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Analysis of the Ginzburg-Landau equation with non-local coupling

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Abstract: The small amplitude Ginzburg-Landau equation, which appropriately describes fields with non-local coupling is studied numerically in the Benjamin-Feir unstable limit. We have found that by changing the reduced system size and its coupling, different behaviors emerge, particularly a metastable one on which small perturbations make the system come back to its chaotic saddle.

Keywords: spatiotemporal chaos, synchronization, metastability.

1. INTRODUCTION

For a qualitative understanding of complex dynamics of real systems, simplified mathematical models of coupled oscillators have proved to be extremely useful. Amongst them, coupled limit-cycle oscillators with high degrees of freedom are known to represent a wide variety of systems. The present study aims at these, by making a non-local coupling, on which the coupling strength is not uniform throughout the network.

As a prototypical equation, we study the Ginzburg-Landau network, which naturally arises in large assemblies of oscillatory elements with indirect coupling[1], mediated by a diffusive scalar field,

$$\frac{\partial W_j}{\partial t} = W_j + k(Z_j - W_j) - (1+iC_2)\,|W_j|^2\, W_j \qquad (1)$$

with:

$$Z_j = \sum_{j'=1}^{N} C\exp(-\gamma\,|\,j-j'|\,\delta)W_j \qquad (2)$$

where which W_j is a complex variable associated with the jth oscillator, C_2 is a real constant, and C normalization constant. In addition to the coupling strength k, we have the reduced system size $L=\gamma N\delta$; δ only scales the length of the system.

This equation has been studied under the stability condition, on which turbulent patterns were discovered[2]. Chaotic behavior has also been discovered[3] when L is increased, making the system more locally coupled. However, its behavior when L is decreased, towards the globally coupled regime has been poorly understood.

We have studied the system for $L=1.0$, and found that depending on its coupling strength, distinguish patterns appear. For weak coupling, the real and imaginary parts of the oscillators are frequency synchronized, oscillating periodically and with the same amplitude, which leads to a constant modulus for the oscillators. By increasing k, the system starts presenting longer chaotic transients, until they finally become perennial, which indicates a crisis.

If L continues to be increased, during a short interval, the oscillators become once more synchronized in frequency with a periodically oscillating modulus. Moreover, they present a metastable behavior, on which a great number of asymptotic orbits, with different frequencies and amplitudes occur. At this moment, small perturbations to one or more oscillators make the system return to its chaotic saddle preventing it from synchronizing.

We have also found a relation between the reduced system size and the coupling strength, making it possible to compensate a weak coupling with a smaller γ.

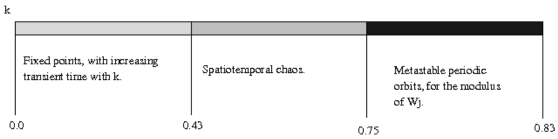
2. PURPOSE

Our aim is to study the Ginzburg-Landau equation through numerical simulations and determine the different behaviors that emerge by varying its parameters. Such behavior can be used as a prototype and can be useful for a large class of systems.

We also intend to understand how pinning one or more oscillators may desynchronize the system by keeping it in the chaotic saddle.

3. RESULTS AND DISCUSSION

We firstly refer to the first transition for $L=1$ which happens at a critical value of the coupling strength (as it is shown on the scheme below), and it is probably caused by a crisis in the system, given the power-law relation between the average transient time and the coupling strength (figure 1b).



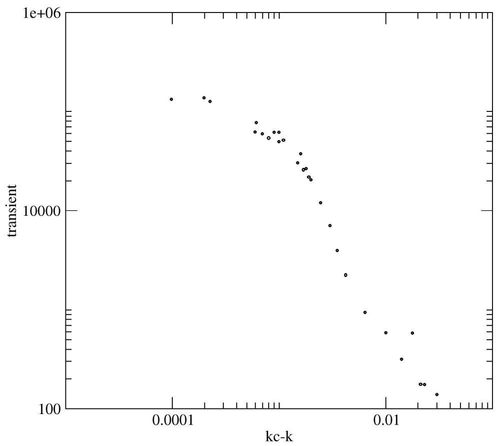


Figure 1 a: Different behaviors as a function of k.
Figure 1b: Relation between the transient time and the critical value of *k.*

In order to characterize the different kinds of the asymptotic orbits, the frequency dispersion has also been determined numerically;

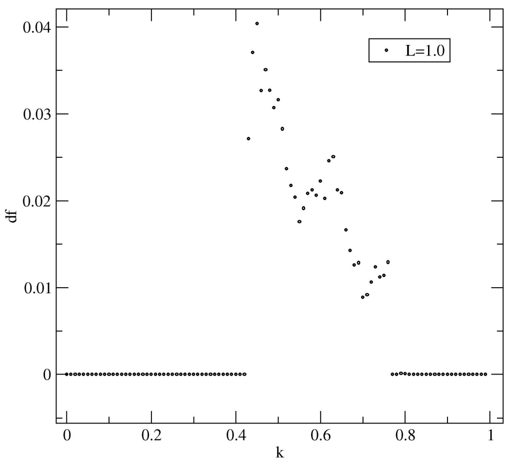


Figure 2: Frequency dispersion calculated form the modulus of *Wj.*

An instantaneous frequency has been calculated by using the time between two consecutive maxima on the time evolution of $|W_j|$ by using these, a time averaged frequency was calculated for each oscillator and the dispersion from the spatial average was determined for the integration with sixty four oscillators.

On the metastable limit, we have found that by pinning one site, we send the entire system to a desynchronized state, on the chaotic saddle. The following graphic presents the time spent on a chaotic transient after a perturbation if given to one of the oscillators.

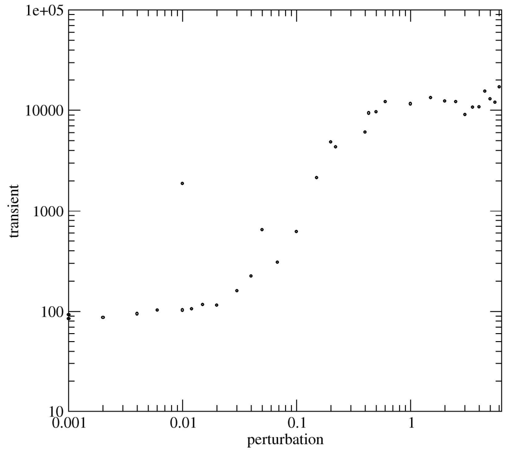


Figure 3: Transient time versus amplitude of perturbation for fixed *L* and *k.*

It can be seen from it that small perturbations are incapable of causing desynchronization, while from an upper limit, the amplitude of perturbation no longer alters the transient time.

We have also found that by increasing the number of perturbed sites or the frequency of perturbations, it is possible to obtain a similar result with lower amplitude of perturbation.

4. CONCLUSIONS

It has been shown that the Ginzburg-Landau equation presents a wide variety of behaviors, witch depend strongly on the reduced system size and on the coupling strength.

Particularly, for a certain interval of *k,* when the metastability appears, the network could be used to model applications where synchronization should be avoided such that manners to prevent it can improve the behavior of the system.

References

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