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# HIGH RESOLUTION PARAMETER-SPACES FOR A FORCED CHUA'S CIRCUIT

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### **1. INTRODUCTION**

The interest in codimension-two bifurcations in flows, when we vary simultaneously two of the system's parameters, have grown substantially in last year's<sup>(3)</sup>. This is due to the observation of complex periodic structures, immersed in chaotic regions, until recently just observed in discrete time maps<sup>(1)</sup>. Recently, some works reported the existence of those periodic structures inside the chaotic phases in some systems described by continuous-time models<sup>(2)</sup>. In this work we report two highresolution experimental parameter space for a chaotic circuit, in this case, a Chua's Circuit.

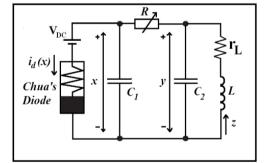


Fig.1 Chua's Circuit forced by an DC voltage source in series with the Chua's Diode.

## 2. EXPERIMENTAL AND METHODS

The Chua's Circuit used in this work is forced by a voltage source d.c., in series with the Chua's Diode. The high resolution in the parameter spaces was propitiated by the use of a 0.5 mV step d.c. voltage source as the new control parameter. The voltage applied  $V_{DC}$ 

change the equilibrium points, defined by the intersection of the d.c. load line (DCLL) and the I(V) curve of Chua's Diode.

The voltage  $V_{DC}$ , shifts the DCLL vertically in the coordinated axe of this I(V) curve and the resistance R (fig.1) change the slope (-1/R) of the DCLL. So we have different intersections points for different control parameters ( $V_{DC}$ ,R).

In the forced Chua's Circuit we varied the control parameters R, from 1600.00  $\Omega$  to 1720.00  $\Omega$  with 1.00  $\Omega$  step, and V<sub>DC</sub>, from -0.7500 V to +0.7500 V with 0,5 mV step. Because of this large amount of experimental measurements (order of  $5 \times 10^5$  time series), was necessary the development of programs to automating the calculus of the periodicity and the Lyapunov exponent of the time series.

We measure the Lyapunov exponent from the time series using the standard method of Sano and Sawada<sup>(5)</sup>. And for detection of the periodicity, we developed programs that do the *Unitary Fourier Transformation* in the time series (UFFT), as described below.

# Periodicity Detection -Unitary Fourier Transformation (UFFT)

From one of the original time series of Chua's Circuit, S(t), we select the points of maximum and minimum,  $S_{máx}(i)$  and  $S_{mín}(j)$ , that define the Poincare Section of S(t). We interca-late this two unitary series and made the  $S_{total}(k)$ , for:

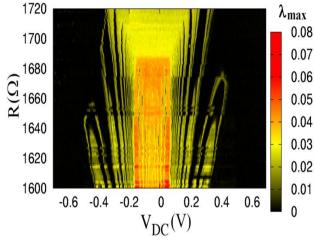
$$S_{total}(k) = [S_{máx}(i), S_{min}(j), S_{máx}(i+1), S_{min}(j+1) \dots]$$

In the time series  $S_{\text{total}},$  we performed the Fourier transformation FFT. In the FFT of  $S_{\text{total}}$ 

we obtain just the distribution of the principal peaks, of the original time series S(t). So just counting the number of peaks, we are able to determine the periodicity of the time series of the Chua's Circuit, x, y or z, automatically.

### **3. RESULTS**

Two high-resolution codimension-two parameter-spaces are present in this work, one for the periodicity and one for the largest Lyapunov Exponent, from our Chua's Circuit forced by a voltage d.c source.

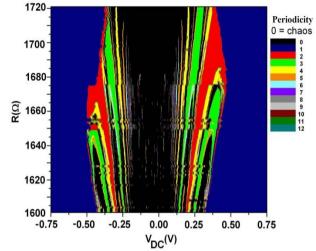


**Fig.2** High Resolution experimental parameter-space for the largest Lyapunov expoent<sup>(3)</sup>  $(\lambda_{máx})$ .  $V_{DC}$  from -0.7000V to +0.7000V with 0.5mV step, and R from 1600  $\Omega$  to 1720  $\Omega$  with 1  $\Omega$  step.

In fig.2 we present our experimental parameter-space for the largest lyapunov exponent of the time series<sup>(3)</sup>. The ordinate stands for the R parameter and the abscissa for the applied D.C. voltage level. A color scale is provided with color ranging from black to yellow and form yellow to red in a continuous 16-bit color palette resolution and corresponding to the measured of the highest Lyapunov exponent,  $\lambda_{máx}$ , in the range of 0.0000 s<sup>-1</sup> to 8.00 x10<sup>-2</sup> s<sup>-1</sup>. And the black stands for periodic behavior of the circuit.

In fig.3 we present, for the first time, our experimental version of the parameter space for the periodicity. A color scale is provided with color ranging of 13-colors resolution, from blue to dark-cyan, and each solid color corresponding to the periodicity detected in the time series. And the black stands for chaotic-behavior of the circuit.

The figures fig.2 and fig.3 show abundance of complex periodic structures that organize themselves in a period-adding bifurcation cascade, as (period-2) - (chaos) -(period-3) - (chaos) - (period-4) - and so on ..., that accumulates in the chaotic region, for  $V_{DC} = 0.0000$  V. Numerical investigations on the dynamical model of this forced circuit were also carried out<sup>(3)</sup> to corroborate several new features observed in those experimental high-resolution parameter-space.



**Fig.3** High Resolution experimental parameter-space for the periodicity.  $V_{DC}$  from -0.7500 V to +0.7500 V with 0.5mV step, and R from 1600  $\Omega$  to 1720  $\Omega$  with 1  $\Omega$  step. Periodicity from period-1 to period-12.

### **4. CONCLUSION**

This forced circuit consists in a platform for the study of this intricate periodic networks formed by periodic self-similar structures surrounded by chaotic phases. Regarding chaos based communication systems<sup>(4)</sup>, the knowledge of what exactly is embedded in the regions of chaos, in dynamical systems, is an important question since clean and extended domains of chaos are important for applications in secure communications<sup>(6)</sup>.

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