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SQUEEZING OF THERMAL NOISE IN A PARAMETRICALLY-DRIVEN DUFFING OSCILLATOR

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A Microelectromechanical system (MEMS) device consisting of a doubly-clamped beam resonator is used as a very sensitive detector of vibrations, weak forces and as a mass spectrometer [1]. Therefore, the reduction of noise is a very important issue for the sensitivity of these systems. Here, we propose a theoretical model that studies the first normal-mode oscillations of this kind of resonator as a parametrically-driven Duffing oscillator under the action of thermal noise. We investigate the effect of noise on the parametric amplification mechanism , experimentally observed and studied in[1–4], with the equation

$$\ddot{x} + \omega^2 x = -\Omega x - \gamma\dot{x} + F_p \cos(2\omega t)\, x + R(t)$$

where $x(t)$ is the position of the deflections of the middle of a doubly-clamped beam, $\Omega = \omega_0^2 - \omega^2$ is the detuning between the driving frequency and the natural frequency of the oscillator, γ is the dissipation coefficient, F_p is the pump drive amplitude, and $R(t)$ is a random function that satisfies $\langle R(t)\rangle = 0$ and $\langle R(t)R(t')\rangle = 2T\gamma\delta(t-t')$ according to the fluctuation-dissipation theorem [5]. We calculate the average fluctuations of these oscillations, via an averaged dynamics for the slowly-varying functions $u(t)$ and $v(t)$ given by $x(t) = u(t)\cos(\omega t) - v\sin(\omega t)$. We show that squeezing of noise occurs by solving a system of coupled autonomous differential equations for $u(t)$ and $v(t)$. In the long time limit, we obtain the average squared deviation from equilibrium of u and v

$$\begin{aligned}
\langle u^2\rangle &= T\left[1 + \frac{2(a+b)a}{\gamma^2 - 4ab}\right] = T\left[1 + \frac{F_p(F_p - 2\Omega)}{4\omega^2\gamma^2 - F_p^2 + 4\Omega^2}\right], \\
\langle v^2\rangle &= T\left[1 + \frac{2(a+b)b}{\gamma^2 - 4ab}\right] = T\left[1 + \frac{F_p(F_p + 2\Omega)}{4\omega^2\gamma^2 - F_p^2 + 4\Omega^2}\right].
\end{aligned}$$

We notice that by varying the pump amplitude F_p and the detuning Ω, we can create a continuous family of classical thermo-mechanical squeezed states, generalizing the experimental results of Rugar and Grütter [1]. We find that quadrature squeezing of noise only when there is detuning between ω and ω_0, with the squeezing and stretching of each quadrature changing according to the sign of Ω. When there is no external parametric driving, $F_p = 0$, we obtain the same results predicted by the equipartition theorem as expected. We also mention that this model breaks down if we are very close to the instability zone boundary.

Furthermore, in the linear regime, we theoretically derive an expression for the amount of mechanical noise squeezing and notice that detuning of the first parametric resonance is a necessary condition for noise squeezing. We derive a family of quadrature average amplitude fluctuations of the oscillator, whose degree of squeezing depends on the strength of the parametric force, detuning and the quality factor Q of the resonator. We also study the nonlinear contributions to noise squeezing by doing linear response and full-numerical integration of the equations of motion. Furthermore, we apply our technique to the calculation of gain for classical parametric amplifiers [6]. We make theoretical predictions for linear gain, nonlinear gain and the effect of noise on gain. We achieve very good agreement between numerical simulations and theoretical predictions.

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