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Hyperbolic kaleidoscopes and chaos in Hele-Shaw cells

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1. INTRODUCTION

In foams, a number of patterns are generated spontaneously due to the reflection and refraction of light. One of these patterns is observed by the formation of some triangular images inside Plateau borders in a single layer of bubbles between two plates, known as the Hele-Shaw cells. By inspection, these patterns bear a resemblance to those observed in some systems involving chaotic scattering and multiple light reflections between spheres. A similar image can be obtained using Christmas balls ornaments (see Fig. 1). The goal of this work is to explain these patterns by the light of chaotic scattering. Basically the image obtained is due to the fact that the three different spheres are mirrored in each of the other spheres giving rise to two primary mirror images. The primary images contain again two subimages, and this repeated image structure continues down to arbitrary scale. The complex organization of these repeated images show a fractal structure [1], and the scattering of light rays of spheres is chaotic.

2. EXPERIMENTAL APPARATUS

The experiments were performed in a transparent Hele-Shaw cell, the setup I, consisting of two plain parallel Plexiglas plates separated by a gap (20 x 20 x 0.2 cm^3). The cell contains only air and an amount of commercial dishwashing liquid (V = 30 cm^3). This liquid is manufactured by Bombril, and is used without dilution. The essential surfactant is Linear Alkylbenzene Sulfonate (LAS). The surface tension is γ = 25 dyne/cm, and the density of this detergent is ρ = 0.95 g/cm^3. The configurations were constructed with two sets of reflective surfaces, the setup II is a set of three reflective spheres. The setup III is a set of three reflective spherical shells in order to construct hyperbolic kaleidoscopes. A small sphere was placed between the spherical shells as the real object of a hyperbolic kaleidoscope.

3. HYPERBOLIC KALEIDOSCOPES

The three-dimensional optical billiards consist of cavities between polished, reflective surfaces of spheres. Exploiting multiple reflections of light between the spheres, a variety of fractal optical effects can be observed. A three dimensional optical billiard can be considered as a discrete time dynamical system mapping a vector describing an incident ray at the n*th* reflection from the scatterer to the (n+1)*th* reflection, and these reflections are mirror images. Korsch and Wagner [2] studied the fractal dimension D of mirror images obtained by repeated lateral reduction, which depends on the geometry of the setup. The mirror images are constructed by a repeated lateral reduction by a factor $p = [2(L/r\text{-}1)]^{-1}$. For the case of three spheres the number of mirror images m=2, the fractal dimension can be obtained from the definition of self-similar set as

$$D = \frac{\ln 2}{\ln[2(L/r - 1)]} \qquad (1)$$

in which L is the distance between each sphere and r is the radius of the spheres. The value of D for this system varies from 0 to 1.

The mathematics of kaleidoscopes in N dimensions is the study of these finite groups of orthogonal NxN real matrices that are generated by reflection matrices. In that way, these patterns are related to Möbius transformations such as reflection symmetries in a non-Euclidean space, if we consider that the three mirrored spheres represent a stereographic projection of a three mirror kaleidoscope.

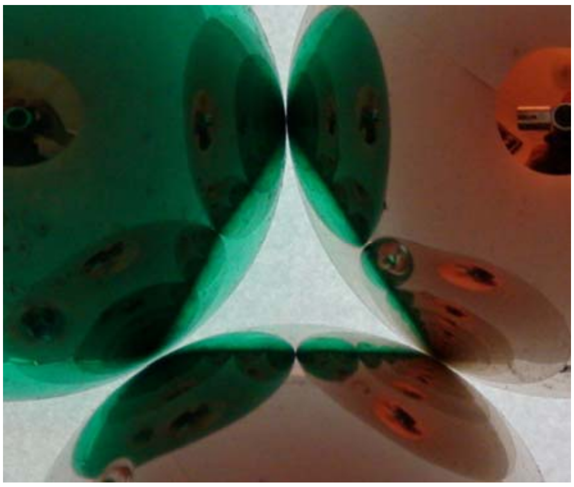

Figure 1 – Example of experimental hyperbolic kaleidoscope with the triangular pattern.

4. REFLECTIONS AND REFRACTIONS

However, hyperbolic kaleidoscopes in liquid systems (Fig. 2) present reflections and refractions of light, different from the pure reflective systems. The question at this point is: is there chaos in a system with refraction and reflection? Due to refraction and reflection at the interfaces, the direction of the rays leaving the liquid bridges can vary greatly for the same incident and only a small positional offset, and consequently there is the possibility of chaotic scattering, but how to characterize this chaotic effect? In the case of a 2D foam, it is not possible to define exits between the interface in a similar way as in the case of spheres, because the transparency of the foam. So we have to explore other ways to characterize this dynamical system.

In systems with escape, the rate of escape of particles from an open system is expressed in terms of the sum of the positive Lyapunov exponents and the Kolmogorov-Sinai entropy on the repeller, can be found in Ref. [3]. Some models of physical systems present chaotic scattering involving both reflection and refraction. Such physical systems represent the propagation of rays passing from one medium to another, such as composite billiards or soft billiard systems [4]. The general case of a geometrical optical system, where a light ray travels through a lattice of circular scatterers in a medium with different values of index of refraction $n=(1-U)^{1/2}$, in which the pure reflective case is when U=1. The signature of chaos is obtained by the computation of the Kolmogorov-Sinai entropy h. The Kolmogorov-Sinai entropy h is the measure of the information loss in N-dimensional phase space. The phase space is divided in cells of size {ε}, and takes the measurements at time increments, {τ}:

$$h = \ln\left[1+\frac{U}{R_l^2}+1.29\left(\frac{U}{R_l^2}\right)^{0.4}\right] \qquad (2)$$

where R_l is the radius of each scatterer. The existence of finite positive values of h indicates the existence of chaotic behavior. Using eq. (2), and R_l =1.0, for the pure reflective case the value of h is 1.19, while in a system with n_g = 1.0 and n_l = 1.33, the value of h is around 1.13, almost at same order of magnitude of the former case.

5. CONCLUSION

In conclusion, We have presented the analogy between chaotic scattering and the effects of light rays in liquid bridges observed in Hele-Shaw cells. We have discussed some aspects of the three spherical shells and compared them to the case of non-linear kaleidoscopes using the Poincaré disk model. This three-mirrored sphere system can be regarded as a chaotic system where a wave or a particle is scattered by a cavity composed by surfaces with negative curvature. In that way, the incoming wave undergoes successive Möbius transformations, such as translations, rotations, inversions, and dilations. The effects of the refraction and reflection of the light rays were studied using some properties of soft billiards. The existence of finite positive values of Kolmogorov-Sinai entropy is an indicative light can be channeled through the network of Plateau borders.

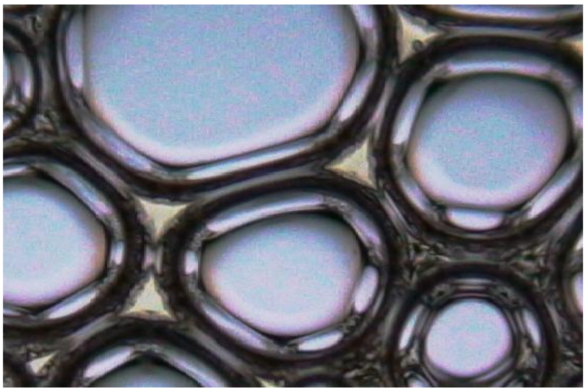

Figure 2 – Example of foam containing multiples hyperbolic kaleidoscopes with the triangular pattern from liquid bridges in a Hele-Shaw cell.

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