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Statistical Study of logistic Map Orbits In None Chaotic And Chaotic Orbits

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1. Introduction

Chaotic behavior as a disordered state is a special manner of nonlinear systems. Nonlinear dynamic can be explained by discrete dynamic systems. One dimensional nonlinear return maps are simple relations by chaotic behavior in specific conditions. Mathematical theory of these relations plays an important role in developing the chaos theory.

Logistic map is one of the well known discrete chaotic systems. It may be expressed as follow

$$x_{n+1} = ax_n(1-x_n) \qquad (1)$$

Where x_0 is the initial value of orbit and a as a control parameter can determine the maps behavior [1].

In the study of dynamical systems, some statistical methods are reported [2,3]. In this paper we study the mean and standard deviation of various orbits obtain from Logistic map for different control parameters and initial values of orbits.

2. The Method

The average and the standard deviation plots as a function of initial values of the orbits obtained as follow. 100 different x_0 from the interval $(0,1)$ with 0.01 increments choosed calculate the orbits up to x_{100} using logistic map, then calculate and plot the average and standard deviations for any orbit. In other word, any point on the average curve shows the $(x_0, \frac{\sum_{i=0}^{99} x_i}{100})$ and any point on the standard deviation curve is $(x_0, \sqrt{\frac{\sum_{i=0}^{99} x_i - \overline{x}^2}{100-1}})$. The study shows that increasing .

the length of the orbit doesn't changes the shape of the curves significantly.

The average and standard deviation plots of the orbits are plotted versus different initial values for some selected control parameters.

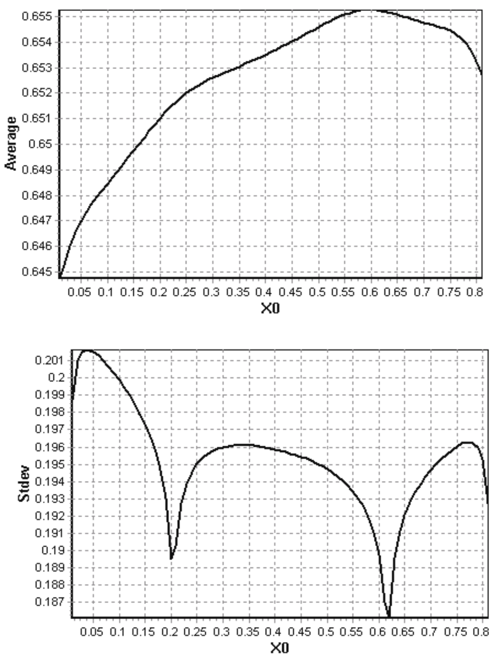

Figure 1–The mean and standard deviation for hundred repeated Logistic map to x0 for a=2.9

Figure 1 shows the plots for a=2.9, at which the system has an attractor. This figure shows that there isn't any relation between average and standard deviation.

The plots given in figure 2, are due to the first bifurcation of logistic map.

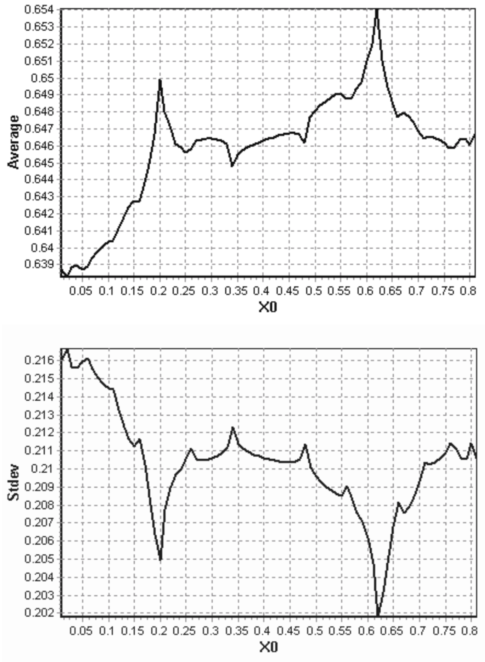

Figure 2 – The mean and standard deviation for hundred repeated Logistic map to x0 for a=3.

Increasing the control parameter value causes the next bifurcations. In figure 3, one can see the plots of a=3.7.

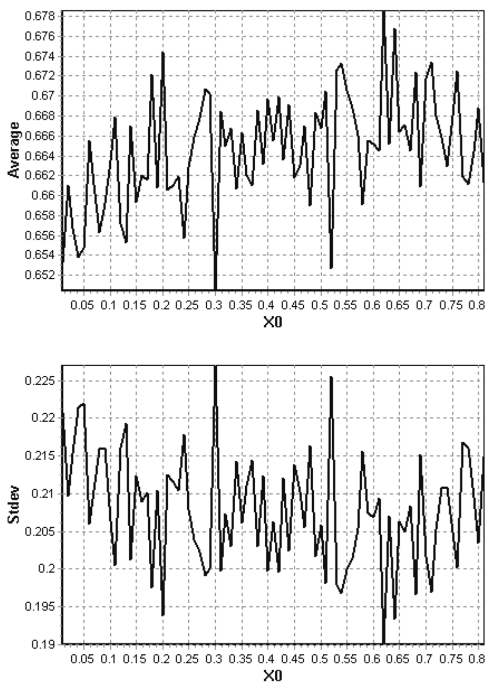

Figure3– The mean and standard deviation for hundred repeated Logistic map to x0 for a=3.7.

When a reader reaches to 3.83, the chaotic state of the system can be seen. The plots are more related and symmetric in this state as can be seen in figure 4.

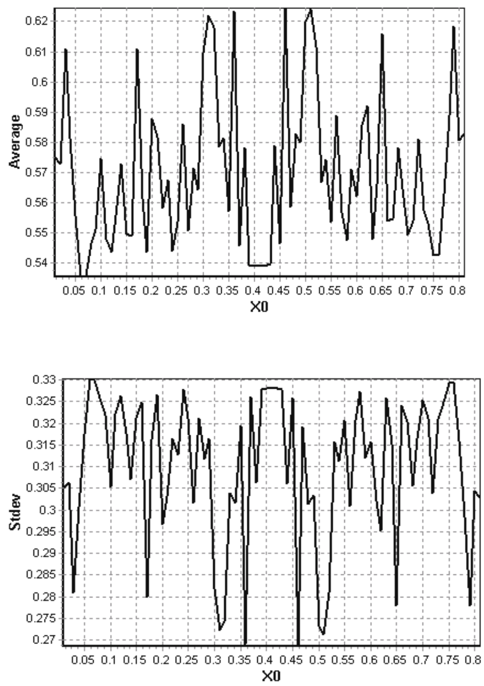

Figure 4 – The mean and standard deviation for hundred repeated Logistic map to x0 for a=3.83.

Study shows that mean and standard deviations are symmetric, relative to a horizontal line.

3. Conclusion

The study show that the periodic and chaotic states of the system can be identified by considering the sensitivity of the average and the standard deviation of the orbits to initial conditions. For the control parameters which the system bifurcates, there is a similarity between the average and the standard deviation plots. The plots loss their smoothness when the bifurcation proceeds. At the start of chaotic state, the curves have a vertical symmetry line.

References

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