USING MATLAB AS A TOOL FOR THE TEACHING OF NONLINEAR SYSTEMS IN ENGINEERING: THE CASE OF THE INVERTED PENDULUM

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Abstract: Matlab is a powerful tool in engineering education and widely used in simulation and modeling of dynamic systems. In the study of nonlinear systems, students use various algorithms and theoretical methods that require a strong mathematical background and analysis of complex graphics. This article elaborates on the use of Matlab in engineering as a way to help the study of dynamical systems using the case of the inverted pendulum.

keywords: engineering education, nonlinear systems, inverted pendulum

1. INTRODUCTION

The control theory is characterized by the involvement of complex calculations for dealing with chaotic systems and unpredictable, and this fact is often responsible for the lack of interest among students in the area. Currently, several strategies are used to assist in the teaching of nonlinear systems, and the problem of inverted pendulum is often cited to illustrate the dynamics of these systems.

The use of computational tools is essential for the simulation of dynamical systems and this article presents the Matlab as a powerful study tool to help teachers in the teaching of non-linear systems. The case of the inverted pendulum is used as a case study because it is a classic problem in the teaching of control [1], and will address the design of PID control parameters for identifying and modeling the system. Besides Matlab, another important program used as tools of virtual education in the control linear and nonlinear is LabVIEW [2].

Matlab is widely used in scientific applications and offers a great robustness in situations that require a lot of calculations. In addition, Matlab has a great flexibility as to combine its robust programming language with its capacity for data acquisition.

Many control experiments are characterized by a high cost in its implementation and use of virtual tools is a great way to see the results of the parameters, serving as an excellent form of education on issues that have great complexity, particularly the systems inverted pendulum.

The problem of inverted pendulum is characterized by being an inherently unstable system and present a complex mathematical representation, which makes a mechanism widely used to represent inherently unstable systems [3].

In literature, different techniques are used to control the inverted pendulum [4], and the choice of PID control is because there is a great approach to theory.

2. DYNAMIC MODEL OF THE SYSTEM OF INVERTED PENDULUM

The first step in modeling the inverted pendulum problem is to obtain its parameters. Figure 01 illustrates the model used in this study to represent the inverted pendulum on a rolling basis.
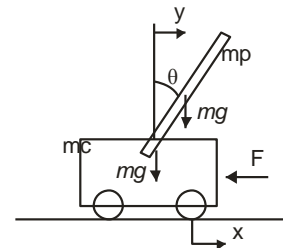

Figure 01 - Parameters of inverted pendulum system

From the parameters obtained based on Lagragian mechanics, have the following dynamic equations [5].

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -\dfrac{((m.l)sen\theta\,.cos\theta\,.\omega)}{det} & 0 & \dfrac{m\,.l\,.b\,.cos\theta}{det} \\ 0 & 0 & 0 & 1 \\ 0 & \dfrac{(m\,.l^2\,.l)m\,.l\,.sen\theta\,.\omega}{det} & 0 & -\dfrac{b\,(m\,.l^2+I)}{det} \end{bmatrix} \quad (1)$$

$$vet = \begin{bmatrix} 0 \\ -\dfrac{m\,.l\,.cos\theta\,.F}{det} + \dfrac{(mc+mp)\,.mp\,.g\,.l\,.sen\theta}{det} \\ 0 \\ \dfrac{(mp\,.l^2+I)}{det}\,.F - \dfrac{(mp\,.l)^2\,.g\,.sen\theta\,.cos\theta}{det} \end{bmatrix} \quad (2)$$

$$\frac{d}{dt}\begin{bmatrix} \theta \\ \omega \\ x \\ v \end{bmatrix} = A\,.\begin{bmatrix} \theta \\ \omega \\ x \\ v \end{bmatrix} + vet \quad (3)$$

Where mp is the mass of the rod, mc the mass of the

car, I the moment of inertia, *l* is half the length of the rod, b the coefficient of viscous force, θ the angle of the pendulum, ω the angle of the pendulum, x position of the car, v is mobile speed and g the acceleration of gravity. The block diagram of the system with the inclusion of the PID control is shown in Figure 02.
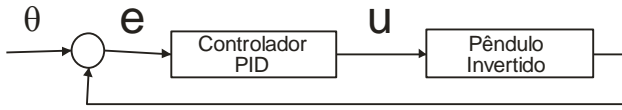

Figure 02 - Block diagram of the system

Where:

$$u = e\,(\theta - \theta d) \qquad (4)$$

The user sets the value of demand θd.

Thus, the PID equation is implemented, and the dynamic equations are generated: system matrix, input vector and the equation of internal dynamics of inverted pendulum with the PID controller, respectively:

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & -\dfrac{((m.l)sen\theta\,.\cos\theta\,.\omega)}{det} & 0 & \dfrac{m.l.b\,.\cos\theta}{det} & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & \dfrac{(m.l^2.I)m.l\,.sen\theta\,.\omega}{det} & 0 & -\dfrac{b\,(m.l^2+I)}{det} & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix} \qquad (5)$$

$$vet = \begin{bmatrix} 0 \\ -\dfrac{m\,.\,l\,.\,\cos\theta\,.\,F}{det} + \dfrac{(mc+mp)\,.\,mp\,.\,g\,.\,l\,.\,sen\theta}{det} \\ 0 \\ \dfrac{(mp\,.\,l^2+I)}{det}\,.\,F - \dfrac{(mp\,.\,l)^2\,.\,g\,.\,sen\theta\,.\,\cos\theta}{det} \\ -\theta \end{bmatrix} \qquad (6)$$

$$\frac{d}{dt}\begin{bmatrix} \theta \\ \omega \\ x \\ v \\ i \end{bmatrix} = A\,.\begin{bmatrix} \theta \\ \omega \\ x \\ v \\ i \end{bmatrix} + vet \qquad (7)$$

3. USING MATLAB

You can see the complexity of the equations that describe the problem of inverted pendulum, however, Matlab has several features that can facilitate the understanding of the system, including allowing the student to observe the effects of changes in system parameters.

Simulink is a tool that comes with Matlab and allows working with several systems, algorithms and numerical methods, since the processing of simple calculations to the deployment of artificial intelligence algorithms.

Besides this, other tools that have great potential in the teaching of control systems are: sisotool and vrpend that enable systems to handle input and single output (SISO) and visualize the action of changing parameters of PID controller in a system of pendulum inverted, respectively.
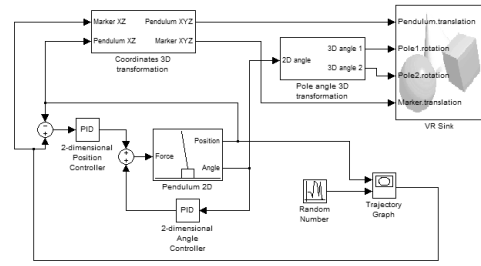

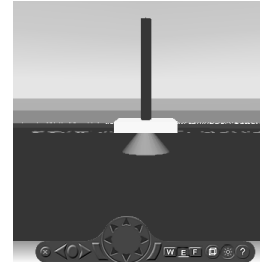**Figura 03- PID Controller of Inverted Pendulum in Simulink**

Figure 04- Model of the virtual inverted pendulum

4. CONCLUSION

This article discussed the potential of Matlab for modeling and simulation of dynamical systems to develop and stimulate the teaching of nonlinear systems in engineering courses, addressing the problem of inverted pendulum, demonstrating their complex mathematical representation, but in a simple simulated through Matlab, giving the student a better understanding of the system. Thus, the simulation shows to be a good tool to motivate the learning of nonlinear systems.

References

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