathcal{L}$ given by

$$\mathcal{L} = \frac{A}{2}({n_x}^2+{n_y}^2)+\frac{1}{4}({n_x}^2+{n_y}^2)^2+\frac{k}{2}[(\frac{dn_x}{dz})^2+(\frac{dn_y}{dz})^2] -\frac{\chi}{2}\mathcal{H}^2{n_x}^2-\Lambda(n_x\frac{dn_y}{dz}-n_y\frac{dn_x}{dz}). \quad (1)$$

We use a Hamiltonian formulation derived of Landau's Lagrangian which reproduces the linear results of Michelson [2] and Pitanga et al. [3].

Furthermore, to perform a global analysis we use suitable parameter scaling and nonlinear transformations. The system reduces then to 2 d.o.f with two parameters. This enables us to make: a) a complete analysis of the topology of a related integrable system; b) to introduce Poincaré sections and obtain the main families of periodic orbits; c) to compute Lyapunov diagrams on the phase space depending on parameters; d) to detect the existence of invariant tori and chaotic domains. The methodology is similar, e.g., to [4], i.e., a combination of local analytic tools, numerical detection of invariant objects and massive simulations. This allows a general view of the full problem and we hope that completion of this ongoing work will contribute to understanding the smectic transitions.

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