DYNAMICS DAYS SOUTH AMERICA 2010
International Conference on Chaos and Nonlinear Dynamics

INPE – National Institute for Space Research
São José dos Campos – SP – Brazil – July 26-30, 2010

EXTREME FRACTAL STRUCTURES IN CHAOTIC MECHANICAL SYSTEMS: RIDDLED BASINS OF ATTRACTION

Sabrina Camargo[1], *Sérgio R. Lopes*[2], *Ricardo L. Viana*[3]

[1]Instituto de Física da Universidade de São Paulo, São Paulo, Brazil, scamargo@if.usp.br
[2]Departamento de Física da Universidade Federal do Paraná, Curitiba, Brazil, lopes@fisica.ufpr.br
[3]Departamento de Física da Universidade Federal do Paraná, Curitiba, Brazil, viana@fisica.ufpr.br

keywords: Applications of Nonlinear Sciences, Chaotic Dynamics.

1. INTRODUCTION

Riddling is an intrinsic phenomenon of dynamical systems which fulfill specific conditions and leads to basins of attraction that never exhibit a disk in the phase space. We say that a dynamical system has a chaotic attractor whose basin of attraction is riddled with "holes" belonging to the basin of another (non necessarily chaotic) attractor. Riddling means that every point in the basin of attraction of attractor A has pieces of the basin of attraction of attractor B arbitrarily nearby. Physical consequences can be very serious in terms of predicting the final state of a given initial condition in a system which presents riddled basins [1, 2].

2. MATHEMATICAL CONDITIONS

For riddled basins of attraction the system must satisfy the following conditions [2]:

1. There is an invariant subspace whose dimension is smaller than the phase space dimension.

2. The dynamics on the invariant subspace has a chaotic attractor.

3. There is another attractor, chaotic or not, and not belonging to the invariant subspace.

4. The chaotic attractor is transversely stable in the phase space.

5. A set of unstable periodic orbits embedded in the chaotic attractor becomes transversely unstable and at least one of the Lyapunov exponents along directions transverse to the invariant subspace, although negative for almost any orbit in the attractor, experiences positive finite-time fluctuations.

3. MECHANICAL SYSTEM

The mechanical system we have choose for the riddling study consists of a particle of unit mass moving under a dissipative friction, and a periodic forcing in the x direction,

$$\ddot{\boldsymbol{r}} + b\dot{\boldsymbol{r}} + \boldsymbol{\nabla} V = A[\sin(\omega t)]\mathbf{x} \tag{1}$$

where $\mathbf{r} = x\mathbf{x} + y\mathbf{y}$, b is the coefficient of dissipation, A and ω are the amplitude and frequency of the periodic forcing. $V = V(x,y)$ is the y-symmetric bidimensional potential,

$$V(x,y) = -a\cos x + (x+\chi)y^2 - y^4. \tag{2}$$

The choice of parameters $a = 1.0, b = 0.22, \omega = 1.0, A = 2.7$ ensures that at the invariant subspace there is a chaotic attractor [3]. Fig. 1(a) shows the riddled basin of attraction of the system given by Eq. 1, with $\chi = 2.5$. The maximal transversal Lyapunov exponent and the distribution of the finite time largest transversal Lyapunov exponent are shown in Figs. 1(b) and (c).

4. SCALING LAWS

Qualitatively descriptions of the riddling have already been presented by the positive fluctuations of the finite time Lyapunov exponent while the infinity Lyapunov is negative, associated to the fractal form of the basin of attraction. Quantitative analysis of the riddled basins are presented through the scaling laws.

The first scaling law relates the fraction $P_\star$ of the basin of attraction that escapes from the chaotic attractor to the distance $y = y_0$ of the initial condition to the invariant subspace where the chaotic attractor is placed. $P_\star$, is expected to scale with y_0 as a power law [4], $P_\star \sim |y_0|^\eta$, where η is given in terms of the finite time Lyapunov exponent, $\eta = \frac{2|\lambda_\perp|}{n\sigma^2}$. The second scaling law plays the role of the measuring the error in the prediction of the final state: the fraction of ε-uncertain points, $\langle p \rangle$, is the probability of making a mistake when attempting to predict which basin the initial condition x_0 is in, given a measurement uncertainty ε, and is expected to scale as $\langle p \rangle \sim \varepsilon^\phi$, $\phi = \frac{\lambda_\perp^2}{2n\sigma^2\lambda_\parallel}$.

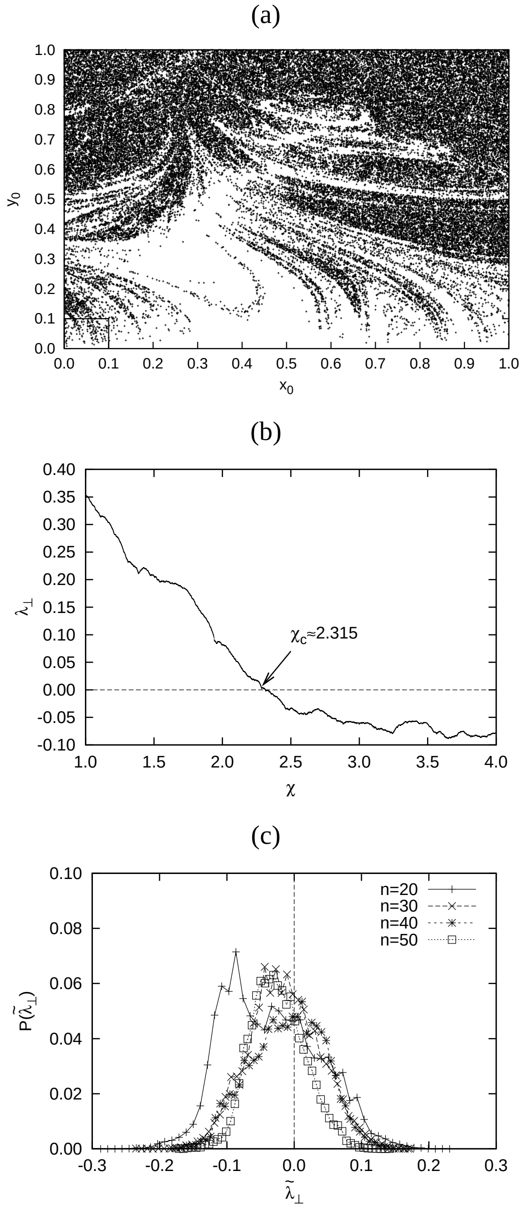

Figure 1 – (a) Riddled basin of attraction. Black dots are the basin of attraction of the attractor placed at $y = p_y = 0$. (b) Maximal transversal Lyapunov exponent with the control parameter χ. (c) Distribution of the finite time largest transversal Lyapunov exponent for a typical orbit in the $y = 0$ invariant subspace for different finite time computations.

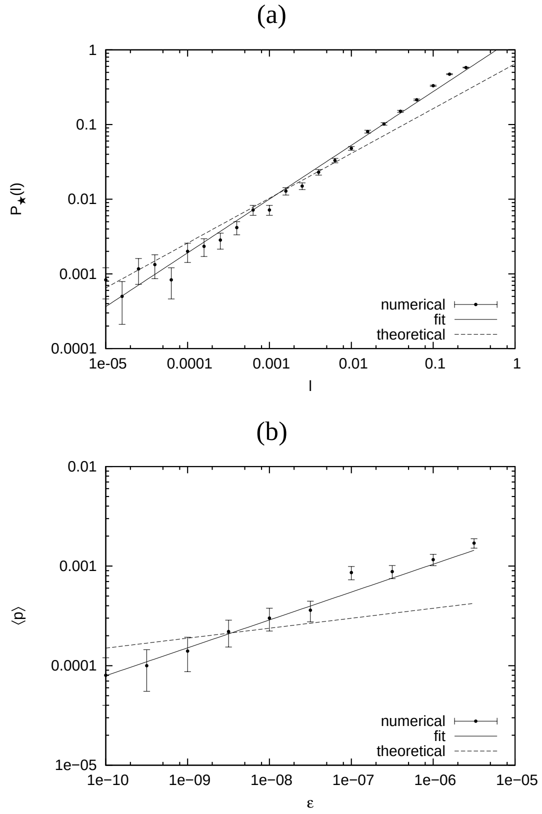

Figure 2 – (a) Numerical calculation of the fraction $P_\star$ of orbits belonging to the basin of the chaotic attractor in $y = p_y = 0$ with the distance y_0 to the invariant subspace. (b) Numerical calculation of $\langle p \rangle$, which plays the role of the measuring the error in the prediction of the final state.

5. CONCLUSIONS

In our forced damped pendulum the riddling indeed will be controlled via the parameter χ, for $\chi > 2.32$, approximately. For a suitable choice of χ there will be unstable periodic orbits in the chaotic attractor contained in $y = p_y = 0$, exhibiting positive fluctuations in the finite time Lyapunov exponent. As the basin of the attractor in $y = 0$ is riddled with holes belonging to the basin of the final state $y \to \infty$, it follows that for any distance y_0, no matter how small, there is always a nonzero value of $P_\star$. The important practical applications of systems that exhibit a forced damped pendulum behavior, like clocks or mechanical arms, requires an investigation of the presence of riddled basins of attraction.

References

[1] J. C. Alexander, J. A. Yorke, and Z. You, Riddled basins, International Journal of Bifurcation and Chaos , 795 (1992).

[2] J. Aguirre, R. L. Viana, and M. A. F. Sanjuán, Fractal structures in nonlinear dynamics, Rev. Mod. Phys. **81**, 333 (2009).

[3] C. Grebogi, E. Ott, and J. A. Yorke, Chaos, strange attractors, and fractal basin boundaries in nonlinear dynamics, Science , 632 (1987).

[4] E. Ott, J. C. Alexander, I. Kan, J. C. Sommerer, and J. A. Yorke, The transition to chaotic attractors with riddled basins, Physica D , 384 (1994).