DYNAMICS DAYS SOUTH AMERICA 2010
International Conference on Chaos and Nonlinear Dynamics

INPE – National Institute for Space Research
São José dos Campos – SP – Brazil – July 26-30, 2010

DECENTRALIZED OBSERVER FOR A CLASS OF NONLINEAR NORM BOUNDED SYSTEMS

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keywords: Applications of Nonlinear Sciences.

INTRODUCTION

The problem of state estimation of nonlinear systems is a very important issue in practical applications where the system state to be controlled is unavailable or its measurement is very expensive as in power systems, design of spacecraft, vehicle control, etc [3]. This paper develops a methodology for designing a decentralized observer for a class norm bounded nonlinear systems. For this, we use the second method of Lyapunov, LMIs (*Linear Matrix Inequalities*) and a procedure we propose.

DESIGN OF DECENTRALIZED OBSERVER

Consider a nonlinear system composed by N subsystems

$$\begin{aligned} \dot{x}_i &= A_i x_i + B_i u_i + h_i(x,t) \quad i = 1,\ldots,N \\ y_i &= C_i x_i \end{aligned} \tag{1}$$

where x_i, u_i, A_i, B_i and C_i have appropriate dimensions. Consider the pairs (A_i, C_i) are observable. The nonlinear terms $h_i(t,x)$ are norm bounded [1]:

$$h_i^T h_i \leq \alpha_i^2 x_i^T H_i^T H_i x_i \tag{2}$$

where the scalar α_i and the matrix H_i of appropriate dimension are known. The state representation of the overall system is given by

$$\begin{aligned} \dot{x} &= A_D x + B_D u + h \\ y &= C_D x \end{aligned} \tag{3}$$

where $A_D = diag(A_i)$, $B_D = diag(B_i)$, $C_D = diag(C_i)$ and $h^T = [h_1^T, \ldots, h_N^T]^T$. We propose the following procedure to design a decentralized observer via LMIs for system (3). The observer has the following dynamics:

$$\begin{aligned} \dot{\widehat{x}} &= A_D \widehat{x} + B_D u + L_D(y - \widehat{y}) \\ \widehat{y} &= C_D \widehat{x} \end{aligned} \tag{4}$$

where $L_D = diag(L_i)$ is the decentralized observer gain, $\widehat{x}$ is the state estimate and $\widehat{y}$ is the output estimate. For the state estimation error $e = x - \widehat{x}$ the dynamics is:

$$\dot{e} = (A_D - L_D C_D)e + h \tag{5}$$

For (5) to be asymptotically stable, a Lyapunov function $V(e) = e^T P_D e > 0$ should exist such that $\dot{V}(e) < 0$. Using this fact we obtain:

$$\begin{aligned} \dot{V}(e) = e^T(&A_D^T P_D + P_D A_D - C_D^T L_D^T P_D \\ &- P_D L_D C_D)e + e^T P_D h + h^T P_D e < 0 \end{aligned} \tag{6}$$

We use the following Lemma to obtain a quadratic form for $e^T P_D h + h^T P_D e$:

Lemma 1 *For any matrices (or vectors) X and Y with appropriate dimensions, we have the following inequality:*

$$X^T Y + Y^T X \leq X^T J X + Y^T J^{-1} Y \tag{7}$$

where $J = J^T > 0$.

Using the Lemma 1 and (2) we obtain

$$e^T P_D h + h^T P_D e \leq x^T H_D H_D x + e^T P_D P_D e \tag{8}$$

Replacing (8) in (6), making the change of variable $P_D L_D = T_D$ proposed in [2], writing an augmented system for e and x, using the Schur complement and limiting the observation gain L_D through restrictions on T_D and P_D [**?**], we construct the following convex optimization problem in terms of LMIs:

$$\begin{aligned} &\min \quad k_{T_D} + k_{P_D} \\ &\text{s.a} \\ &P_D > 0, \\ &\begin{bmatrix} W_D & 0 & P_D \\ 0 & H_D^T H_D & 0 \\ P_D & 0 & -I \end{bmatrix} < 0 \end{aligned}$$

$$\left[\begin{array}{cc} -k_{T_D}I & T_D^T \\ T_D & -I \end{array}\right] < 0$$

$$\left[\begin{array}{cc} k_{P_D}I & I \\ I & P_D \end{array}\right] > 0 \qquad (9)$$

where $W_D = A_D^T P_D + P_D A_D - C_D^T T_D^T - T_D C_D$, $H_D = diag(\alpha_i H_i)$ and $L_D = P_D^{-1} T_D$. To illustrate the proposed method we use the following example

Example 1

$$\dot{x} = \left[\begin{array}{cccc} -0.4534 & 0.6946 & 0 & 0 \\ 0.4449 & -0.3787 & 0 & 0 \\ 0 & 0 & -0.2052 & 0.5226 \\ 0 & 0 & 0.9568 & -0.1199 \end{array}\right] x +$$

$$\left[\begin{array}{cc} 0.1730 & 0 \\ 0.9797 & 0 \\ 0 & 0.2714 \\ 0 & 0.2523 \end{array}\right] u + \left[\begin{array}{c} h_1 \\ h_2 \end{array}\right]$$

$$y = \left[\begin{array}{cccc} 0.8939 & 0.1991 & 0 & 0 \\ 0 & 0 & 0.2987 & 0.6614 \end{array}\right] x \qquad (10)$$

where

$$h_1 = \cos(x_{11}) \left[\begin{array}{cc} 0.0284 & 0.0065 \\ 0.0469 & 0.0988 \end{array}\right] x_1,$$

$$h_2 = \cos(x_{21}) \left[\begin{array}{cc} 0.0950 & 0.0607 \\ 0.0231 & 0.0486 \end{array}\right] x_2. \qquad (11)$$

and $x = (x_1^T, x_2^T)^T$, $x_1 = (x_{11}, x_{12})^T$ *and* $x_2 = (x_{21}, x_{22})^T$ *with* $x(0) = (1, -2, 1, -2)^T$.

Using the proposed method we find

$$L_D = \left[\begin{array}{cc} 126.6368 & 0 \\ 116.4463 & 0 \\ 0 & 145.6937 \\ 0 & 114.8357 \end{array}\right] \qquad (12)$$

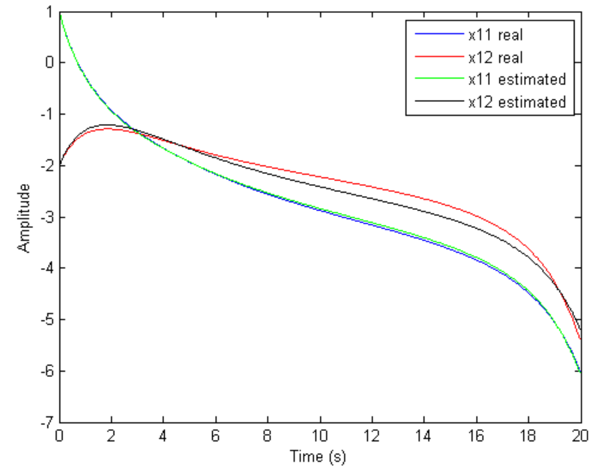

Figure 1 – State x_{11} and x_{12} real and estimated.

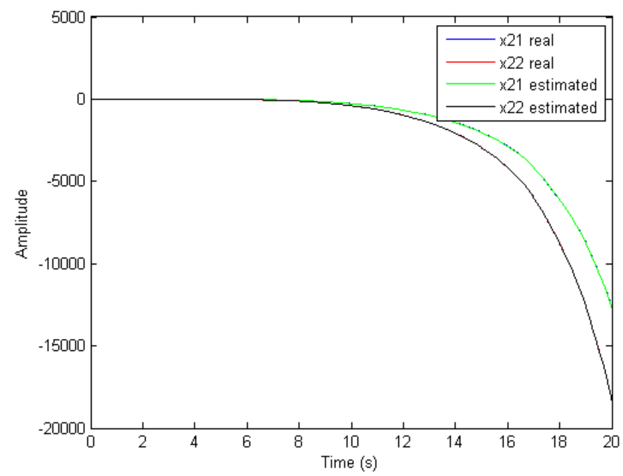

Figure 2 – State x_{21} and x_{22} real and estimated.

CONCLUSIONS

In this paper we proposed a new methodology for the design of a decentralized observer for a class of norm bounded nonlinear systems that use Lyapunov theory and a alternative procedure to $S - procedure$ [6] to deal with the nonlinearities. From the numerical example we notice that the estimated state follows the actual state with high quality. Thus we can say that this work reached its goal.

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