DYNAMICS DAYS SOUTH AMERICA 2010
International Conference on Chaos and Nonlinear Dynamics

INPE – National Institute for Space Research
São José dos Campos – SP – Brazil – July 26-30, 2010

THE SKEW TENT MAP ENTROPY PRODUCTION

Fabiano Alan Serafim Ferrari[1] *and Sandro Ely de Souza Pinto*[2]

[1]Departament of Physics, Universidade Estadual de Ponta Grossa, Ponta Grossa, Brasil, fabianosferrari@gmail.com
[2]Departament of Physics, Universidade Estadual de Ponta Grossa, Ponta Grossa, Brasil, desouzapinto@pq.cnpq.br

Abstract: The tent map shows important dynamical properties. Through the coupled map lattices, one can measure the entropy, and consequently, the uncertainness present in this system. The study of tent map is relevant, it describes a triangular wave very useful in electronic systems.

keywords: Entropy,probability,maps

1. INTRODUCTION

Using entropy, one can measure the mean value of uncertainness associated to a random variable. The entropy measure is expressed in bits and provides the number of the bits necessary to describe a random variable. The entropy is directly linked to the probability density, and proportional to the uniformity of probability distribution associated to the random variable [1].

The information theory is applied to study of the dynamical systems, in cases where the dynamic is non-linear. A dynamical system consists of a deterministic rule and a set of the possible states, that characterize the system in a given instant. A dynamical system is described by dependent and independent variables, which vary with the time. The space formed by possible states is called phase space [2].

The study of dynamical systems may be realized through analysis of maps where a later state can be obtained from a former state. The tent map is important for its application specially in eletronics, due to its behavior similar to a trigonal wave [3].

Through a coupled map lattices one may quantify the information transmitted from a site to another, and that causes a change in the probability distribution. In case of uncoupled tent map, the distribution is uniform [4][5].

2. PURPOSE

To modify the p parameter of tent map, study the change in probability distribution and evaluate the entropy.

3. METHODS

The tent map will be redefined as [6]:

$$S(x) = \begin{cases} x/p & \text{if} \quad 0 \leq x \leq p \\ (1-x)/(1-p) & \text{if} \quad p < x \leq 1 \end{cases} \tag{1}$$

where $S(x) = x_{n+1}$ and $x = x_n$, for the unidimensional case.

The type of coupled maps lattice used:

$$x_{n+1}^{i} = (1-\epsilon)S(x_n^i + \frac{\epsilon}{2}[S(x_n^{i-1}) + S(x_n^{i+1})] \tag{2}$$

where the periodic boundary conditions is used

The figure 1 shows that when the p parameter value changes, the dynamics of the system change too, and also shows up a displacement of the discontinuity.

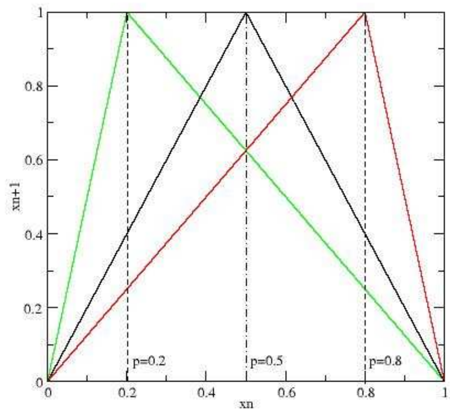

Figure 1 – The consequence of the p parameter change

4. RESULTS AND DISCUSSION

For p parameter values defined in interval $[0.5; 0.9]$ in small lattices, as the coupled increases, the probability distribution piles up near the p parameter, as shown in figure 2.

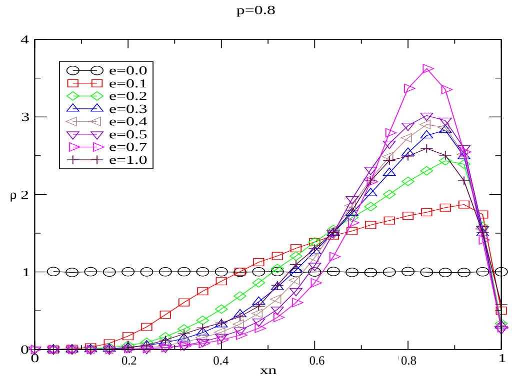

Figure 2 – Probability distribution for a single site of the lattice

Knowing the probability distribution it is possible to determine the entropy associated to each lattice element, that in this case is the same for all sites:

$$H(x) = -\sum p(x) \log p(x) \qquad (3)$$

where the sum is over all acessible states.

The figure 3 shows the entropy for this case:

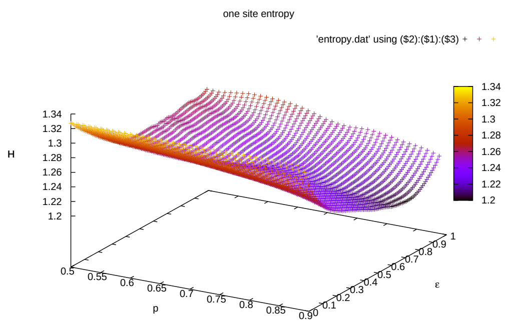

Figure 3 – Entropy by one site of lattice

5. CONCLUSIONS

The tent map behavior for $p \pm \delta$ displacements is symmetrical. Due to this fact, it was expected that probability distribution would behave the same way. But this did not happen. As the points of time series spread near the p, for the interval $[0.5; 0.9]$ and strong coupling, a decrease in system uncertainness occurs.

ACKNOWLEDGMENTS

I wish to acknowledge with many thanks, Fundação Araucária for financial support, and Rodrigo Pereira and Romeu Szmoski for their help.

References

[1] J.A. Cover, T. M.; Thomas. *Elements of Information Theory*. John Wiley and Sonsm Inc, 2006.

[2] G. Brin, M.; Stuck. *Introduction to Dynamical Systems*. University Cambridge Press, 2006.

[3] S.; Nakajima K. Tanaka, H.; Sato. Integrated circuits of map chaos generators. *Analog Integrated Circuits and Signal Processing*, 25(7):329–335, Sep 2000.

[4] H.; Mackey M.C. Hauptmann, C.; Touchette. Information capacity and pattern formation in a tent map network featuring statistical periodicity. *Physical Review E*, 67(5):026217, 2003.

[5] T. Schreiber. Measuring information transfer. *Physical Review Letters*, 85(4):461, Jan 2000.

[6] A.; Badea B.; Frunzete M. Luca, A.; Vlad. A study on statistical independence in the tent map. *Signals, Circuits and Systems, 2009. ISSCS 2009. International Symposium on*, 2009.