STUDY OF THE ATTRACTOR STRUCTURE IN AN AGENT-BASED SOCIOLOGICAL MODEL

André M. Timpanaro[1] *and* <u>*Carmen P. C. Prado*</u>[2]

[1]Universidade de São Paulo, São Paulo, Brazil, timpa@if.usp.br
[2]Universidade de São Paulo, São Paulo, Brazil, prado@if.usp.br

keywords: Stochastic Dynamics, Complex Networks, Formation and Dynamics of Patterns, Applications of Nonlinear Sciences.

1. BASIC INFORMATION

In the last decade, econophysics and sociophysics models have attracted the attention of physicists, due to the current availability of great amounts of data and computing power. The Sznajd model simulates the propagation of opinions in a society, using an agent-based approach. It has been successfully employed in modeling some properties and scale features of both proportional and majority elections [Bernardes et al.] [Costa Filho et al.], but its stationary states are always consensus states. Seeking to explain more complicated behaviours in an unified way, we have modified the *bounded confidence* idea, found in other opinion models [Deffuant et al.], to allow for complex opinion interactions.

The attractor structure of the resulting model can be solved in a mean-field approach and Monte Carlo simulations in a Barabási-Albert network show great similarities with the mean-field, for the tested cases of 3 and 4 opinions [Timpanaro & Prado].

1.1. Model Definition

The society is modeled as a network. Each node is an agent (person), each edge is a social connection (friendship, marriage, acquaintances, etc.) and each node possesses an integer between 1 and M, representing its opinion. At each time step, a node i is chosen at random, and then a neighbour j (another node, connected to i) is also chosen. If they agree (i and j possess the same integer value), then a neighbour of j, k is chosen. If σ_i and σ_k are respectively the opinions of i and k, then k is convinced (meaning it assumes the same value as i and j, σ_i) with probability $p_{\sigma_k \to \sigma_i}$, or nothing happens. If instead, i and j disagree, nothing happens.

The idea is that the first step represents a conversation between two people that know each other and they discuss some issue. If they disagree, none manages to convince the other. But, if they agree, they may set to convince another person, that one of them knows, and this person is convinced

with a certain probability, that depends of its current point of view and of the pair opinion. The reason why this probability must depend on both opinions is that, usualy an opinion includes prejudices about differing points of view.

The mean field version of this model is equivalent to the model simulated in a complete network (all nodes are connected to every other node). If η_σ is the proportion of nodes holding opinion σ and the network is large, then the dynamics is given by the flow

$$\dot{\eta}_\sigma = \sum_{\sigma'} \left(\eta_\sigma^2 \eta_{\sigma'} p_{\sigma' \to \sigma} - \eta_\sigma \eta_{\sigma'}^2 p_{\sigma \to \sigma'} \right), \qquad (1)$$

where one time unit corresponds to N simulation time steps and N is the number of nodes. The phase space of this flow is an $(M-1)$-simplex (that is embedded in an M dimensional space for convenience), where the vertices correspond to consensus states and the other states are convex combinations of the vertices, with coeficients η_σ.

1.2. Mean-Field Results

Using linear stability analysis, we were able to find the qualitative structure of fixed points and attractors. The system has $M(M-1)$ parameters $p_{\sigma \to \sigma'}$, where M is the number of opinions and the parameters are in the interval $[0, 1]$. These parameters can be thought as the terms of the adjacency matrix of a weighted graph, that will be called the confidence rule, and denoted R, which helps to schematize the interactions among the different opinions. We can assign a skeleton $Sk(G)$ to a weighted graph G, which is the directed graph with the same nodes as the weighted graph, and all the arrows that have a non-zero weight (Figure 1). We can also assign to each group of nodes, Δ, in the confidence rule, the manifold M_Δ, where only the opinions in Δ survive. In this way, the flow restricted to M_Δ is equivalent to the model with a confidence rule R_Δ, the subgraph of R induced by Δ.

The linear analysis results in [Timpanaro & Prado] can be summed up as

- If Δ is a set of opinions, such that $Sk(R_\Delta)$ is symmetric, then there exists a repeller, where all the opinions in Δ survive. For 3 opinions, this repeller is a fixed point.
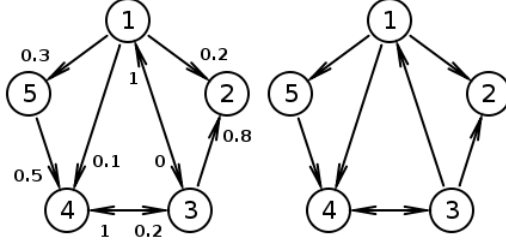
Figure 1 – A confidence rule for 5 opinions and its skeleton.
Here $p_{1\to2} = 0.2$, $p_{1\to4} = 0.1$, $p_{1\to5} = 0.3$, $p_{3\to1} = 1$, $p_{3\to2} = 0.8$, $p_{3\to4} = 1$, $p_{4\to3} = 0.2$, $p_{5\to4} = 0.5$ and $p_{\sigma\to\sigma'} = 0$ otherwise.

- The manifold M_Δ is an attractor, where all of its points are fixed points iff the opinions in Δ are non-interacting, and every opinion not in Δ can be convinced by at least one opinion in it.

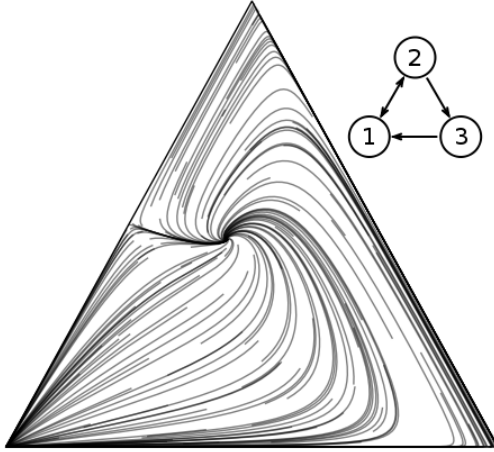A phase portrait that helps to ilustrate the linear analysis results for $M = 3$ can be found in figure 2.

Figure 2 – Phase portrait for the mean-field model in a 3 opinions case. All the weights in the skeleton are 1.

1.3. Simulations

Simulations of the model in a Barabási-Albert network show a very similar phase portrait (Figure 3 shows the simulation results of the previous mean-field case) for 3 opinions, and the attractor structure agrees with what was found in the mean-field analysis. The "phase space" in this case is actually a projection to the $(M-1)$-simplex, where the states are described by η_σ. As such, some odd phenomena are possible, as the crossing of trajectories seen in figure 3. Moreover, the derivation of equation 1 makes it clear that the variables in the flow are actually the expected values for the simulation (in a complete network). So, to draw the trajectories, we made an average over many simulations and the initial conditions were enforced only by changing the probabilities that a site would start with a given opinion.
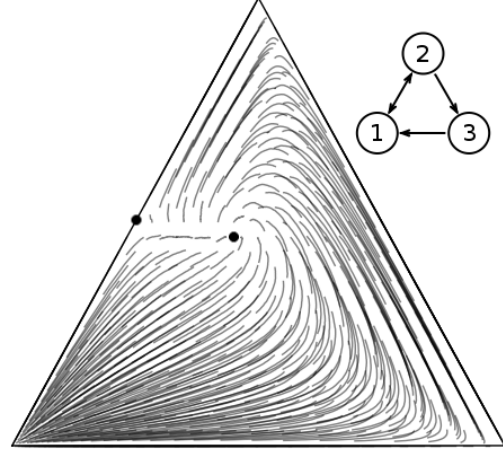

Figure 3 – Phase portrait for the simulated model in the same case as figure 2. The marks indicate the locations of the fixed points in the mean-field model.

References

[Bernardes et al.] A. T. Bernardes, D.Stauffer, and J. Kertész, "Election results and the Sznajd model on Barabasi network", The European Physical Journal B, Vol. 25, No. 1, pp. 123-127, January 2002.

[Costa Filho et al.] R. N. Costa Filho, M. P. Almeida, J. S. Andrade Jr., and J. E. Moreira, "Scaling behavior in a proportional voting process", Physical Review E, Vol. 60, No. 1, pp. 1067(2), July 1999.

[Deffuant et al.] G. Deffuant, D. Neau, F. Amblard, G. Weisbuch, "Mixing beliefs among interacting agents", Advances in Complex Systems, Vol. 3, No. 1, pp. 87-98, 2000.

[Timpanaro & Prado] A. M. Timpanaro, and C. P. C. Prado, "Generalized Sznajd model for opinion propagation", Physical Review E, Vol. 80, No. 2, pp. 021119(8), August 2009.