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CONTROLLING THE INTERACTION BETWEEN SUGARCANE BORER AND IT'S PARASITOID

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The increase in world demand for ethanol will bring an increase of the sugarcane planted in Brazil.

One of challenges of the improvements in the farming and harvesting of cane is the biological pest control. It is known that for each 1% of plant infestation by pests the industries lose 0.2% of the ethanol production, that is, in average 25 liters per ha. It means that a good strategy of biological pest control can increase the ethanol production.

Biological control is the use of living organisms to suppress pest populations, making them less abundant and thus less damaging than they would otherwise be [1]. Pests are species that interfere with human activity or cause injury, loss, or irritation to a crop, stored product, animal, or people. One of the main goals of the pest control is to maintain the density of the pest population in an equilibrium level below economic damages. Natural enemies play an important role in limiting potential pest populations.

In order for biological control to succeed, the dynamics of the pest and its enemy populations have to be understood. Mathematical modeling is an important tool used in studying agricultural problems. Mathematical modeling applied to the problems of biological pest control allows a qualitative and quantitative evaluation of the impact between the pest and its natural enemy populations. In this paper we consider the host–parasitoid relations between the sugarcane borer *(Diatraea saccharalis)* and its parasitoid. The nonlinear feedback control problem for nonlinear systems has been formulated in order to obtain the optimal pest control strategy only through the introduction of natural enemies. Numerical simulations for possible scenarios of biological pest control based on the Lotka–Volterra models are provided to show the effectiveness of this method.

In this paper, a nonlinear control strategy is proposed to indicate how the natural enemies should be introduced in the environment.

We hope to formulate the pest control strategy of the sugarcane borer through natural enemies' (parasitoid) introduction. This control moves the system to the steady state in that the pest density is stabilized without causing economic damages, and that the natural enemies' population is stabilized in a level enough to control the pests.

The ecosystem sugarcane borer – its parasitoid with biological control is described by two following differential equations [2], [3]:

$$\begin{aligned} \frac{dx_1}{dt} &= x_1(r_1 - \alpha_{11}x_1 - \alpha_{12}x_2) \\ \frac{dx_2}{dt} &= x_2(-r_2 + \alpha_{21}x_1) + U \end{aligned} \tag{1}$$

where x_1 and x_2 are respectively sugarcane borer and parasitoid densities.

The goal of the pest control strategy maintains the pest population at level $x_1^* = x_d$ by control *U*, where x_d is a pest population density below economic injury level. In this paper the SDRE method was used to solve the formulated biological pest control problem.

According Mracek and Cloutier [4] the SDRE approach for obtaining a suboptimal solution of the above formulated problem is:

1) use direct parameterization (factorization) to bring the nonlinear dynamics to the state-dependent coefficient (SDC) form

$$\dot{x} = A(x)x + B(x)u \tag{2}$$

where

$$f(x) = A(x)x \; ; \tag{3}$$

2) solve the state-dependent Riccati equation

$$\begin{aligned} &P(x)A(x) + A^T(x)P(x) - \\ &- P(x)B(x)R^{-1}(x)B^T(x)P(x) + Q(x) = 0 \end{aligned} \tag{4}$$

to obtain matrix-valued function *P(x)* which is positive definite for all *x*;

construct the nonlinear feedback controller

$$u = -R^{-1}B^T P(x)\,x \tag{5}$$

Under the assumption $f(0) = 0$ and $f \in C^1(R^n)$, a continuous nonlinear matrix-valued function $A(x)$ always exists such that (3) is satisfied. It is obvious, that $A: R^n \to R^{nxn}$ is found by mathematical factorization and is nonunique for $n > 1$.

Let x_1^* and x_2^* are desired levels of the pest and parasitod populations, respectively. The main objective of the pest control to maintain $x_1^* \le x_d$, were x_d is the economic injury level.

The equilibrium points x_1^* and x_2^* must satisfy the system (1):

$$\begin{aligned} & x_1^*(r_1 - \alpha_{11} x_1^* - \alpha_{12} x_2^*) = 0 \\ & x_2^*(-r_2 + \alpha_{21} x_1^*) + u^* = 0 \end{aligned} \qquad (6)$$

From the first equation of the system (6) we obtain the parasitoid population that can maintain the pest population at level $x_1^* \le x_d$:

$$x_2^* = (r_1 - \alpha_{11} x_1^*)/\alpha_{12} \qquad (7)$$

From the second equation of the system (6) we obtain the control

$$u^* = x_2^*(r_2 - \alpha_{21} x_1^*) \qquad (8)$$

Defining the new variables as:

$$y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} x_1 - x_1^* \\ x_2 - x_2^* \end{bmatrix}, \ u = U - u^* \qquad (9)$$

we get the following error system:

$$\dot{y} = Ay + Bu \qquad (10)$$

were matrices A e B are

$$A = \begin{bmatrix} -\alpha_{11}(y_1 + x_1^*) - a_{12} y_2 & -\alpha_{12} x_1^* \\ \alpha_{21} x_2^* & \alpha_{21} y_1 \end{bmatrix},$$

$$B = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \qquad (11)$$

The feedback control u can be calculated by SDRE method in form (5). The control strategy of the introduction of parasitoid species is

$$U = u^* + u \qquad (12)$$

The dynamics of the system (1), controlled by nonlinear control strategy (12), is presented in Figure 1.

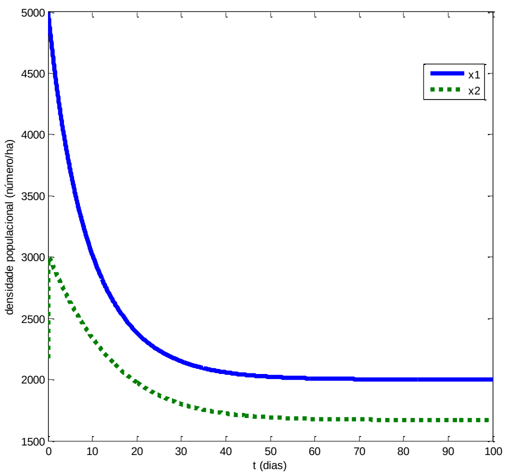

Figure 1 – Dynamics of the controlled system with nonlinear control

It is interesting to compare this result with the linear control strategy, proposed in [4]. The Figure 2 shows the dynamics of the system (1), controlled by linear control strategy.

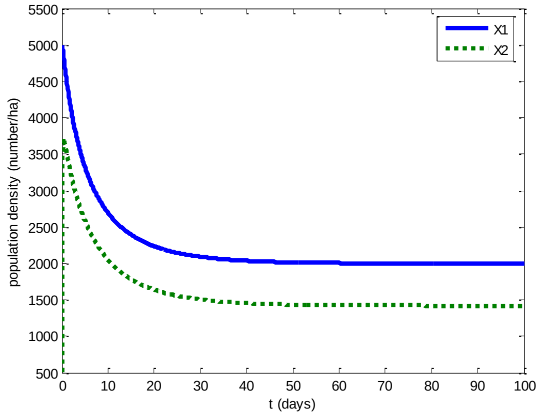

Figure 2 – Dynamics of the controlled system with linear control

One can see that both, linear and nonlinear, controls make the same general result: the system (1) becomes stable. But the linear control strategy is more expensive. According to this control more than 3700 parasitoids/ha have to be introduced in initial time. For the nonlinear control this quantity is 2700 parasitoids/ha.

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