INPE – National Institute for Space Research
São José dos Campos – SP – Brazil – July 26-30, 2010

INFLUENCE OF DAMPING IN THE LONG TERM BEHAVIOR OF A CARDANICALLY SUSPENDED BODY

Danny Hernán Z. Carrera, Hans Ingo Weber

Pontifical Catholic University of Rio de Janeiro, Rio de Janeiro, Brazil, hernanz@aluno.puc-rio.br
Pontifical Catholic University of Rio de Janeiro, Rio de Janeiro, Brazil, Hans@puc-rio.br

Keywords: Nonlinear Dynamics, Limits of Stability, Damping, Orientation in the space.

INTRODUCTION

The contribution of this work besides showing the effect of friction and asymmetry of inertia on a cardanically suspended body is relevant for the analysis of tumbling bodies in space. Curiously, if the initial energy is enough to start the tumbling, the movement does not damp out and remain inside a basin of attraction, but continues changing basin of attraction regardless to the kinetic energy becoming lesser due to friction. Newton-Euler laws were used to obtain the equations of movement, with a procedure of normalization for to reduce one parameter in the equation and a dimensionless model of a gyroscope is obtained. This work uses tools of nonlinear dynamics and chaos theory to characterize the rotational motion.

INFLUENCE OF DAMPING

The present work focuses on the influence of friction in the dynamics of the Gyroscope, composed by two gimbals and one non axi-symmetric rotor, studies the long term behavior, following the loss of kinetic energy due to dissipation. To describe the dynamics of the system sequential rotations with cardanic angles were used, each elementary rotation corresponds to the displacement of one component. Only the rotation dynamics of the body is considered, since its center is fixed by the suspension. To define the sequential rotations it is necessary to work with 4 reference frames (RF), one RF in the space (the RF (F) is fixed) and the others attached to the components of the gyroscope (in movement).

$$\underset{x,y,z}{F} \xrightarrow{\alpha_x} \underset{x,y',z'}{Q} \xrightarrow{\beta_y} \underset{x'',y',z''}{R} \xrightarrow{\gamma_{z''}} \underset{x''',y''',z''}{S}$$

Total kinetic energy is the addition of the energies of each body. The specific problem studied consists in: the rotor which is initially turning with constant angular momentum, suddenly an external force produces an impact on the gyroscope at one of its axles, x or y, of RF (R). This impact generates an instantaneous change in the angular momentum; with this impact the nutation motion is started. If the impact is strong enough the body starts to tumble, alternating in a random order the basin of attraction.

An instant initial before the impact, the rotor has constant angular momentum $\mathbf{H}_G$ in direction of the symmetry axle z_0 of the rotor in a RF (x_0, y_0, z_0). With the impact it changes in module and direction to a new angular momentum $\mathbf{h}_{G_0}$: it now has the direction of axle z of the RF (x, y, z) formerly defined as (F) but the rotor keeps its position in space.

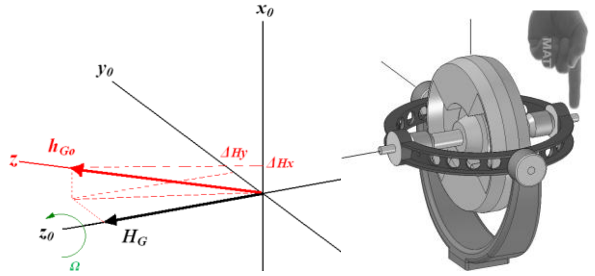

Figure 1. Diagram of the angular momentum during the impact

Equations of motion were obtained using Newton-Euler laws. The three bodies of the Gyroscope are separated, and analyzed. The forces are due to the bearings between the components. Right away, the internal forces that act between the bodies are action and reaction, and the differential equations that describe the behavior of the Gyroscope are obtained. To better analyze the results, and also to diminish one parameter in the motion equations, it is advisable to count on dimensionless motion equations: therefore the initial value (just after the impact) of the angular momentum $h_G = I_3\, v$ is considered for the normalization. The time will be changed for a dimensionless value $\tau = vt$, where v represents an angular speed (instantaneously after the impact). The temporal derivatives will be changed to:

$$v = h_G / I_3 \qquad \dot{\alpha} = v\alpha' \qquad \dot{\beta} = v\beta' \qquad \dot{\gamma} = v\gamma'$$

In this work it is assumed the hypothesis of viscous friction in the bearings of the Gyroscope. All the moments of inertia of the components of the Gyroscope were obtained with a CAD software. Inertial configurations: external gimbal: $0{,}258 = I_x/I_3$, internal gimbal: $0{,}116 = I_p/I_3$ and

$0{,}058 = I_d/I_3$, rotor: $0{,}775 = I_1/I_3$ and $0{,}489 = I_2/I_3$ ($I_3 > I_1 > I_2$).

Bifurcation diagrams are generated using the friction coefficient as control parameter, which is taken equal in all the bearings of the Gyroscope, as $k_{AF} = k_{AB} = k_{BC} = k$.

Figure (2) presents the dissipation of kinetic energy during the motion, for each value of coefficient k varying in a range. Two regions are observed, a region of almost linear variation where the energy is affected in homogeneous form, another region where the variation of the energy is not very intense but is very heterogeneous, k varies from *0,0085* to *0,0200*. In the figure we see that lesser dissipation of kinetic energy with greater friction coefficient is possible. This occurs when the motion follows a trajectory where more energy goes to the gimbal that has fewer bearings. The behavior of α confirms this fact: in the second region angle α is more scatter, since in this region the external gimbal presents greater amplitudes and the internal gimbal is quieter.

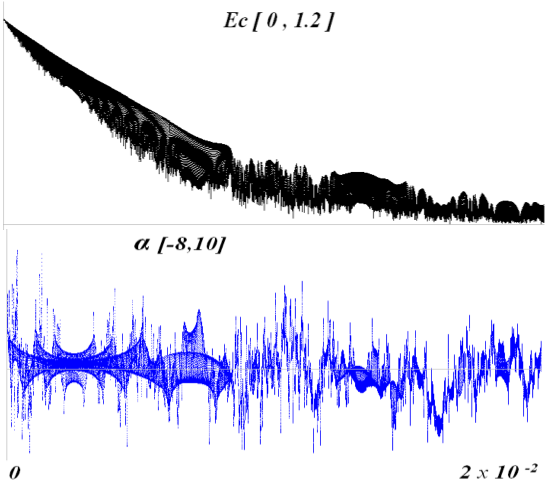

Figure 2. Bifurcation Diagram for k and Energy.

In the next step a simulation is shown eliminating the friction in the connections between the two gimbals to observe the influence on the dynamics. The other values are $k = 0{,}008$ and the initial condition $\alpha_0 = 0{,}35$.

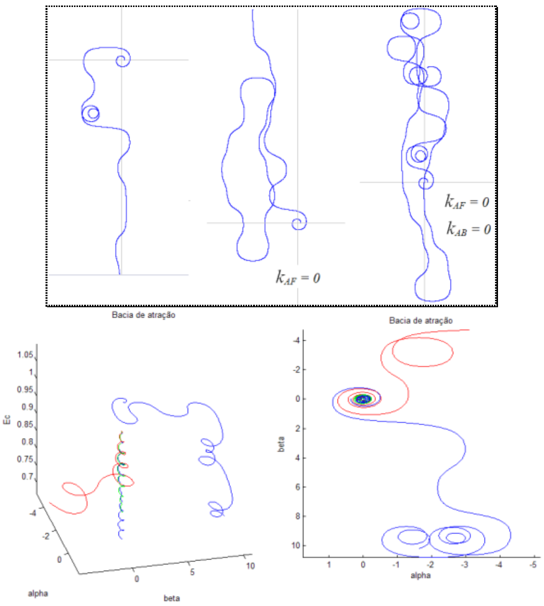

Figure 3. Orbit α vs β and Basins of attraction.

In Figure (3) the trajectory will be increasing in longitude (β direction) at the same time it will form more basins of attraction. When eliminating two friction elements the motion in β increases, and the motion in α remains oscillating.

In the study of the basins of attraction [4], the minimum energy for the basin change is *1,0025* ($\alpha_0 = 0{,}0545$) and was numerically calculated, but now, this minimum energy is responsible only for the first change, since the change of basin continues occurring even if the energy level is lesser that the initial minimum energy necessary for the change of basin of attraction. The numerical value of the initial minimum energy necessary for the change of basin of attraction is not very influenced by the friction, for example, in case that $k=0$ the energy is $Ec_0 = 1{,}0023$ ($\alpha_0 = 0{,}05238$), if $k = 0{,}008$ the energy is $Ec_0 = 1{,}003$ ($\alpha_0 = 0{,}0595$), and if $k = 0{,}02$ the energy is $Ec_0 = 1{,}006$ ($\alpha_0 = 0{,}085$), for higher values of k the motion is damped quickly.

CONCLUSION

The internal gimbal has a more intense motion when adding the effect of friction in the Gyroscope dynamics, which is attributed to its lesser inertia. For small values of friction, the rotor continues to have the greater speed in the motion, but for bigger values of k, the gimbals can reach greater speeds during the movement.

There exists a minimum value of the kinetic energy necessary for the change in basin of attraction, but after the first change, the motion will continue changing basins of attraction even if the energy it is lesser than the initial minimum necessary energy, until the gyroscope is motionless. The friction causes a minimal influence in value of the necessary initial energy for the change of basin of attraction.

In general, we can modify the behavior of the Gyroscope, manipulating the friction between their components. Friction could also be substituted by torque and this a good way to control the movement.

REFERENCES

[1] Magnus, K., Der Kreisel, Industrie Druck, Göttingen, 1965.

[2] Schaub, H., Junkins, J. L., “Analytical Mechanis of space systems”, AIAA Education Series, 2003.

[3] Weber, Hans I., “Large Oscillations of Non-AxiSymmetric Rotation Rigid Body in Space”, Proc. of the Workshop on Nonlinear Phenomena: Modeling and their Applications, Ed. J.M. Balthazar et al., UNESP, Brazil. 2005,

[4] Carrera, Danny. H. Z. & Weber, Hans I., “Influence of the Cardanic Suspension in the Dynamic of a Gyroscope”, 20th International Congress of Mechanical Engineering, COBEM 2009.

[5] Savi, Marcelo A., “Dinâmica Não-linear e Caos”, E-Papers Serviços Editoriais, Rio de Janeiro, Brazil, 2006.

[6] Rimrott, F.P.J., “Introductory Attitude Dynamics, Mechanical Engineering Series”, 1989.