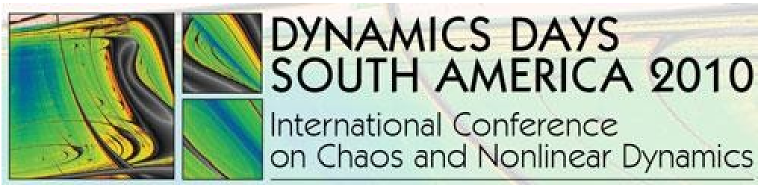

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SYMMETRY BREAKING EFFECTS IN ESCAPE BASIN ANALYSIS

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In this contribution the main goal is to investigate the symmetry breaking effects in the fractal structures [1] associated to the escape basins of a two-dimensional Hamiltonian system, the Hénon-Heiles (HH) model [2]. This system presents different symmetries [3]. The dihedral group D_3 is the spatial group, where the D_3 x τ is the full symmetry group, at which τ, a $\mathbf{Z}_2$ symmetry, is the time reversal symmetry.

The well-known classical Hénon-Heiles Hamiltonian was introduced in 1964 in the study of galactic dynamics to describe the motion of stars around a galactic center, constituting a symmetric perturbation to a two-dimensional harmonic oscillator. It is written as

$$H=\frac{1}{2}(\dot{x}^2+\dot{y}^2)+\frac{1}{2}(x^2+y^2)+x^2y-\frac{1}{3}y^3 , \quad (1)$$

where the two first potential terms form a potential well responsible for the oscillations of the particle and the last two potential terms are responsible for the existence of the exits.

An open Hamiltonian or a Hamiltonian with escapes [1] is a Hamiltonian system at which a particle can escape toward infinity from a certain bounded region. Since there are no attractors for Hamiltonian systems, we cannot define basins of attraction. However, we can define an exit basin in an analogous way to the basins of attraction in a dissipative system. The exit basin associated with the exit *k* of an open Hamiltonian system is the set of initial conditions that escape from the bounded region through the exit *k*.

The escape basins and associated fractal structures of the HH Hamiltonian have been studied for energy values above the escape energy E_e=1/6= 0.1666, at which the three system exits open together and the trajectories may escape from the bounded region and go to infinity through these exits. The three exits present in this system are usually defined as: exit 1, the upper exit $(y\rightarrow\infty)$, exit 2, the left exit $(y\rightarrow-\infty, x\rightarrow-\infty)$, and exit 3, the right exit $(y\rightarrow-\infty, x\rightarrow+\infty)$.

A chaotic saddle, also known as nonattracting chaotic set, is formed by a set of orbits of zero Lebesgue measure that will never escape from the scattering region for both $t\rightarrow\infty$ *or* $t\rightarrow-\infty$. Its stable manifold contains the orbits that will never escape if $t\rightarrow\infty$, while the unstable manifold is formed by the orbits that never escape if $t\rightarrow-\infty$, while the chaotic saddle, being the intersection of its stable and unstable manifolds, does not escape at any time. These fractal invariant sets can be numerically built by the sprinkler algorithm, which works fine for two-dimensional systems.

At the energy escape, the three exit basins present a fractal boundary, constituted by the stable manifold of the chaotic saddle, and satisfy the properties of the Wada basin definition, as shown by [2]. As the energy grows the fractality of the boundaries diminishes, but never disappears. This information is corroborated by the fractal dimension computed as a function of the energy for the fractal invariant sets.

This contribution is devoted to explore the spatial symmetry breaking of the Hamiltonian, including a new control parameter, in such a way that the opening of the three exits do not occur at the same energy level, allowing to verify the effects on the competition of the basins and on the fractal properties of the associated invariant sets. Diffusion and scaling laws in statistical properties of escape analysis for short time and long term behavior in a parametric HH Hamiltonian were previously investigated by Kandrup *et al* [6,7]. However our investigation is concerned with symmetry breaking effects on the properties of phase space trajectories and of the involved dynamical system invariant sets.

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