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Robust Tori in a Double-Waved Hamiltonian Model

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A Hamiltonian system perturbed by two waves with particular wave numbers can present robust tori, barriers created by the vanishing of the perturbed Hamiltonian at some defined positions. When robust tori exist, any trajectory in phase space passing close to them is blocked by emergent invariant curves that prevent the chaotic transport. Our results indicate that the considered particular solution for the two waves Hamiltonian model shows plenty robust tori blocking radial transport.

The effect of transport barriers in Hamiltonian systems is a subject of global interest in different branches of physics [1, 2, 3]. W. Horton introduced one type of Hamiltonian model with two waves, relevant for particle transport in plasma physics [4]. The Hamiltonian describes drift waves, originated by particles drift proportional to $\vec{E} \wedge \vec{B}$ in nonuniform plasmas, propagating in a magnetic toroidal and an electric radial fields. The model has been explored to describe the onset of stochasticity for test particles driven by these drift waves in tokamaks. The model has been applied in many works as to investigate the influence of the equilibrium electric and magnetic fields on the radial transport as to analyze experimental results [5, 6 ,7].

We observed that this model could present infinite robust tori (RT) which correspond to dynamical barriers that may appear in Hamiltonian systems [8, 9, 10, 11]. In this work, we start with a Hamiltonian with only one wave in order to emphasize the abundance of RT and next with the addition of other wave these RT could be broken giving rise to anomalous radial transport. Our goal in this work is to present a particular solution for this wave Hamiltonian model that prevents the breaking of the RT, even if we add as many waves as we want in the perturbation. This is an important fact since the creation of barriers in Hamiltonian systems has been considered an important subject in several areas of physics especially in plasma confinement in tokamaks [2, 7, 12, 13].

When the Hamiltonian cited above presents only one wave, the system is globally integrable. However, when we consider two waves for the system, the integrability will be broken and chaos will be observed around the hyperbolic fixed points. We used a nonperturbed Hamiltonian of the flowing kind: $H_0(x) = \alpha\, x$, then we have:

$$
\begin{aligned}
&H(x,y,t) = (\alpha - u_1)x + A_1 \sin(k_{x1}x)\cos(k_{y1}y) + \\
&A_2 \sin(k_{x2}x)\cos(k_{y2}(y-ut))
\end{aligned}
\tag{1}
$$

We can observe that when $\sin(k_{x1}x) = 0 = \sin(k_{x2}x)$ the perturbation vanishes. Looking at the new equations of motion:

$$
\begin{aligned}
&\dot{x} = A_1 k_{y1} \sin(k_{x1}x)\sin(k_{y1}y) + \\
&A_2 k_{y2} \sin(k_{x2}x)\sin(k_{y2}(y-ut)) \\
&\dot{y} = (\alpha - u_1) + A_1 k_{x1} \cos(k_{x1}x)\cos(k_{y1}y) + \\
&A_2 k_{x2} \cos(k_{x2}x)\cos(k_{y2}(y-ut))
\end{aligned}
\tag{2}
$$

We note that the motion in the x-direction can disappear if the wave numbers obey the condition $k_{x1} = m.k_{x2}$. If *m* is an integer number, RT will appear as in the integrable case, but if *m* is a non-integer number, only fewer RT will survive. In the former case there will be lines with *x=constant* in the positions $x = \frac{n_1 \pi}{k_{x1}} = \frac{n_2 \pi}{k_{x2}}$ for all n_1 and n_2 integers and RT will continue intact even with the addition of the second wave what allows us to expect to block the radial transport.

We present in Fig. 1 two different situations for the waves model of Eq. (1). We see in Fig. 1(a) the Poincaré map for the case $k_{x1} \neq m.k_{x2}$, known in the literature [4, 5, 6, 7]. The addition of the second wave breaks the integrability of the system and chaos may fills the phase space. The particles can move along the radial and poloidal coordinates making a chaotic web, as is shown in Fig. 1(a). In order to understand the dynamics we choose different colors for the initial conditions representing different regions. The blue, green, and red colors are mixed in Fig. 1(a) showing that there are not barriers for the radial transport developed by the particles. On the other hand, in Fig. 1(b) we show the Poincaré map for the particular case presented in this paper $k_{x1} = m.k_{x2}$ for *m* an integer. RT (black lines) are intact even after the addition of the second wave and there is not mixing of the colors along the phase space. As was expected RT block the radial diffusion.

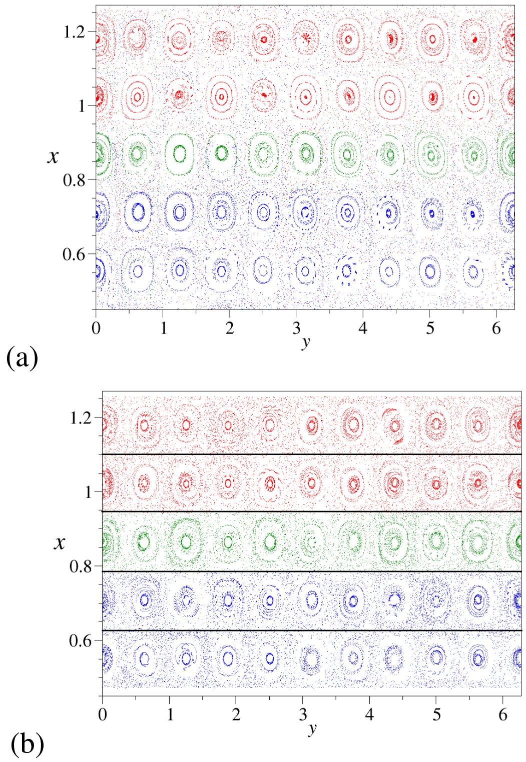

Figure 1 - Poincaré maps for the Hamiltonian with two waves Eq. (1) (a) for $k_{x1} \neq m.k_{x2}$ (without RT) (b) for $k_{x1} = m.k_{x2}$ with m integer (with RT in black color).

Previous studies [2, 3] have showed the importance of decreasing the radial transport induced by drift waves to improve the plasma confinement in tokamaks. It is also reported that similar Hamiltonians to the one presented in this paper have been used to study transport but only few works were dedicated to control chaos in these systems [5, 14]. Even though there is not an effective way to control the wave numbers of the drift waves in tokamaks neither to measure the radial wave number k_{xn}, our contribution shows a direction to block the radial transport with the particular solution presented here for the two wave Hamiltonian model.

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