DYNAMICS DAYS SOUTH AMERICA 2010
International Conference on Chaos and Nonlinear Dynamics

INPE – National Institute for Space Research
São José dos Campos – SP – Brazil – July 26-30, 2010

LAGRANGIAN COHERENT STRUCTURES IN A NONLINEAR MHD DYNAMO

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keywords: Applications of Nonlinear Sciences; Fluid Dynamics, Plasma and Turbulence

The analysis of passive scalars is a powerful way to trace the turbulence in hydrodynamical and magnetohydrodynamical flows. When probing turbulent transport of passive scalars, both Eulerian and Lagrangian tools can be employed. In the Eulerian approach, for a given velocity field, one can solve an advection–diffusion equation for the passive scalar concentration, from which turbulent transport coefficients can be computed. Also, instantaneous snapshots of tracer and velocity fields can be used to extract coherent structures such as eddies, convection cells, filaments and other structures that may act as transport barriers. Alternatively, in the Lagrangian approach the dynamics of fluids is studied by following the trajectories of a large number of fluid elements or tracer particles. The Lagrangian description has been gaining increasing attention in the past decade and it has been suggested that Lagrangian tools are more appropriate to identify tracer patterns than their Eulerian counterparts, since they don't rely on single instantaneous snapshots of the velocity field. By following particle trajectories, the spatiotemporal evolution of the fluid is taken into account to extract transport properties, including the detection of structures that are smaller than the resolution of the velocity field.

This work presents the detection of transport barriers called Lagrangian coherent structures in direct numerical simulations of a 3–D magnetohydrodynamic (MHD) dynamo. An MHD dynamo is the process responsible for the growth of an initially weak magnetic field in a magnetized fluid due to the conversion of kinetic energy to magnetic energy. We focus on the change in transport and mixing properties of the flow when the system undergoes a transition whereby a large–scale spatially coherent magnetic field loses its stability due to change in the magnetic diffusivity η. The transition results in a magnetic field with complex spatiotemporal dynamics.

We use the Pencil code (`http://www.nordita.org/software/pencil-code`) to solve the continuity equation for the gas density ρ, the momentum equation for the velocity field $\mathbf{u}$ and the induction equation for the magnetic field $\mathbf{B}$ for an isothermal gas in a box with sides $L = 2\pi$ and periodic boundary conditions. An external driver given by a periodic helical wave provides an energy injection at the wave number $k = 5$. Following [1], a numerical resolution of 64^3 mesh points is chosen. We choose the magnetic diffusivity η as the control parameter and fix the kinematic viscosity $\nu = 0.005$, which in the absence of magnetic fields corresponds to a spatiotemporally chaotic flow with the Reynolds number $Re \approx 100$.

The magnetic field is initialized with a small random perturbation and depending on the value of η, it can decay to zero or grow until it saturates in a nonlinear dynamo regime. For $\eta = 0.01$ the average magnetic energy is close to the kinetic energy and there is a coherent large–scale B_y component accompanied by small–scale turbulent fluctuations. Figure 1 shows the patterns in B_y for four different times. The large–scale field is robust and propagates in the vertical direction. For $\eta = 0.05$ the coherent state becomes unstable and the magnetic field displays an intermittent switching between coherent and incoherent large-scale structures (Fig. 2).

The magnetic field affects the velocity field due to the action on the Lorentz force in the momentum equation. The details of what happens with the velocity field and passive scalars is best described in terms of Lagrangian coherent structures, which are computed using the finite–time Lyapunov exponents (FTLEs). The FTLEs quantify the average rate of attraction and repulsion of two nearby particles passively advected by the flow, being a measure of Lagrangian chaos. It was shown by Haller [2] that the FTLE field can also be used to identify repelling and attracting material lines that are time–dependent analogous to the stable and unstable manifolds of saddle points in dynamical systems theory. These material lines form transport barriers in flows with chaotic streamlines and are called Lagrangian coherent structures (LCS). For a time-dependent velocity field, regions of maximum material stretching generate local maximizing curves (*ridges*) in the FTLE field. Thus, repelling material lines (finite-time stable manifolds) produce ridges

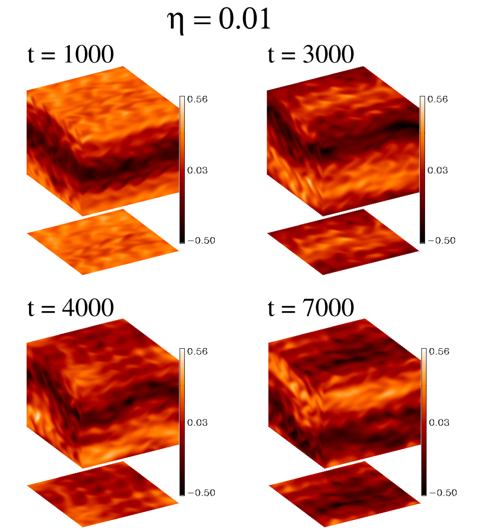

Figure 1 – Magnetic field component B_y at $\eta = 0.01$.

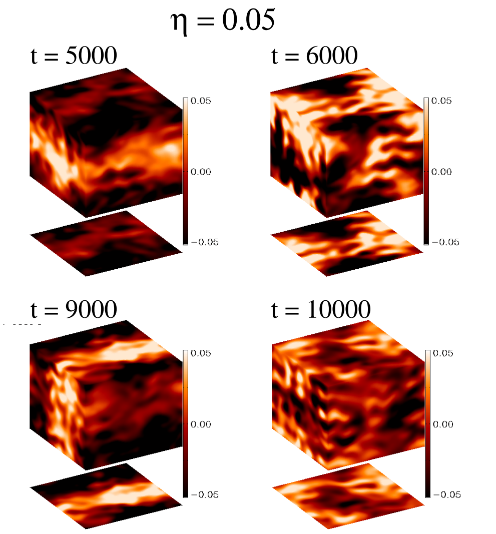

Figure 2 – Magnetic field component B_y at $\eta = 0.05$.

in the maximum FTLE field in the forward-time system and attracting material lines (finite-time unstable manifolds) produce ridges in the backward-time system [2].

Figure 3 displays the forward–time (green) and backward–time (red) maximum FTLE fields at $x = 0$ for $\eta = 0.01$ (upper panel) and $\eta = 0.05$ (lower panel). Figure 3 was computed using 254×254 fiducial particles uniformly distributed on the plane $x = 0$. Bright colors correspond to large values of the maximum FTLE and dark regions to low values. The ridges in the FTLE fields can be clearly seen and represent the Lagrangian coherent structures, which form the Lagrangian "skeleton" of turbulence. The higher complexity in the distribution of material lines in the flow at $\eta = 0.05$ indicates that transport of passive scalars is highly enhanced due to the frequent crossings of attracting and repelling lines. The enhanced chaoticity at $\eta = 0.05$ leads to stronger line stretching and field amplification, and the "competition" between this effect and destruction of magnetic flux due to high magnetic diffusion seems to be the cause of the intermittent regime.

The Lagrangian coherent structures are a powerful tool to trace the preferred pathways for passive scalars, detect transport barriers and explore the chaotic mixing properties of the flow. We believe LCS can be used in conjunction with classical tools to provide a better description of MHD turbulence in space and fusion plasmas.

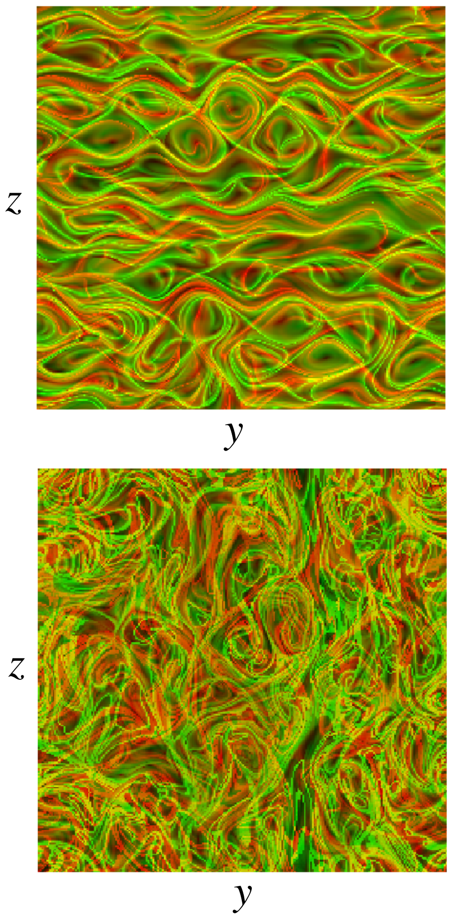

Figure 3 – Lagrangian Coherent Structures at $\eta = 0.01$ (upper panel) and $\eta = 0.05$ (lower panel).

References

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