DYNAMICS DAYS SOUTH AMERICA 2010
International Conference on Chaos and Nonlinear Dynamics

INPE – National Institute for Space Research
São José dos Campos – SP – Brazil – July 26-30, 2010

STABILIZING EQUILIBRIUM BY LINEAR FEEDBACK FOR CONTROLLING CHAOS IN CHEN SYSTEM

Costa, Viviana Angélica 1, González, Graciela Adriana 2,3

1 Departamento de Ciencias Básicas, Facultad de Ingeniería (UNLP), La Plata, Argentina, vacosta@ing.unlp.edu.ar
2 Departamento de Matemática, Facultad de Ingeniería (UBA), Ciudad de Buenos Aires, Argentina, ggonzal@fi.uba.ar
3 Consejo Nacional de Investigaciones Científicas y Técnicas (CONICET), Argentina

Abstract: Stabilization of a chaotic system in one of its unstable equilibrium point by applying small perturbations is studied. Feedback control, Lyapunov stability and ergodicity are combined to improve performance.

Keywords: Control of Chaos and applications, Feedback control, Lyapunov stability, Ergodicity, Observability

1. INTRODUCTION

Although several authors resorted to the well developed machinery of Modern Control Theory to solve control chaos problems, it is known that they often did not take full account of the special aspects of chaotic motion neither of achieving improved performance by applying only small perturbations on some accessible system parameter. For example, with the objective of suppressing chaotic behaviour, [1], [2], [3] and [4] explored the linear feedback control while [5] dealt with a controller based on a PI regulator control. However, these approaches only take care of local stabilization.

This work concentrates on Chen system but its ideas are straightful applicable to other chaotic systems like Lorenz, Chua, Rossler etc. Our purpose is to stabilize the system in one of its (unstable) equilibrium by using linear feedback control. Improvement of system performance is dealt with by exploiting the ergodicity of the original dynamics and using Lyapunov stability results for control design ([6]).

2. PURPOSE AND METHOD

Let us assume that we have a chaotic dynamical system determined by: $\dot{X} = F(X)$ and that E is one of its unstable equilibrium point embedded in its chaotic attractor.

Our aim is controlling chaos by applying feedback control. For this, we will construct a linear feedback control, depending on a gain parameter k, to stabilize the system in the equilibrium E. The following facts will be of relevance: i) due to ergodicity, for the free system, (almost) every trajectory initialized in the strange attractor reaches a chosen E neighbourhood, $B(E,\delta)$; ii) fixed k such that locally asymptotic convergence is guaranteed, the corresponding region of attraction Ω_k of the controlled system may be estimated. We look for a control law such that $B(E,\delta) \subset \Omega_k$ and which remains bounded by a desired fixed bound. The control strategy consists of making the free system run till it reaches the neighbourhood $B(E,\delta)$. Once the trajectory reaches it, the control is activated. Note that once the trajectory enters Ω_k, it will never leave it, and so the feedback control will remain under the desired bound.

Chen dynamical system is determined by:

$$\begin{aligned} \dot{x}_1 &= a(x_2 - x_1) \\ \dot{x}_2 &= (c-a)x_1 + cx_2 - x_1x_3 \\ \dot{x}_3 &= -bx_3 + x_1x_2 \end{aligned} \qquad (1)$$

being x_1, x_2 and x_3, the state variables and a, b, and c, positive real constants. For $a=35$, $b=3$ and $c=28$, it has a chaotic attractor and its unstable equilibrium points are: $E_1 = (0,0,0)$, $E_2 = (\sqrt{63},\sqrt{63},21)$ and $E_3 = (-\sqrt{63},-\sqrt{63},21)$.

Let $E=E_2$. As in [1], we assume that all the state variables are observable and that each system equation may be affected by an additive control, i.e.

$$\begin{aligned} \dot{x}_1 &= a(x_2 - x_1) + u_1 \\ \dot{x}_2 &= (c-a)x_1 + cx_2 - x_1x_3 + u_2 \\ \dot{x}_3 &= -bx_3 + x_1x_2 + u_3 \end{aligned} \qquad (2)$$

Put: $u_1 = -k(x_1 - \sqrt{63})$, $u_2 = -k(x_2 - \sqrt{63})$ and $u_3 = -k(x_3 - 21)$.

By mean of Lyapunov function construction ([6]), we estimate the region of attraction of system (2) as Ω_k, an ellipsoid centered in E.

To state the strategy we must choose k that verifies:

$$(x_1,x_2,x_3)(t) \in \Omega_k \Rightarrow \|(u_1,u_2,u_3)(t)\|_2 \le U \qquad (3)$$

being U the desired fixed control bound.

As

$$\|(u_1,u_2,u_3)(t)\|_2 \le k\|(x_1,x_2,x_3)(t) - E\|_2 \le k \cdot s_M(k) \qquad (4)$$

where $s_M(k)$ is the length of the ellipsoid mayor axis, (3) is verified by k such that

$$k \cdot s_M(k) \le U\,. \qquad (5)$$

Due to numerical experience, we claim that every sphere $B(E,\delta)$ with $\delta > \Delta = 0.82$ will be visited by any trajectory of system (1) at any time. On the other hand, convergence is guaranteed if $B(E,\delta) \subset \Omega_k$ which is valid if $\delta \le s_m(k)$, the length of the ellipsoid minor axis. So, we need to choose δ such that

$$\Delta \le \delta \le s_m(k)\,. \qquad (6)$$

Hence, let us fix k and δ according to (5) and (6). The algorithm consists of two stages. In the first, the system runs free (control no activated). Let t_f the first time at which its trajectory reaches $B(E,\delta)$. The second stage begins at time $t=t_f$ at which the control is activated. Note that differs from the OGY method ([7]) in that the control is kept activated for all $t>t_f$.

3. RESULTS AND EXTENSION

We emphasize that under the stated requirements, trajectories convergence and U-bounded controls are formally proved, for (almost) initial conditions in the strange attractor. Note that smaller δ is, the smaller k may be chosen and hence, the control effort may be reduced. However, a great reduction on δ will probably translate into a dramatic increase of the waiting time (first stage time). Besides, in general, a too small k delays too much convergence in the second stage. Therefore, parameters values will be chosen from a compromise between control effort, and total convergence time.

We claim the conservative feature of our estimation by mean of an example. According to theoretical results, for the choice $\delta = 2$ and $k=30$, convergence is guaranteed and $\|(u_1(t),u_2(t),u_3(t))\| \le 185.7$ is predicted. Lack of space does not allow us to show graphically the control performance but it is worth to comment that taking *(-20,-20,30)* as initial condition, it is seen that the control is around 58.

We wonder if this methodology applies when not all states are observable. Suppose that we have system (1) but only the output y is at our disposal, being $y=CX$. The objective is to make the output converge to $y_E=CE$. We implement a two stages-algorithm as in the complete observability case, save that: *i)* the criteria for control activation is $|y - y_E| < \delta$, *ii)* the condition for control activation must be verified at every time. This is because in this case, we do not have estimated region of attraction.

In spite of this limitation, we obtain experimental evidence of the algorithm success. Let us show it through the next example. The output of system (2) is determined by $C=(0,1,0)$. The linear feedback control is: $u_1=0$, $u_2=-k(y-y_E)$ and $u_3=0$ and the initial state is *(5,-15,40)*. The parameters values are chosen: $\delta = 2$ and $k=10$ so we predict $\|(u_1(t),u_2(t),u_3(t))\| \le 20$. Convergence is verified through simulation. In Figure 1 the corresponding output and controls are displayed.

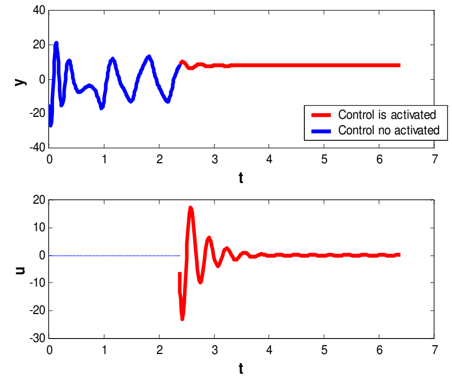

Figure 1 –Output and control

4. DISCUSSION AND CONCLUSIONS

Chen system has been stabilized while considering fundamentals on controlling chaos so making progress with respect to some previous works ([1], [2], [3], or [4]).

As in OGY method, the on-line implementation only requires data on system linearization. But, for control design, extra system information is needed (for estimation of region of attraction) to choose control parameters which guarantee convergence. Then, fixed these values, not only convergence but also no "kicking"' of the trajectory out of the neighbourhood of the equilibrium point is assured. On the other hand, simulated results show us that our theoretical estimation is too conservative. This drawback will be object of future investigation as well as the extension of these ideas to other plausible situations like stabilization of periodic orbits, controllability restrictions, system affected by noise, etc,

We have also considered the case of uncompleted observability as in [5]. Bounded controls (under the desired bound) have been achieved by our approach. The theoretical proof of convergence or any other property of the control strategy promises interesting research.

References

[1] M. T. Yassen, "Chaos control of Chen chaotic dynamical system", Chaos, Solitions & Fractals 15, pp. 271-283, 2003.

[2] X. Wang., L. Tian and L. Yu, "Linear Feedback Controlling and Synchronization of the Chen's Chaotic System", International Journal of Nonlinear Science, vol.2, no.1, pp.43-49, 2006.

[3] L. Cui-Xiang. and T. Jia-Shi., "Equilibrium points and bifurcation control of a chaotic system", Chinese Physics B, Vol.17, No.1, 135-38, 2008.

[4] T. Plienpanich, Niamsup and Y. Lenbury, "Controllability and stability of the perturbed Chen chaotic dynamical system", Applied Mathematics and Computation 171, pp. 927-947, 2005.

[5] G. P. Jiang, and S.P. Wang, "Stabilizing Unstable Equlibrium of Chaotic Systems using PI regulator", Proceedings of IEEE International Symposium on Circuits and Systems, III, 64-67, 2003.

[6] H. Khalil, Nonlinear Systems, Prentice Hall; 2001.

[7] E. Ott, C. Grebogi and J. Yorke, Controlling Chaos, Phys. Rev. Lett. Vol.64, pp. 1196, 1990.