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RATCHET CURRENT IN THE TOKAMAP WITH MIXED PHASE SPACE

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Based on the tokamap, we investigate characteristic features of magnetic field lines and zeroth-order guiding-center particle motion in the whole body of a magnetically confined plasma, e.g. a tokamak plasma. We show that the tokamap exhibits a poloidal transport that can be regarded as a Hamiltonian ratchet. The observed mean velocity is in agreement with the value predicted by the so-called sum rule for Hamiltonian ratchet currents [1, 2] .

Recent progress in non-equilibrium statistical mechanics show a new mechanism for the generation of currents in stochastic systems. The ratchet effect, i.e. the generation of transport with a preferential direction [3], in systems with mixed phase space is now widely discussed [1, 2, 4]. Stochastic ratchets provide a new understanding of transport phenomena far from thermal equilibrium [5]. The generic model of a stochastic ratchet consists of a sawtooth potential with a noisy driving. The potential, while being periodic, must be spatially asymmetric, and the noise should result from non-equilibrium situations. When the noisy driving is replaced by a purely deterministic one, one applies the notion deterministic ratchet [6, 7]. Purely Hamiltonian dynamics with both regular and chaotic phase space structures or with complete chaos may cause Hamiltonian ratchets.

Ratchet currents are found in many different systems, from nanotechnology [8] to plasma physics. In the latter field, the ponderomotive ratchet in a uniform magnetic field [9] and the impurity pinch in tokamak plasmas [10, 11] were discussed. A ratchet-type average velocity was demonstrated for test particles moving *radially* in a stochastic potential when the magnetic field is space-dependent [10]. This constitutes a possible explanation for impurity behavior in tokamak plasmas [11]. The control of impurities in magnetically confined plasmas is a very important issue for the development of fusion reactors. Experimental results show that there is accumulation of these particles in the central region of the plasma, which appears to be a directed transport (a pinch) rather than a diffusive one. In this work we shall investigate whether the tokamap also allows for a directed chaotic transport in the poloidal direction.

The tokamap, and also its variants and generalizations [13], can be assigned to a Hamiltonian H. The kinetic energy has a generalized form compared to the standard one. Checking the tokamap against other (simple) models, e.g. the kicked rotator and its variants like the standard map [14], the potential energy is significantly different. The amplitude of the periodic potential is momentum-dependent. That behavior is typical for plasma maps modeling toroidal systems. Thus, we have a new type of potential which has not yet been investigated in view of directed chaotic transport via a Hamiltonian ratchet process. Directed chaotic poloidal transport, if it occurs, may have a important consequences for the interpretation of poloidal rotation in ergodized plasmas.

The phase portrait of the tokamap has already been fully depicted in [12]. In our work, we set the stochastic parameter to a value for which the phase space is mixed: large island chains coexist with a chaotic sea and these are bounded, from below and above, by KAM surfaces. These different invariant sets render different velocity distributions of ensemble of particles.

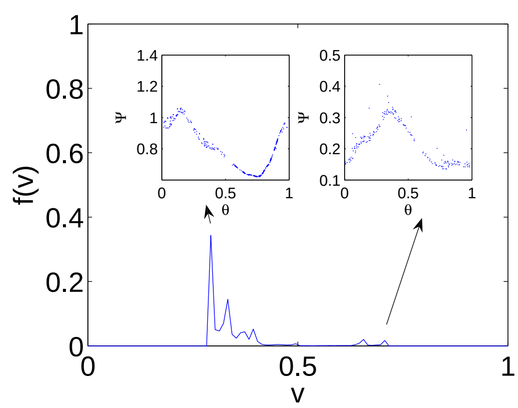

Figure 1 – Distribution of time-averaged velocities in the chaotic sea.

We focus here on the ensembles of particles inside the chaotic sea. Fig. 1 shows the velocity distribution, $f(v)$, after a period of time $t \sim 10^3$. The peaks in $f(v)$ are due to stickiness of the chaotic trajectories to island chains. There are many peaks because different groups of particles follow different island chains. As time evolves, each trajectory eventually overcomes the stickiness and leaves, moving around the chaotic sea, until it is trapped by another island. Since the system is bounded, trajectories of the chaotic sea are destined to wander between the different island chains of the layer. After an amount of time t_f (for the forward tokamap $t_f \sim 10^5$), most particles orbits will have visited all islands and, thus, the distribution $f(v)$ will resume to a narrow peak around the velocity $\overline{v_\theta}$. This velocity is, naturally, independent of initial conditions.

A Hamiltonian ratchet current exists when transport is ballistic, while spreading is not. There should be a locking of the average velocity to a specific, non-zero value that does not depend on initial conditions. As this is the case for the ensemble of chaotic trajectories, we argue here that the poloidal motion of particles in the tokamap can be regarded as a ratchet-like transport. The phenomenon is a direct consequence of the intrinsic asymmetry of the phase space. The tokamap, thus, can be thought as an application of Hamiltonian ratchets.

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