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SPATIOTEMPORAL FEATURES OF A COUPLED MAP LATTICE

Romeu M. Szmoski[1], *Rodrigo F. Pereira*[2], *Sandro E. de S. Pinto*[3]

[1]Universidade Estadual de Ponta Grossa, Ponta Grossa, Brazil, rmszmoski@gmail.com
[1]Universidade Estadual de Ponta Grossa, Ponta Grossa, Brazil,pereira.rf@gmail.com
[2]Universidade Estadual de Ponta Grossa, Ponta Grossa, Brazil, desouzapinto@pq.cnpq.br

Abstract: We investigate the spatiotemporal dynamics of a coupled map lattice focusing on the experimentally observed features in neural networks. The temporal evolution is characterized by Lyapunov exponents, whereas the spatial patterns are analyzed through clustering and locking-time, which is the necessary time for the lattice to achieve the equilibrium state. We also calculate the entropy per network element as a measure of information capacity of the network. Our results show that CML exhibits similar features to one observed in neural networks when coupling strength is weak.

keywords: Activity Patterns, Lyapunov exponents, Entropy.

1. INTRODUCTION

There are in the nature a wide class of systems whose elements can interact with each other and due to this, they can exhibit interesting spatiotemporal patterns. Such systems are studied from networks models, in which the nodes correspond to parts or individual elements and the links to interactions between them. In neuron populations case coupled maps lattices (CMLs) has been considered suitable models, since they allow to reproduce some experimentally observed features such as: a rapid response to stimuli, an irregular or chaotic behavior of individual elements and a high variability of activity patterns [1]. However, although the irregular behavior is a map characteristic, the two other seem to compete, since strong interactions increase the stimuli spreading whereas decrease the multiplicity of activity patterns. Thus, understanding how the interaction between the maps affects both processes is important to identify situations in which the system is consistent with the experimental observations. In this work we study this influence in a bidimensional CML with coupling Laplacian-type local.

2. THE MODEL

We examine a CML consisting of $N \times N$ maps arranged on a square lattice and whose dynamics is given by

$$x_{n+1}^{(i,j)} = (1-\varepsilon)T(x_n^{(i,j)}) + \frac{\varepsilon}{4}\left[T(x_n^{(i\pm1,j)}) + T(x_n^{(i,j\pm1)})\right] \quad (1)$$

where i, j correspond to position of the sites on network, $\varepsilon \in [0,1]$ is the coupling strength and $\pm$ denotes both the sum and subtraction.The term T (x) refers to the local dynamics for which we use tent map

$$T(x) \equiv \begin{cases} ax & , \quad 0 < x < \frac{1}{2} \\ a(1-x) & , \quad \frac{1}{2} \leq x \leq 1 \end{cases} \quad (2)$$

where $a \in [0,2]$ is a control parameter. For $a < 1$ all orbits converge to the fixed point $x^* = 0$, whereas for $a = 1$ all initial points are fixed points. On the other hand, for $a > 1$ there is one unstable fixed point in $x^* = a/(a+1)$ and the dynamic presents chaotic regimes. Here we use $a = \sqrt{2}$. To such parameter value the density of an ensemble of non-interacting maps can converge to a stationary distribution or oscillate with certain period. Fig. (1) show the density temporal evolution for $x_0 \in [0.1; 0.9]$. We can observe that density oscillates between a pattern and its inverse for $n > 5$.

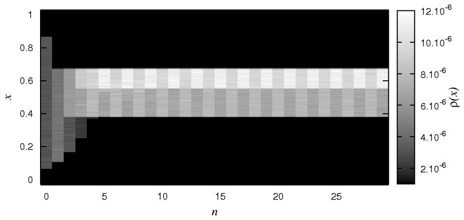

Figure 1 – Density evolution of an ensemble of 100.000 tend maps with $a = \sqrt{2}$. The initial values were uniformly distributed on the interval [0.1,0.9].

3. RESULTS

The dynamical behavior of the system is characterized by its spectrum of Lyapunov exponents. For the CML (1) with periodic boundary conditions we verified that such spectrum is given by

$$\lambda_m = \lambda_u + \ln\left|1 - \varepsilon + \frac{\varepsilon}{2}\sum_{j=1}^{2}\cos\left(\frac{2\pi jm}{N^2}\right)\right|, \quad (3)$$

where $\lambda_u = \ln|a|$ is the Lyapunov exponent of the uncoupled map and $m = 0, 1, \cdots, N^2$. Thus, since the CML has at least one positive Lyapunov exponent its evolution is consistent with the chaotic dynamic.

Fig. 2 exhibits the evolution of two activity patterns of the CML for $\varepsilon = 0.2$. We can observe a periodic behavior as one that we found for the density of a ensemble of uncoupled maps (Fig. 1).

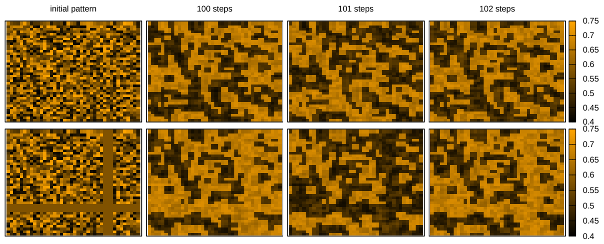

Figure 2 – Activity patterns of the CML for $\varepsilon = 0.2$ and different inputs: (a) random initial distribution in [0,1]. (b) same as (a) but with an additional cross of value 0.5.

In order to examine the activity patterns we use a coarse graining approximation. We express the original pattern as a binary pattern according to fixed point $x^* = a/(a+1)$. Fig. 3 exhibits both patterns for different initial distributions. Due to the similarity between them we can claim that that point x^* is a suitable threshold for the coarse graining. Considering

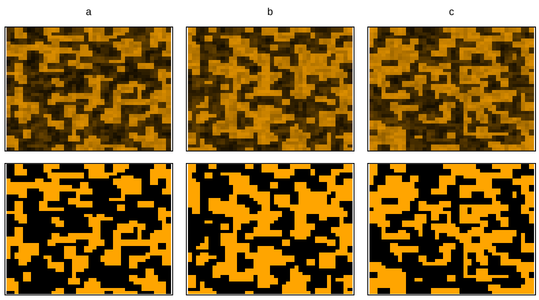

Figure 3 – Original and binary patterns for $\varepsilon = 0.2$, $n = 20$ and different initial patterns in $[0, 1]$.

the binary pattern we use the Hoshen-Kopelman algorithm [1] in order to determine the number of clusters and mean cluster size. Fig. 4a and 4b show both quantities with respect to the coupling strength. We can observe that the number of clusters (mean cluster size) decreases (increases) when the coupling strength is increased. We also computed the probability $p(x|x_{nn}) = p(x, x_{nn})/\sum_x p(x, x_{nn})$ which is the probability of finding a site in the up state $(x = 1)$ or in down $(x = 0)$ when its four-neighbors are in a given configuration. Then, we calculate the entropy per network element

$$H = -\sum_{x_{nn}} \sum_{x} p(x|x_{nn}) \ln p(x|x_{nn}) \qquad (4)$$

which corresponds to the uncertainty of state of one site in relation to the states of the neighboring elements [1]. Fig. 5a shows the entropy H with respect to the strength coupling. We can see that it has a maximum value for $\varepsilon = 0$ and decreases when ε is increased.

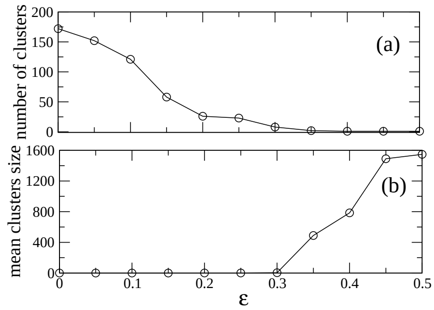

Figure 4 – Number of clusters (a) and mean cluster size (b) with respect to strength coupling for $N = 40$ and 1.000 random initial distributions in $[a(1 - a/2), a/2]$.

Finally, we study the locking-time through correlation function

$$r_n = \frac{1}{N^2} \sum_{i,j} \left| x_s^{(i,j)} - x_n^{(i,j)} \right| \qquad (5)$$

where $x_s^{(i,j)}$ is the state of i, j-th element in the binary pattern stationary. Since in stationary state r_n is 0 or 1, due the period-2 oscillation of the density, we consider the locking-time when $r_n < 0.01$. We can observe in Fig 3b that locking-time is larger for strong strength coupling.

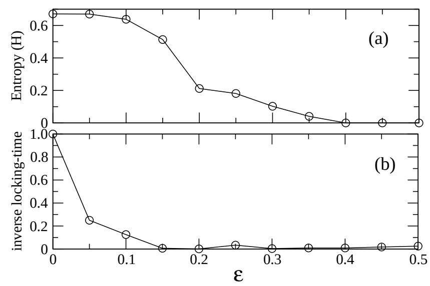

Figure 5 – Entropy per element (a) and inverse locking time (b) as a function of strength coupling.

In conclusion, we investigate some spatiotemporal aspects of a CML with periodic activity pattern. We observe that the spatial patterns depends on initial states distribution. Using the coarse graining approximation we express the original pattern in binary and we analyze it with respect to the coupling strength. We analyze the clusters, the entropy per element unit and the locking-time. Our results show that the CML has a faster convergence to the steady periodic pattern and it has higher entropy when the coupling strength is weak. Thus, this is the condition for the model exhibits the experimentally observed features in neural networks.

References

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