DYNAMICS DAYS SOUTH AMERICA 2010
International Conference on Chaos and Nonlinear Dynamics

INPE – National Institute for Space Research
São José dos Campos – SP – Brazil – July 26-30, 2010

BIPARAMETRIC INVESTIGATION OF THE PHASE SPACE STRUCTURES OF THE GENERAL STANDARD MAP IN DISSIPATIVE REGIME

Priscilla A. de Sousa Silva[1] *and Maisa de Oliveira Terra*[2]

[1]Instituto Tecnológico de Aeronáutica - Departamento de Matemática, São José dos Campos, Brazil, priandss@ita.br
[2]Instituto Tecnológico de Aeronáutica - Departamento de Matemática, São José dos Campos, Brazil, maisa@ita.br

keywords: Bifurcation theory and analysis, general standard map, multistability.

Generally the systematic study of the long term behavior of the solutions of deterministic systems is developed by investigating the dynamics of such systems as a control parameter set is varied. By doing so, one is able to track and analyze local and global bifurcations that account for the appearance and disappearance of distinct asymptotic solutions and for the changes undergone by the phase space structures.

In this contribution, we investigate global bifurcations of a biparametric two-dimensional map, known as the standard map, derived from a model for the periodically kicked mechanical rotor. This archetypical system is adequate for the study of several fundamental features such as multistability, crises and chaotic transients.

Departing from the conservative case of the map, as dissipation builds up, we describe the evolution of periodic solutions and their basins, paying special attention to the effects of the dissipative and forcing parameters in the feature of multistability. Also, we relate numerically the mechanism of global bifurcations associated to small chaotic attractors in the multistable system. Through the characterization of an interior, a merging and a boundary crisis, we study the crucial role played by fundamental invariant structures, such as unstable periodic orbits (UPO's) and their invariant manifolds, in the mechanisms by which the phase space is globally transformed.

The understanding and characterization of such mechanisms and of the structures involved are essential to the comprehension of complex behavior of nonlinear systems in several areas of science and engineering.

The standard map derives from a model for the motion of a periodically kicked mechanical pendulum - a bar attached at one end to a pivot and subjected to a vertical periodic impulsive force at the other end of the bar. In the conservative case the pivot is frictionless. The discrete time system is related to the state of the continuous time system, described by a set of two first order differential equations, just after each periodic impulse. The map is usually described by

$$\begin{aligned} \theta_{k+1} &= (\theta_k + p_k) mod(2\pi), \\ p_{k+1} &= (1-\nu)p_k + f_0 sin(\theta_k + p_k), \end{aligned} \tag{1}$$

where θ and p correspond, respectively, to the angular position and momentum of the mechanical pendulum and f_0 and ν are the parameters of the system. The first parameter, f_0, accounts for a forcing of constant strength applied in the continuous time system at times nT, $n = 1, 2, ...$, with T as a constant period. The second one, ν, is the damping parameter and varies from 0 (no dissipation, Hamiltonian system) to 1 (very strong damping).

As the two parameters are varied, this map presents a wide assortment of dynamical phenomena. For $\nu = 0$ the dynamics is ruled by the typical phenomenology of Hamiltonian systems depending exclusively of the forcing parameter. For $f_0 = 0$, the system is integrable and p is a constant of the motion. When $f_0 > 0$, the integrability of the system is broken and the phase space consists of regions of chaotic sea interspersed with periodic islands [1].

For $0 < \nu \leqslant 1$ the map is dissipative. When some amount of dissipation is introduced, the stable periodic orbits of the conservative system become sinks and the chaotic motion is replaced by long chaotic transients before the trajectory eventually settles down in one of the sinks. For very small damping the dynamical behavior is mainly dominated by the appearance, disappearance and coexistence of periodic attractors of several different periods. The complexity of the bifurcation structure depends strongly on f_0 and ν and the feature of multistability is prevalent for very small and moderate dissipation [2, 3]. In the case of maximum dissipation, the map reduces to the one-dimensional circle map with zero rotation number [4].

Typically, the number of coexistent periodic attractors can be made arbitrarily large as $\nu \to 0$, for intermediate f_0. Most of the detected attractors have low periods because high period and chaotic attractors have very small basins which makes their detection more complicated. The basins of the multiple coexistent attractors are strongly intertwined and have fractal boundaries which appear to spread over most

of the phase space, resulting in a high sensitivity to the final state [2]. Figure 1 displays the bifurcation structure as ν is varied for $f_0 = 4.5$.

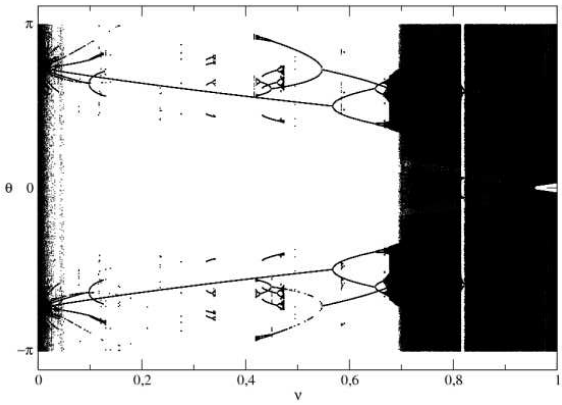

Figure 1 – Bifurcation diagram for $f_0 = 4.5$ and $0 < \nu < 1$.

As ν increases, chaotic attractors persist for larger intervals in parameter space and their basins are large enough as to allow them to be more easily found throughout phase space. Figure 2 displays the basins of attraction of a small chaotic attractor and three period-two attractors which coexist for $f_0 = 4.5$ and $\nu = 0.47330$. Generally this small attractors appear through a period doubling cascade starting at a saddle-node bifurcation and they disappear when the attractor collides with an UPO on the boundary of its basin of attraction in a global bifurcation known as boundary crisis.

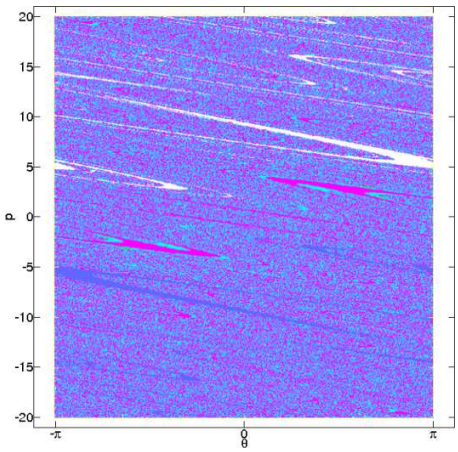

Figure 2 – Basins of attraction of three period-two attractors (white, cyan and blue) and of the chaotic attractor (magenta) which coexist for $f_0 = 4.5$ and $\nu = 0.47330$.

We have numerically followed and characterized a boundary crisis (displayed in Figure 3) for a typical chaotic attractor, as well as a merging and an interior crisis, depicting the role of the invariant structures involved, for the case of intermediate f_0.

We also describe the case of weak dissipation and forcing.

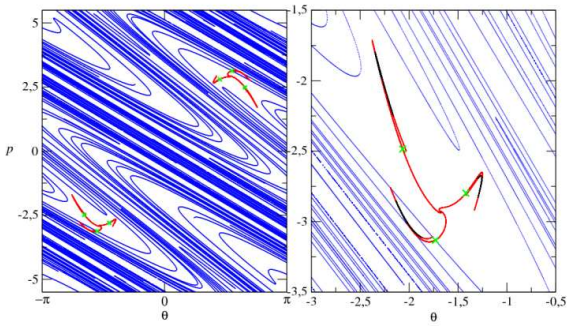

Figure 3 – Left side: stable (blue) and unstable (red) manifolds of the period-18 UPO which collides with the chaotic attractor (black) at the boundary crisis ($\nu = 0.47333695$) shown for $f_0 = 4.5$ and $\nu = 0.47330$. Right side: magnification of the lower part of the phase space shown in left side along with nine points (green crosses) of the period-18 UPO.

Unlike the previously described situation, the total number of attractors is limited for low f_0 and ν. In this case, one is able to observe the evolution of the basin boundary structure as the parameters are varied.

The performed numerical study and characterization of the structures involved in the global transformations of the phase space is important to the comprehension of multistable systems and the observed phenomenology is common to a wide variety of dynamical systems.

ACKNOWLEDGEMENTS

This work was partially supported by CAPES, Brazil.

References

[1] E. Ott, “Chaos in Dynamical Systems,” 1993, Cambridge University Press, Canada, 385 p.

[2] U. Feudel, C. Grebogi, B. R. Hunt, and J. A. Yorke, “Map with more than 100 coexisting low-period periodic attractors,” Phys. Rev. E, Vol. 54, No. 1, July 1996.

[3] L. C. Martins, J. A. Gallas, “Multistability, phase diagrams and statistical properties of the kicked rotor: a map with many coexisting attractors”, Int. J. of Bifurcation and Chaos, Vol. 18, No. 6, November 2008.

[4] G. Schmidt, B. W. Wang, “Dissipative standard map”, Phys. Rev. A, Vol. 32, No. 5, November 1985.