DYNAMICS DAYS SOUTH AMERICA 2010
International Conference on Chaos and Nonlinear Dynamics

INPE – National Institute for Space Research
São José dos Campos – SP – Brazil – July 26-30, 2010

Time Dependent Shocks and Forcing Effects in the Velocity Probability Distribution Functions of Burgers Turbulence

Luca Moriconi [1] , *Antônio Francisco Neto* [2]

1Universidade Federal do Rio de Janeiro, Rio de Janeiro, RJ, moriconi@if.ufrj.br
2 Universidade Federal de São João Del-Rei, Ouro Branco, MG, antfrannet@gmail.com

Keywords: Astrodynamics; Fluid Dynamics; Plasma and Turbulence; Burger Turbulence; Shocks.

In the Burgers model, the dynamics of the one dimensional velocity field, $u \equiv u(x, t)$, is described by:

$$\partial_t u + u\, \partial_x u = \nu\, \partial_x^2\, u + f, \qquad (1)$$

where ν is the kinematical viscosity and $f \equiv f(x,t)$ is the external force (usually, Gaussian and white noise in time). It is worth mentioning that Burgers turbulence [1] has been studied not only as a toy model which reproduces general turbulence properties of three-dimensional flows, such as intermittency. The multidimensional version of Burgers turbulence, for instance, plays an important role in the description of several realistic problems. Some interesting applications are related to nonlinear acoustics, cosmology, critical interface growth, traffic flow dynamics, etc. See, for instance, Ref. [2] for a comprehensive review. Among the statistical properties of interest is the determination of the behavior of the velocity-difference probability distribution function (pdf) $\rho(z)$, where

$$z = u(x + \zeta , t) - u(x - \zeta ,t). \qquad (2)$$

A remarkable feature of $\rho(z)$ is that it can be regarded as a "probe" of intermittency, as indicated by non-gaussian fluctuations of local galilean-invariant observables in the high Reynolds number regime. The determination of the velocity-difference pdf tail asymptotics ($|z| >> 1$)

$$\rho(z) \sim 1 / |z|^{\alpha}, \qquad (3)$$

has been a matter of intense debate leading to some controversy on the value of the exponent α. Regarding the left pdf tail, in one direction, a Fokker-Planck approach to the computation of velocity-difference pdfs, with closure given by an operator product expansion treatment of the dissipative anomaly was put forward by Polyakov [3]. This method provides a fine description of the pdf's right tail, and yields a power law form for the left tail with $5/2 \le \alpha \le 3$ [4]. Extensive numerical simulations performed by Gotoh and Kraichnan [5] indicate that $\alpha = 3$. At variance with such findings, an analytical study based on the velocity field profiles in space-time neighborhoods of shocks, the so-called preshock events, gives $\alpha = 7/2$ [6]. In another direction, Boldyrev, Linde and Polyakov, in Ref. [7], have suggested that the left tail exponent is not universal, departing from $\alpha = 3$ if flow realizations fail to satisfy a strong form of galilean invariance [7], which holds, by definition, if usual galilean invariance is observed in the bulk, regardless the boundary conditions at infinity. In rephrased form, the whole point of Ref. [7] is that finite-size effects which break strong galilean invariance would lock larger fluctuations of shock jumps and negative velocity derivatives, reducing intermittency. In [8], an analytical approach suitable to analyze the left tail of the velocity-difference pdf of Burgers turbulence driven by external stochastic forcing (Gaussian and white noise in time) was conceived. In [8], support was given to the conjecture that the left tail exponent is $\alpha = 3$ when the strong form of galilean invariance is fulfilled in accordance with the previous analytical/numerical work of Ref. [7]. It was shown analytically, that the aymptotic form of $\rho(z)$ is given by

$$\rho(z) \sim a / |z|^3, \qquad (4)$$

where a is a prefactor that can be approximately computed in principle. Those issues were addressed with the help of the Martin-Siggia-Rose (MSR) field theory approach to classical mechanics [9]. This approach, in the same spirit of Feynman's path integral description of quantum electrodynamics [10], allows us to rewrite the classical transition probability as a path integral with the weight given by the exponential of i ($i^2=-1$) times an appropriate action S. This formulation provides an interesting stage for the use of standard non-perturbative quantum field theory (QFT) techniques, such as the instanton calculus [10], which among all possible trajectories contributing to the path integral weight selects the ones corresponding to extrema of the action. In this way, a functional Taylor expansion of the action S around solutions of the classical equations of motion (this terminology is borrowed from QFT), obtained by fixing the first functional variation of the action to zero, i.e. $\delta S = 0$, can be advanced (within the Feynman's space-time approach to QFT, $\delta S = 0$, gives the classical equations of motion, i.e., the Euler-Lagrange equations). We note that although the time independent shock profiles considered in [8] can be used to find the correct power law profile of ρ(z), they are not very realistic. It has been assumed that shocks are time independent and totaly dissipated at the origin during the time interval L/U, where U is an estimate of the shock velocity jump and L is the mean shock-to-shock distance. In this work, as a refinement of the strategy of Ref. [8], we investigate how velocity-difference pdf are corrected from the consideration of more realistic time-dependent shock profiles (still analytically tractable), necessarily subject to external forcing fluctuations. We find, furthermore, under general circumstances, and up to second order in the stochastic force f, that the asymptotic scaling form of the velocity-difference pdf is stable, i.e. $\alpha = 3$, although its prefactor will depend on the particular structure of the ensemble of evolving shocks.

References

[1] J.M. Burgers, "A mathematical model illustrating the theory of turbulence", Advances in Applied Mechanics Vol. 1 pp. 171-199, 1948.

[2] J. Bec and K. Khanin, "Burgers Turbulence", Physics Reports Vol. 447, Nos. 1-2, pp. 1-66, August 2007.

[3] A.M. Polyakov, "Turbulence without pressure", Physical Review E Vol. 52, No. 6, pp. 6183-6188, December 1995.

[4] S. Boldyrev, "Velocity-difference probability density functions for Burgers turbulence", Physical Review E Vol. 55, No. 6, pp. 6907-6910, June 1997.

[5] T. Gotoh and R.H. Kraichnan, "Steady-state Burgers turbulence with large-scale forcing", Physics of Fluids Vol. 10, No. 11, pp. 2859-2866, November 1998.

[6] W. E, K. Khanin, A. Mazel and Y. Sinai, "Probability Distribution Functions for the Random Forced Burgers Equation", Physical Review Letters Vol. 78, No. 10, pp. 1904-1907, March 1997.

[7] S. Boldyrev, T. Linde and A. Polyakov, "Velocity and velocity-difference distributions in Burgers turbulence", Physical Review Letters Vol. 93, No. 18, pp. 184503-1-184503-4, October 2004.

[8] L. Moriconi, "Instanton Theory of Burgers Shocks and Intermittency", Physical Review E Vol. 79, No. 4, pp. 046324-1-046324-7, April 2009.

[9] P.C. Martin, E.D. Siggia and H.A. Rose, "Statistical Dynamics of Classical Systems", Physical Review A Vol. 8, No. 1, pp. 423-437, July 1973.

[10] J. Zinn-Justin, "Quantum Field Theory and Critical Phenomena", Claredon Press – Oxford.