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PARTIAL ANNIHILATION OF COUNTER-PROPAGATING PULSES

Orazio Descalzi[1], *Jaime Cisternas*[1], *Daniel Escaff*[1], *Helmut R. Brand*[2]

[1]Complex Systems Group, Facultad de Ingeniería y Cs. Aplicadas, Universidad de los Andes, Santiago, Chile, odescalzi@miuandes.cl
[2]Department of Physics, University of Bayreuth, 95440 Bayreuth, Germany.

Abstract: Partial annihilation of two counter-propagating dissipative solitons has been widely observed in different experimental contexts, from hydrodynamics to chemical reactions. Based on our results for coupled complex cubic-quintic Ginzburg Landau equations as well as for the FitzHugh-Nagumo equation we conjecture that noise induces partial annihilation of colliding dissipative solitons in many systems.

keywords: Formation and Dynamics of Patterns, Stochastic Dynamics, Bifurcation theory and analysis

1. INTRODUCTION

Partial annihilation of two counter-propagating pulses, with only one pulse surviving the collision, has been observed in dissipative-excitable chemical and dispersive-dissipative hydrodynamical systems. In the former case, the catalytic oxidation of carbon monoxide on a Pt(110) single-crystal surface, collisions waves formed by elongated dark regions due to enhanced oxygen coverages lead mostly to partial annihilation [1]. In the latter case, convection in binary mixtures, collision of counter-propagating convective pulses lead to bound states and partial annihilation, depending on the approach velocity [2, 3].

Our conjecture is that the noise (always present in real systems) induces partial annihilation. During the collisions fluctuations break the parity symmetry allowing the system to choose one of the two pulses.

In order to test our conjecture we investigate coupled complex Ginzburg-Landau equations (stochastic version), which arise as prototype equations near the weakly hysteretic onset of an oscillatory instability to traveling waves. These equations are suitable to study collisions of dispersive-dissipative solitons. We also study our conjecture using the stochastic version of the FitzHugh-Nagumo equation, a prototype equation for dissipative-excitable systems.

2. METHODS

We investigated two coupled complex subcritical cubic-quintic Ginzburg-Landau equations for counter-propagating waves with noise [4]:

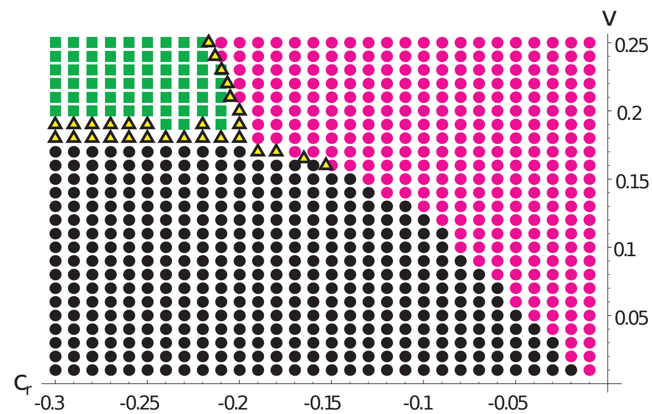

Figure 1 – Phase diagram in the plane approach velocity v versus strength of cubic cross-coupling of counter-propagating waves, c_r, for stabilizing (negative) values of c_r, and for $\eta = 0$. Marked in green (grey solid squares) is the annihilation of two pulses. Marked as black solid circles are bound states of pulses and in pink (grey solid circles) the interpenetration of two pulses. As open triangles we have depicted the outcome of partial annihilation. (Fig. 1 of [4])

$$
\begin{aligned}
\partial_t A - v\partial_x A &= \mu A + (\beta_r + i\beta_i)|A|^2 A \\
&+ (\gamma_r + i\gamma_i)|A|^4 A + (c_r + ic_i)|B|^2 A \\
&+ (D_r + iD_i)\partial_{xx} A + \eta\, \xi_A, \qquad (1)\\
\partial_t B + v\partial_x B &= \mu B + (\beta_r + i\beta_i)|B|^2 B \\
&+ (\gamma_r + i\gamma_i)|B|^4 B + (c_r + ic_i)|A|^2 B \\
&+ (D_r + iD_i)\partial_{xx} B + \eta\, \xi_B, \qquad (2)
\end{aligned}
$$

where $A(x,t)$ and $B(x,t)$ are complex fields, η the noise strength, and we have discarded quintic cross-coupling terms for simplicity. A and B are slowly varying envelopes and the stochastic forces $\xi_A(x,t)$ and $\xi_B(x,t)$ denote white noise with the properties $\langle \xi_{A,B} \rangle = 0$, $\langle \xi_A(x,t)\, \xi_A(x',t') \rangle = \langle \xi_B(x,t)\, \xi_B(x',t') \rangle = \langle \xi_A(x,t)\, \xi_B(x',t') \rangle = 0$ and $\langle \xi_A(x,t)\, \xi_A^*(x',t') \rangle = \langle \xi_B(x,t)\, \xi_B^*(x',t') \rangle = 2\delta\,(x-x')\,\delta\,(t-t')$. Our numerical simulations were done for $\mu = -0.112$, $\beta_r = 1$, $\beta_i = 0.2$, $\gamma_r = -1$, $\gamma_i = 0.15$, $D_r = 1$, $D_i = -0.1$ and $c_i = 0$, while varying the approach velocity v and the

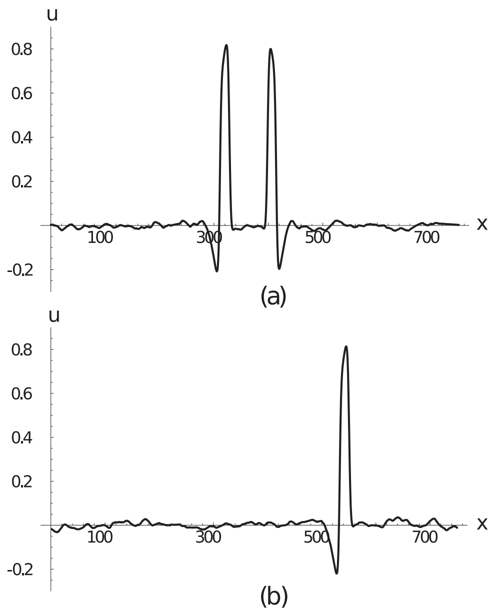

Figure 2 – The process of partial annihilation for the stochastic FitzHugh-Nagumo equation: a) before the interaction two excitable waves approach each other, b) partial annihilation after a transient. Noise strength $\eta = 0.0095$, $a = -0.044$. (Fig. 4 of [4])

strength of the cubic cross-coupling, c_r. Here we focus on stabilizing (negative) values of c_r.

In addition we studied the stochastic one-dimensional FitzHugh-Nagumo model

$$\partial_t u = -v - u(u-1)(u-a) + \partial_{xx} u + \eta\, \xi_u, \quad (3)$$
$$\partial_t v = \epsilon(u - bv) + \delta\, \partial_{xx} v + \eta\, \xi_v, \quad (4)$$

where the variables u and v represent the activator and inhibitor, respectively. The parameter ϵ, the ratio of the time-scales of u and v is taken as 0.015, δ as 1.25 and b as 3.5. The stochastic forces $\xi_u(x,t)$ and $\xi_v(x,t)$ denote white noise with the properties $\langle \xi_{u,v} \rangle = 0$, $\langle \xi_u(x,t)\ \xi_v(x',t') \rangle = 0$, $\langle \xi_u(x,t)\ \xi_u(x',t') \rangle = \langle \xi_v(x,t)\ \xi_v(x',t') \rangle = \delta\,(x-x')\,\delta\,(t-t')$.

We used periodic boundary conditions and the Heun method applied to a spatially discretized version of the stochastic equations. Typically we used $N = 2 \cdot 10^3$ points and a grid spacing of $dx = 0.4$ corresponding to a box size $L = Ndx = 800$. The typical time step used was $dt = 0.05$.

3. RESULTS

For the two coupled complex subcritical cubic-quintic Ginzburg-Landau equations and for $\eta = 0$, that is, only considering the numerical error (which cannot be avoided), varying v and c_r, we obtained essentially three different results after collision: interpenetration, bound states of pulses, and annihilation. Only in a very narrow region, in the vicinity of the boundaries between the three above mentioned results, partial annihilation was observable. We conclude that the small amount of numerical noise causes the partial annihilation confined in a thin region (depicted with open triangles in Fig.1). In order to test our conjecture: *noise induces partial annihilation of colliding dissipative solitons*, we varied the strength noise η over six orders of magnitude along the diagonal $c_r = v$. For $\eta = 10^{-6}$ we got similar results to those obtained in Fig.1. As the noise strength is further increased to $\eta = 10^{-5}$, the range of $c_r = v$ over which partial annihilation occurs increased by a factor 3 and the outcomes of collisions, for fixed values of parameters, are not unique. There is a coexistence of annihilation and partial annihilation, and a coexistence of interpenetration and partial annihilation. For $\eta = 10^{-2}$ the only coexistence which still remains is annihilation with partial annihilation, and the range where partial annihilation is observed increased by a factor 60 respect to $\eta = 10^{-5}$. For the FitzHugh-Nagumo equation and for $\eta = 0$, that is, only considering the numerical error, collisions for counter-propagating pulses lead to interpenetration for $a < -0.0433$, while for $a > -0.0433$ both pulses died after collision (annihilation). For small noise strength and close to $a = -0.0433$ we obtained that after a transient the parity symmetry becomes broken and only one excitable wave survived (see Fig. 2).

4. CONCLUSIONS

The conjecture: *noise induces partial annihilation of colliding dissipative solitons* has been tested in two systems of different nature, namely, the coupled stochastic complex cubic-quintic Ginzburg-Landau equations, and the stochastic FitzHugh-Nagumo model. The former is suitable to study collisions of dispersive-dissipative solitons and the latter is a prototype equation for dissipative-excitable systems. In both systems, we have demonstrated that noise plays a crucial role for the range of existence of partial annihilation and thus can be used to control the outcomes of collisions.

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