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NOISE-INDUCED INTERMITTENCY IN A MEAN-FIELD DYNAMO

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Mean-field dynamo is currently used to model the solar magnetic cycle [1, 2]. In particular, low-dimensional models derived from mean-field dynamo equations are useful to understand the underlying mechanisms of observed signatures in solar and stellar magnetic activities [3, 4]. In this context, and considering that a low-dimensional mean-field modelling neglects the small-scale dynamics, the study of stochastic effects on such systems becomes relevant. In particular, the onset of the intermittency induced by noise is important to understand how stochastic perturbations can change the global dynamics of the system, and help to understand in a simpler manner relevant features of solar activity such as the Maunder-type minima [5]. Rempel et al. [6] studied the effect of noise on nonlinear Alfvén waves in a regime with multistability, including coexistence of periodic attracting sets and a chaotic nonattracting set (chaotic saddle), showing that an external stochastic source can destroy attractors, as well as induce chaotic transients and extrinsic intermittency. Moreover, the effects of Gaussian and non-Gaussian noise were compared [7]. The theoretical framework to study the role of additive noise in nonlinear dynamical systems was developed by Lai et al. [8]. They showed that the key mechanism for the onset of noise-induced unstable dimension variability, transition to chaos and intermittency, is the presence of a chaotic saddle. External noise induces the interaction between a random attractor with the stable manifold of a chaotic saddle, changing the global properties of the dynamical system, measurable by the Lyapunov exponents.

In this work, we study the random chaotic dynamics and intermittency of a mean-field dynamo model with α-quenching, originally proposed by Schmalz and Stix [10], including an additive noise. The induction and α-quenching equations for the mean-field dynamo are given by the following system of nonlinear equations:

$$\frac{dA_n}{dt} = -n^2 A_n + \frac{D}{2}(B_{n-1} + B_{n+1}) + \sum_{j=1}^{N}\sum_{k=1}^{N} F_{njk} B_j C_k + \sigma\xi_{A_n}(t), \quad (1)$$

$$\frac{dB_n}{dt} = -n^2 B_n + \sum_{j=1}^{N} G_{nj} A_j + \sigma\xi_{B_n}(t), \quad (2)$$

$$\frac{dC_n}{dt} = -\nu^2 C_n - \sum_{j=1}^{N}\sum_{k=1}^{N} H_{njk} A_j B_k + \sigma\xi_{C_n}(t), \quad (3)$$

where A_n and B_n are the Fourier amplitudes of the poloidal and the toroidal components of the mean magnetic field, and C_n is the Fourier amplitude of the dynamical α-quenching [5, 10]. The variables $\xi_A(t)$, $\xi_B(t)$ and $\xi_C(t)$ are vectors whose components are Gaussian random variables of zero mean and unit variance, representing additive noise. The control parameters are the dynamo number D, proportional to the square of angular velocity gradient and to the square of the turnover time of the turbulent convection eddies [12], and the noise amplitude σ. Choosing the truncation order $N = 4$, and limiting our study to the antisymmetric subspace of solutions, as in [13], Eqs. (1)–(3) reduce to a six-dimensional dynamical system. Note that when $N = 1$ Eqs. (1)–(3) reduce to the Lorenz system [11]. In order to study the influence of noise on the attractors of the system, we consider the case where an attractor (periodic or chaotic) coexists with a chaotic saddle, which can be expected in any periodic window of a nonlinear dynamical system [8, 9]. We focus on a periodic window for Eqs. (1)–(3) in the deterministic case ($\sigma = 0$), near $D \sim 206$, and study the effects of noise by fixing the parameter D for two different values within the periodic window: a) $D = 205.88$, corresponding to a periodic attractor, and b) $D = 205.8525$, corresponding to a chaotic attractor, before the onset of an interior crisis.

The stochastic differential equations are solved using the Heun's method [14]. The dynamical properties of the system can be characterized by the Lyapunov exponents. As an example of the effects of noise, the upper panel of Fig. 1 shows the first and second Lyapunov exponents as a function of σ, for the case b). When σ is small the random chaotic attractor has the same Lyapunov spectrum as the deterministic chaotic attractor, indicating that the random attractor preserves the features of the deterministic system. Increasing σ, the random attractor undergoes a transition to a more chaotic state, at $\sigma_c \approx 10^{-3.1}$. This transition occurs in a continuous manner. Going to greater values of σ, the first Lyapunov exponent reaches asympotatically the value of the first Lyapunov exponent of the surrounding chaotic saddle (brown dashed line in the upper panel of Fig. 1). Lai et al. [8] found that

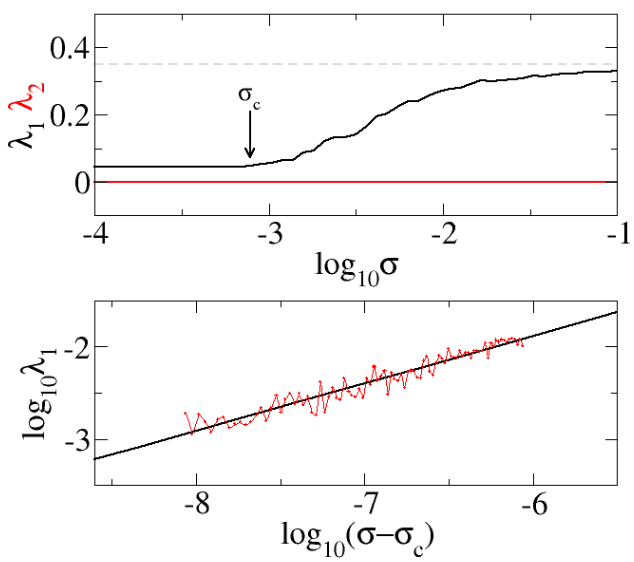

Figure 1 – a) First and second Lyapunov exponent as a function of the noise level σ. b) Power-law scaling of the largest Lyapunov exponent after the onset of the noise–induced enlargement of the stochastic chaotic attractor at $\sigma_c = 10^{-3.5}$.

the first Lyapunov exponent obeys an algebraic scaling law near the transition. The lower panel of Fig. 1 shows the first Lyapunov exponent as a function of $\sigma - \sigma_c$ in log-log scale. A power-law scaling of λ_1 clearly holds for noise amplitude arround σ_c, in agreement with the finding of Lai et al. [8]. Further details of the noise-induced intermittency in a mean-field dynamo will be presented.

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