DYNAMICS DAYS SOUTH AMERICA 2010
International Conference on Chaos and Nonlinear Dynamics
INPE – National Institute for Space Research
São José dos Campos – SP – Brazil – July 26-30, 2010

NUMERICAL BIFURCATION ANALYSIS OF THE WATT GOVERNOR SYSTEM

José C. C. Vieira[1], *Holokx A. Albuquerque*[2]

1 Santa Catarina State University, Joinville, Brazil, zephisicks@gmail.com
2 Santa Catarina State University, Joinville, Brazil, dfi2haa@joinville.udesc.br

keywords: Chaotic Dynamics, Applications in Engineering and Nanoscience, Applications of Nonlinear Sciences.

In this work, we numerically studied the dynamics of the Watt governor system model. The Watt governor is a device that automatically controls the speed of an engine [1]. As commented in Ref. [1], that system is dating to 1788, and landmarks for the study of the local stability of the Watt governor system are the works of Maxwell [2] and Vyshnegradskii [3]. The Watt governor system model is described by a set of three coupled first-order differential equations, which can be derived from Newton's Second Law of Motion [1]. In Eqs. (1) below, we show the model with normalized variables and parameters.

$$\dot{x} = \frac{dx}{dt} = y,$$

$$\dot{y} = \frac{dy}{dt} = z^2 \sin(x)\cos(x) - \sin(x) - \varepsilon y, \qquad (1)$$

$$\dot{z} = \frac{dz}{dt} = \alpha[\cos(x) - \beta].$$

In Eqs. (1), $\alpha > 0$, $0 < \beta < 1$, and $\varepsilon > 0$ are parameters. A complete analytical bifurcation study, regarding codimension one, two, and three Hopf bifurcations, was done in Ref. [1] and references therein. In this sense, the aim of our work is to extend those studies, carrying out a numerical study of the global bifurcations of the Watt governor system, modeled by Eqs. (1).

The numerical study carried out in this work consists of to calculate the largest Lyapunov exponent, numerically solving the Eqs. (1) with the fourth-order Runge-Kutta method with time step equal to 10^{-2}, for each pair of parameters (α, ε), with $\beta = 0.8$. The range of parameter values was discretized in a mesh of 500×500 points equally spaced. We identify for each largest Lyapunov exponent a color, varying continuously from black (zero exponent), passing through yellow (positive exponent), up to red (positive exponent).

Fig. 1 shows the two-dimensional parameter space for the parameters (α, ε) of Eqs. (1). Black regions represent periodic behaviors, and the yellowish and reddish regions represent chaotic behaviors. Inside the chaotic regions, we can observe the existence of immersed periodic structures, represented by the black regions inside of the yellowish and reddish regions.

Fig. 2 shows the amplification of the white box in Fig. 1. Fig. 3 shows the attractors located in the marked structures of Fig. 2. All the attractors are in periodic regions, black colors in Fig. 2. In Fig. 3, we observe attractors with periodic behaviors (limit cycles), for example, attractors with period-12 (attractor-a), with period-6 (attractor-b), with period-8 (attractor-c), with period-16 (attractor-d), with period-20 (attractor-e), and with period-10 (attractor-f).

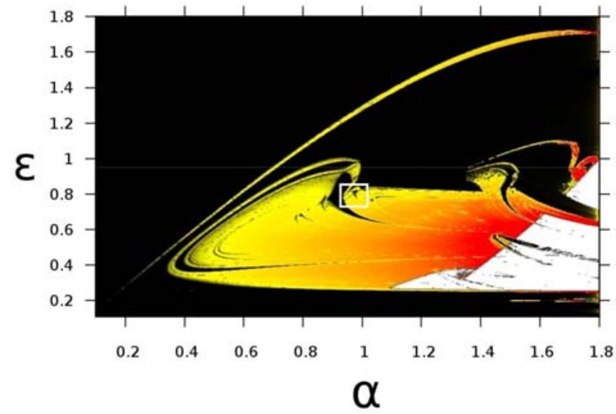

Figure 1 – Global view of the (α, ε) parameter space of Eqs. (1). Black color indicates periodic behavior, yellow and red ones indicate chaotic behavior. The white regions indicate divergence of Eqs. (1). The white box represents the amplification region shown in Fig. 2.

Periodic structures embedded in chaotic regions were reported in recent works [4-6], where the dynamical systems are modeled by a set of first-order differential equations. In those works, the observed periodic structures organize themselves in bifurcation cascades, called period-adding cascades, that accumulate in periodic boundaries. That behavior seems to be a common feature presented in those systems. Indeed, in the Watt governor model, Eqs. (1), we observe a new sequence of accumulation in a region of the parameter space, the white box in Fig. 1. The amplification of that box, shown in Fig. 2, presents a sequence of pairs of periodic structures, where the first pair, i.e., structures ‘a’ and ‘b’, has a period-half cascade (period-12 to period-6). The second pair, i.e., structures ‘c’ and ‘d’, has a period-doubling cascade (period-8 to period-16), and the third pair, i.e., structures ‘e’ and ‘f’, has a period-half cascade (period-20 to period-10). That alternating sequence of pairs of periodic structures continues until the periodic boundary, which is a large periodic region with period-4.

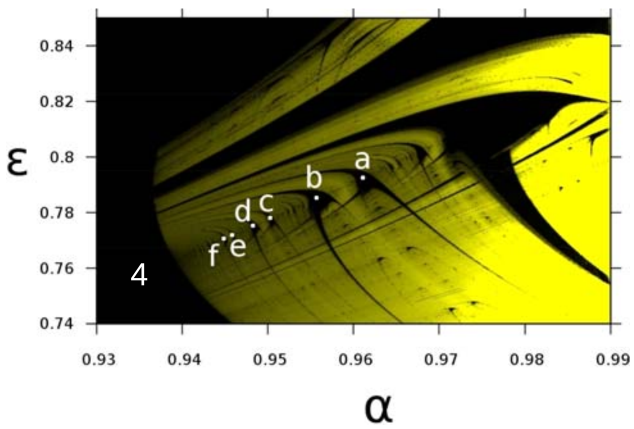

Figure 2 – Amplification of the white box in Fig. 1. Black regions are periodic behaviors, and yellow ones are chaotic behavior. The letters indicate the positions of the attractors shown in Fig. 3. The black region in the left side is a periodic boundary with period-4.

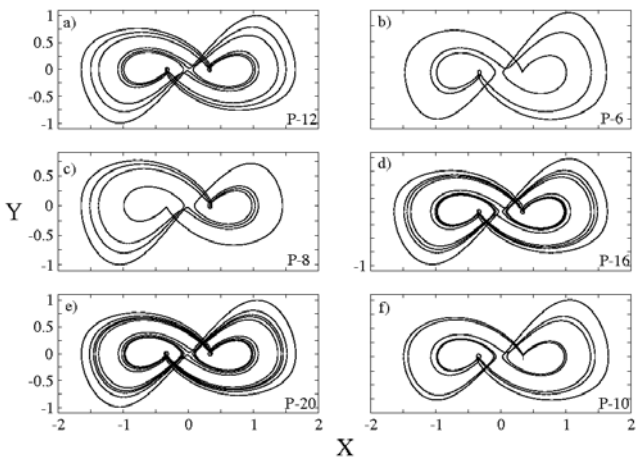

Figure 3 – Periodic attractors of the selected structures in Fig. 2. In the right side below of each attractor we show the period of the limit cycles.

A two-dimensional parameter space, using the largest Lyapunov exponent codified in a continuous range of colors, for the Watt governor system model was reported. We observed a diversity of self-similar periodic structures immersed in the chaotic regions. A new sequence of bifurcation cascade was observed, with pairs of periodic structures alternating from period-half to period-doubling bifurcation and accumulating in a periodic boundary.

ACKNOWLEDGMENTS

The authors thank Coordenação de Aperfeiçoamento de Pessoal de Nível Superior-CAPES, Conselho Nacional de Desenvolvimento Científico e Tecnológico-CNPq, and Fundação de Apoio à Pesquisa Científica e Tecnológica do Estado de Santa Catarina-FAPESC, Brazilian agencies, for the financial support.

References

[1] J. Sotomayor, L.F. Mello, and D.C. Braga, “Bifurcation Analysis of the Watt Governor System”, Comp. and Appl. Math. Vol. 26, No. 1, pp. 19-44, 2007.

[2] J.C. Maxwell, “On governors”, Proc. R. Soc. London Vol. 16, pp. 270-283, 1868.

[3] I.A. Vyshnegradskii, “Sur la Théorie Générale des Régulateurs”, C. R. Acad. Sci. Paris Vol. 83, pp. 318-321, 1876.

[4] E.R. Viana Jr., R.M. Rubinger, H.A. Albuquerque, A.G. de Oliveira, and G.M. Ribeiro, “High-resolution Parameter Space of an Experimental Chaotic Circuit”, to appear, Chaos, June 2010.

[5] C. Stegemann, H.A. Albuquerque, and P.C. Rech, “Some Two-dimensional Parameter Spaces of a Chua System with Cubic Nonlinearity”, to appear, Chaos, June 2010.

[6] C. Bonatto, and J.A.C. Gallas, “Accumulation Boundaries: Codimension-two Accumulation of Accumulations in Phase Diagrams of Semiconductor Lasers, Electric Circuits, Atmospheric and Chemical Oscillators”, Phil. Trans. R. Soc. A Vol. 366, pp. 505-517, August 2007.