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PERIODIC, CHAOTIC, AND HYPERCHAOTIC STATES IN PARAMETER-SPACES OF A FOUR-DIMENSIONAL CHUA'S SYSTEM

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Motivated by our recent work [1], where we reported a new four-dimensional Chua's system with cubic nonlinearity, we carry out a more detailed numerical study in that system, where we calculate the Lyapunov exponent spectrum. To characterize the hyperchaotic states, the system needs to have more than one positive Lyapunov exponent. In this sense, a four-dimensional system presents hyperchaotic states if it will have two positive Lyapunov exponents [2].

In Eqs. (1), we show the four-dimensional Chua's system model with cubic nonlinearity [1],

$$\begin{aligned} \dot{x} &= \alpha\left(y - ax^3 - (1+c)x\right), \\ \dot{y} &= x - y + z, \\ \dot{z} &= -\beta y - \gamma z + w, \\ \dot{w} &= -sx + yz, \end{aligned} \tag{1}$$

where we added a fourth variable w in the standard three-dimension Chua's system with cubic nonlinearity [3]. In Eqs. (1), α, β, γ, and s are parameters.

The aim of our work is to construct parameter-spaces with the first, and the second largest Lyapunov exponent to show periodic, chaotic and hyperchaotic behaviors of Eqs. (1). With this purpose, we numerically solved Eqs. (1) with the fourth-order Runge-Kutta method, for each pair of parameters (γ, s), with other pair (α, β) kept constant.

In Fig. 1, we show the parameter-space for the first largest Lyapunov exponent, where we codify its values in colors, as shown in the colors palette in right side of figure. Figure 2 shows the parameter-space for the second largest Lyapunov exponent, with the colors code in right side. In Fig. 3, we show the attractors for the parameter values located in the four points of Fig. 1, marked with Pa, Pb, Pc, and Pd. Those points are inside of periodic structures: black regions in Fig. 1, or white regions in Fig. 2.

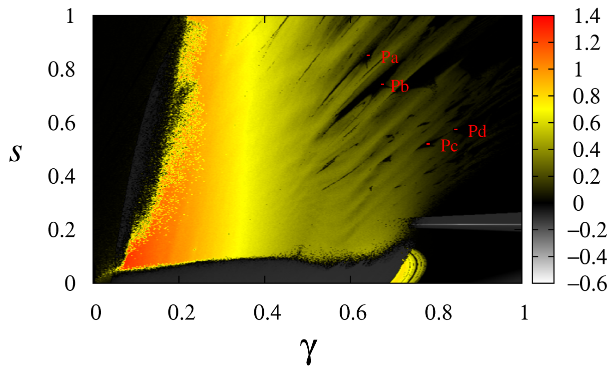

Figure 1 – Parameter space for the first Lyapunov exponent with $0 < \gamma < 1$ and $0 < s < 1$, $a = 0.03$, $c = -1.2$, $\alpha = 30.0$, and $\beta = 50.0$. The points Pa, Pb, Pc, and Pd indicate the parameter values of the attractors shown in Fig. 3.

In hyperchaotic systems, for example, Eqs. (1), with four-dimension, we have four Lyapunov exponents, one exponent related with each dimension. For hyperchaotic behaviors in that system, we have two positive exponent, one zero, and one negative. For chaotic behaviors, we have only one positive exponent, one zero, and two negatives. For periodic behaviors, we have one exponent zero, and the last three negatives [2].

In the description above, we can observe in Figs. 1, and 2, the existence of periodic structures embedded in chaotic and hyperchaotic regions. That feature can be corroborated in Fig. 3, where we show some periodic attractors of marked structures in Figs. 1, and 2. For hyperchaotic regions, we can observe in Fig. 2 reddish and yellowish regions, indicate that we have a second positive Lyapunov exponent in those regions. For chaotic regions in Fig. 2, we have black regions, or the reddish and yellowish regions in Fig. 1.

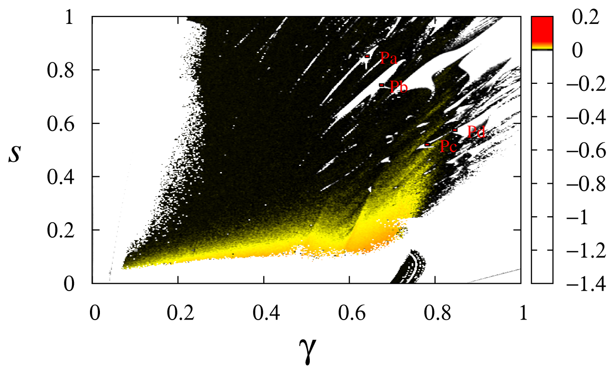

Figure 2 – Parameter space for the second Lyapunov exponent with $0 < \gamma < 1$ and $0 < s < 1$, $a = 0.03$, $c = -1.2$, $\alpha = 30.0$ and $\beta = 50.0$. The points Pa, Pb, Pc, and Pd indicate the parameter values of the attractors shown in Fig. 3.

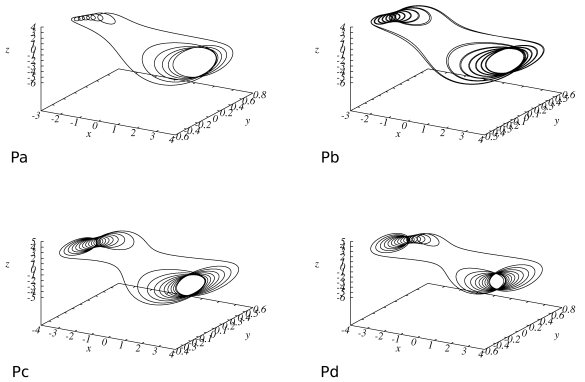

Figure 3 – Periodic attractors for the four marked points in black regions in Fig. 1, or white regions in Fig. 2.

In this work we numerically investigated a hyperchaotic Chua's system with cubic nonlinearity. With the first and the second largest Lyapunov exponent, we constructed two parameter-spaces, and observed the existence of periodic structures, as well as, we identified the chaotic and the hyperchaotic regions of the system for the pair of parameters (γ, s).

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