DYNAMICS DAYS SOUTH AMERICA 2010
International Conference on Chaos and Nonlinear Dynamics

INPE – National Institute for Space Research
São José dos Campos – SP – Brazil – July 26-30, 2010

PARAMETER MODULATION IN THE HÉNON MAP

Gabriela A. Casas[1] *and Paulo C. Rech*[2]

[1]Universidade do Estado de Santa Catarina, Joinville, Brazil, gabriela.a.casas@gmail.com
[2]Universidade do Estado de Santa Catarina, Joinville, Brazil, dfi2pcr@joinville.udesc.br

keywords: Chaotic dynamics, Hénon map, multistability, suppression.

The Hénon map [1] is a well-known two-dimensional discrete-time system given by

$$\begin{aligned} z_{t+1} &= c - z_t^2 + d w_t, \\ w_{t+1} &= z_t, \end{aligned} \tag{1}$$

where z_t , w_t represent dynamical variables, c, d are parameters, and $t = 0, 1, 2, \ldots$ is the discrete time. It was proposed in 1976 as a model to the Poincaré section of the Lorenz system [2] governed by three autonomous nonlinear ordinary first-order differential equations.

In some previous investigations some forms of modulation were applied on the parameters of the Hénon map. For example, in Ref. [3] a low-dissipative Hénon map ($d = 0.9$) with coexisting period-1 and period-3 orbits was considered, and a harmonic modulation was applied on the parameter c. It was shown that, depending on the modulation, the period-3 branch can be expanded, retracted, or even suppressed. In Ref. [4] the parameter c is perturbed by a parametric modulation based on deterministic pseudorandom dynamics, while the parameter d is kept fixed in 0.3. This time it was shown that the dynamics can be aperiodic and not chaotic, *i.e.*, aperiodic with a negative largest Lyapunov exponent.

In this paper we modulate both parameters in Eqs. (1), by using the solutions of another Hénon map. With this aim, we make

$$c = e + f x_t, \quad d = e + f y_t, \tag{2}$$

where e, f are constant parameters, and x_t , y_t are a solution of another Hénon map given by

$$\begin{aligned} x_{t+1} &= a - x_t^2 + b y_t, \\ y_{t+1} &= x_t. \end{aligned} \tag{3}$$

The result of the use of the modulation (2) in Eq. (1) is the four-dimensional system given by

$$\begin{aligned} x_{t+1} &= a - x_t^2 + b y_t, \\ y_{t+1} &= x_t, \\ z_{t+1} &= (e + f x_t) - z_t^2 + (e + f y_t) w_t, \\ w_{t+1} &= z_t, \end{aligned} \tag{4}$$

which can be regarded as a coupling of two maps of the plane: the Hénon map defined by the two first equations in (4), or by Eqs. (3), and the modulated Hénon map, defined by the two last equations in (4). Note that the two first equations do not depend on the two last equations. In other words, we have in (4) an unidirectional coupling of two Hénon maps, namely a master-slave system. The purpose in this paper is to verify possible modifications in the basins of attraction of the Hénon map (3), by virtue of the imposed modulation.

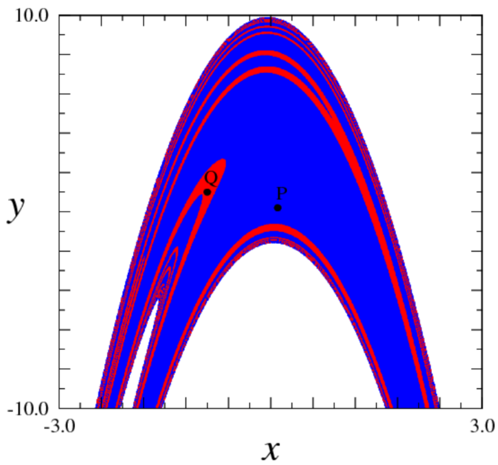

Figure 1 – Basins of attraction of the Hénon map (3), for $a = 1.2$ and $b = -0.3$. Shown are the initial conditions that lead the system to period-1 (blue), period-3 (red), and to infinity (white).

Figure 1 shows the basins of attraction of the period-1 orbit in blue, period-3 orbit in red, and the divergence orbit in white, for the Hénon map with $a = 1.2$ and $b = -0.3$. In this and further plots, a mesh of 750×750 initial conditions was considered. Regarding bounded solutions, the Hénon map exhibits two dynamic equilibrium states, period-1 and period-3 for the parameters $a = 1.2$, $b = -0.3$. This coexistence of more than one attractor has been detected in various fields including, by instance, optics [5] and biology [6], and is called multistability. Recent studies have shown that this phenomenon is a characteristic feature of nonlinear systems, that may be properly explored in more different systems. For instance, multistability may be induced or suppressed, and the emergence may be controlled and limited.

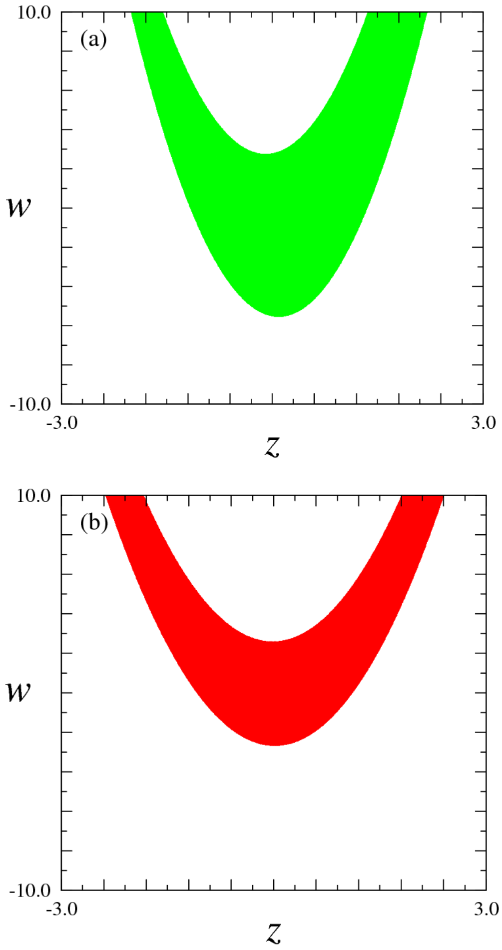

Figure 2 – Basins of attraction of the modulated Hénon map. Shown are the initial conditions that lead the system (a) to period-2 (green), (b) to period-3 (red), and to infinity (white).

Figure 2 is relative to the basins of attraction of the modulated Hénon map, for $e = 0.25$ and $f = 0.2$. In Fig. 2(a), the Hénon map was initialized at $(x_0, y_0) = (0.1, 0.2)$, point P in Fig. 1, corresponding to a period-1 orbit and, consequently, to a fixed-point modulation. In Fig. 2(b) the Hénon map was initialized at $(x_0, y_0) = (-0.9, 1.0)$, point Q in Fig. 1, now corresponding to a period-3 orbit, or a period-3 modulation. Therefore, from plots in Fig. 2 we observe the following phenomena:

- when we choose the initial condition in such way that the orbit of the Hénon map is a period-1 orbit, the modulated Hénon map oscillates in a period-2 orbit, and
- when the initial condition leads the Hénon map to a period-3 orbit, the modulated Hénon map oscillates also in a period-3 orbit.

Hence, we conclude that the procedure of modulation annihilates the multistability, and stabilizes the system in only one periodic orbit.

In this paper we have investigated the effect caused by using the output of a periodic Hénon map to modulate the parameters of another Hénon map. We have shown numerically that multistability may be annihilated by the procedure. Two cases were considered: (i) in the first, we have used a period-1 solution of a multistable Hénon map as the modulation, obtaining a solitary period-2 orbit in the modulated Hénon map; (ii) in the second case, we have used a period-3 solution of the same multistable Hénon map as the modulation, this time obtaining a solitary period-3 orbit in the modulated Hénon map.

ACKNOWLEDGMENTS

The authors thank CNPq and CAPES, Brazilian Agencies, for financial support.

References

[1] M. Hénon, “A two-dimensional mapping with a strange attractor,” Commun. Math. Phys. Vol. 50, pp. 69-77, 1976.

[2] E. N. Lorenz, “Deterministic non-periodic flow,” J. Atmos. Sci. Vol. 20, pp. 130-141, 1963.

[3] J. M. Saucedo-Solorio, A. N. Pisarchik, V. Aboites, “Shift of critical points in the parametrically modulated Hénon map,” Phys. Lett. A Vol. 304, pp. 21-29, 2002.

[4] A. Nandi, S. K. Bhowmick, S. K. Dana, R. Ramaswamy, “Design strategies for the creation of aperiodic nonchaotic attractors,” arXiv:0907.3993v1, pp. 1-9, 2009.

[5] F. T. Arecchi, R. Meucci, G. Puccioni, J. Tredicce, “Experimental Evidence of Subharmonic Bifurcations, Multistability, and Turbulence in a Q-Switched Gas Laser,” Phys. Rev. Lett. Vol. 49, pp. 1217-1220, 1982.

[6] J. Foss, A. Longtin, B. Mensour, J. Milton, “Multistability and Delayed Recurrent Loops,” Phys. Rev. Lett. Vol. 76, pp. 708-711, 1996.