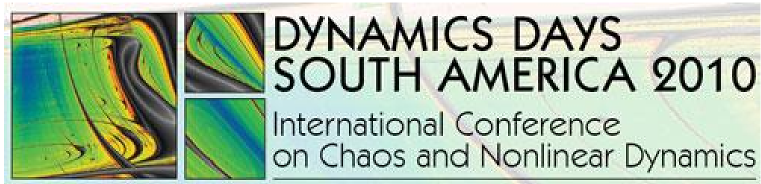

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MULTISTABILITY IN SYSTEMS WITH IMPACTS

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Vibro-impact oscillators have moving parts colliding with either moving or stationary components, and are often found in engineering applications, as vibration hammers, driving machinery, milling, impact print hammers, shock absorbers, and gear rattling [1–3].

In this work we investigate the multistability of a weakly dissipative vibro-impact oscillator. We study a model of a vibro-impact oscillator, known as impact-pair, composed of a point mass m free to move inside a periodically driven box [4].

For the conservative limit of this system, we identify the location of the main islands and their elliptic points on the phase portrait. The multistability of the system is identified by considering a small amount of dissipation [5], the elliptic points giving rise to attractors, i. e., stable fixed points. Moreover, to study the arising attractors we use common tools of chaotic dynamics such as phase portraits, bifurcation diagrams, and basins of attraction [6, 7].

The point mass m displacement is denoted by x, and the box length ν. The mass m is free to move inside the box and the motion of the box is described by a periodic function, $\alpha \sin(\omega t)$, see Figure 1.

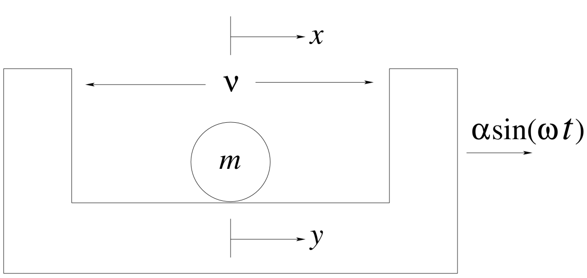

Figura 1 – Model of the impact-pair system

The equation of motion of the point mass m in the absolute coordinate x is:

$$\ddot{x} = 0. \tag{1}$$

Denoting the relative displacement of the mass m by y, we have

$$x = y + \alpha \sin(\omega t). \tag{2}$$

Substituting Eq. 2 into Eq. 1, the equation of motion in relative coordinate y is

$$\ddot{y} = \alpha \omega^2 \sin(\omega t), \quad -\nu/2 < y < \nu/2. \tag{3}$$

Integrating Eq. 3 and invoking initial conditions $y(t_0) = y_0$ and $\dot{(y)}(t_0) = \dot{y}_0$, the displacement y and the velocity $\dot{y}$, between impacts, are

$$y(t) = y_0 + \alpha \sin(\omega t_0) - \alpha \sin(\omega t) + [\dot{y}_0 + \alpha\omega \cos(\omega t_0)](t - t_0). \tag{4}$$

$$\dot{y}(t) = \dot{y}_0 + \alpha \cos(\omega t_0) - \alpha\omega \cos(\omega t). \tag{5}$$

An impact occurs whenever $y = \nu/2$ or $y = -\nu/2$. After each impact, we apply into Eq. 4 and Eq. 5 the new set of initial conditions (Newton's law of impact)

$$t_0 = t, \quad y_0 = y, \quad \dot{y}_0 = -r\dot{y}. \tag{6}$$

where r is a constant restitution coefficient.

The dynamics of the impact-pair system is obtained from Eq. 4 and Eq. 6 and the system depends on the control parameters r, α, ν and ω. As we are interested to study the attractors in the transition from the dissipative to the conservative system, we consider the variation of r and keep the others parameters constant. We vary the control parameter r in the interval $0.8 < r < 1$ and choose $\alpha = 0.05$, $\nu = 2$ and $\omega = 8$.

To evidence the multistable character of the system, in Figure 2 we show a bifurcation diagram for $0.4 < \alpha < 0.55$ and $r = 0.99$. The coexistence of attractors characterizes a multistable system.

The attractors, stable fixed points in the phase space, observed in Figure 2, in the dissipative case correspond to the elliptic points in the conservative phase portrait (obtained for $r = 1$).

To show the onset of attractors, we compare the phase space for a small dissipation, $r = 0.99$, with the

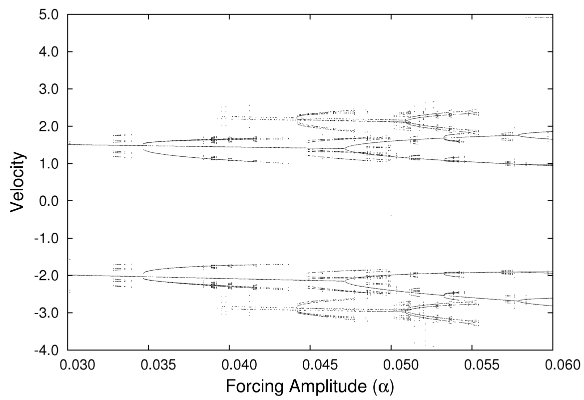

Figura 2 – Bifurcation diagram for varying α and $r = 0.99$.

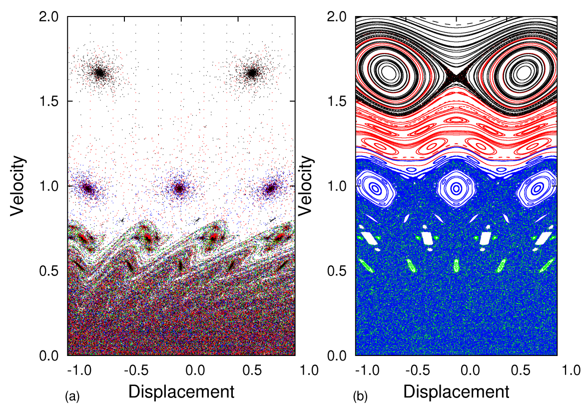

Figura 3 – (a) Partial phase portraits for a weakly dissipative system for $\alpha = 0.05$ and $r = 0.99$. (b) The corresponding conservative limit with $r = 1$ and $\alpha = 0.05$.

one obtained in the conservative limit. Thus, Figure 3(a) shows the attractors formed by the islands shrinking. The correspondent islands, for the conservative case, are shown in Figure 3(b).

Our numerical results indicate the existence of the multistability in the studied impact-pair system. For a small dissipation, we observe the onset of attractors associated to the elliptic points obtained in the conservative limit.

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