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SYNCHRONIZATION CONDITIONS FOR A BANDLIMITED DISCRETE-TIME CHAOTIC SYSTEM

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Since Pecora and Carroll's seminal work [1], much has been written about the potential usefulness of chaotic synchronization in communication systems (e.g., [2]). Much of the impetus for chaotic communications has been the rationale whereby both analog and digital chaotic modulations would have the same properties as conventional spread spectrum techniques [2].

The inherent wideband characteristic of chaotic signals, however, becomes a problem when the communication channel imposes bandwidth limitations. Because of the receiver's nonlinear nature, if any spectral component is amiss, all spectral components at the receiver become affected. Even minute gain or phase changes are enough to fully hinder synchronism [3].

Rulkov and Tsimiring [3] and Eisencraft and Gerken [4] independently proposed a method for synchronizing transmitter and receiver using chaotic signals under bandwidth limitations. The basic idea is to apply an identical filter on both the transmitter and the receiver subsystems in order to circumvent channel impairments. In [5] we have extended this method to discrete-time dynamical systems. Much of the interest in this approach lies in the ease of employing Digital Signal Processors (DSPs) or microcontrollers for their implementation. The proposed scheme is shown in Figure 1.

In this scheme, the information signal $m(n)$ is coded using the x_{k+1} component of the transmitter state vector $\mathbf{x}$ via a coding function $s(n) = c\,(x_{k+1}, m(n))$, so that the information signal can be decoded using the inverse function $m(n) = d\,(x_{k+1}, s(n)) = d\,(x_{k+1}, c\,(x_{k+1}, m(n)))$.

If the channel frequency response $H_C(\omega)$ and the filters in the feedback loop $H_S(\omega)$ are identity systems, i.e., $H_C(\omega) = H_S(\omega) = 1$, for every ω, we have shown in [5] that transmitter and receiver synchronize and $m'(n) \to m(n)$ provided that all the eigenvalues of the matrix A are inside the unit circle. However, if $H_C(\omega)$ is a lowpass or highpass filter

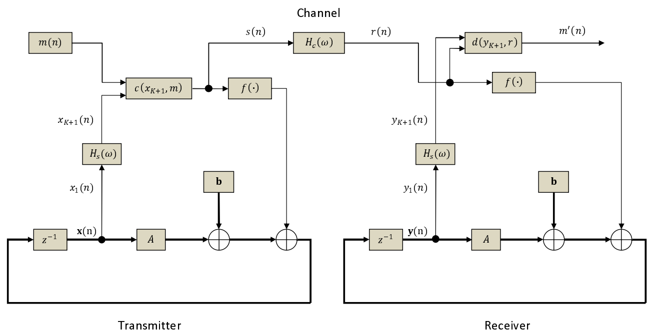

Figure 1 – Discrete-time communication system for bandlimited channels proposed in [5].

with cut-off frequency ω_c we still can attain synchronization and recover $m(n)$ in the receiver if the cut-off frequency ω_s of $H_S(\omega)$ is chosen adequately so that the frequency band affected by $H_C(\omega)$ is previously attenuated. Figures 2 and 3 show examples of transmitted and received signals in the time and frequency domain, respectively when the Hénon map [5] is used.

Although this approach has worked satisfactorily, numerical experiments have shown that depending on the filters employed, the generated signals can cease to be chaotic or diverge. It has also remained to formally prove that transmitter and receiver synchronizes independently of the chosen $H_S(\omega)$.

In the present paper we provide an analytical demonstration that synchronization is not affected when identical, linear, time-invariant, Finite Impulse Response (FIR) filters [5] are included in both the transmitter and receiver subsystems. Furthermore, we numerically investigate for which filter's orders and cut-off frequencies it is possible to obtain chaotic signals. One of our preliminary results is shown in Figure 4. In this figure, we show how the maximum Lyapunov exponent h of the transmitter system varies with the cut-off frequency ω_c when $H_S(\omega)$ is a lowpass Nth-order FIR filter.

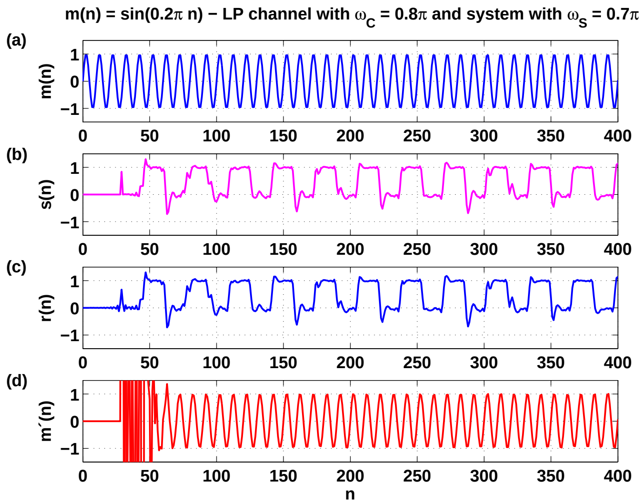

Figure 2 – Examples of transmitted and received signals in time domain for a lowpass channel with $\omega_c = 0.8\pi$ and $\omega_s = 0.7\pi$: (a) message $m(n)$; (b) transmitted signal $s(n)$; (c) received signal $r(n)$; (d) recovered message $m'(n)$ [5].

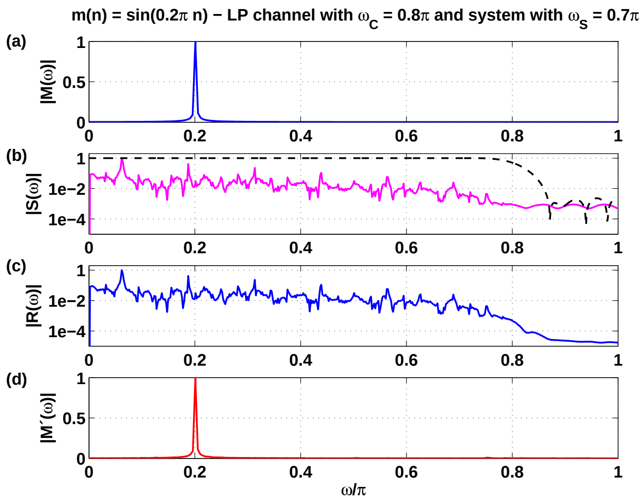

Figure 3 – Examples of transmitted and received signals in frequency domain for a lowpass channel with $\omega_c = 0.8\pi$ and $\omega_s = 0.7\pi$: (a) message $m(n)$; (b) transmitted signal $s(n)$ and frequency response of the channel (dashed line); (c) received signal $r(n)$; (d) recovered message $m'(n)$ [5].

As chaos is characterized by $h > 0$, we can easily see for which values of these parameters we have chaotic signals.

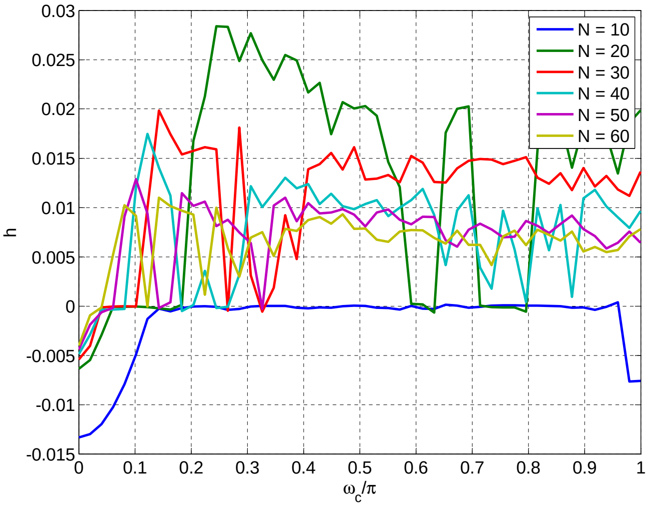

Figure 4 – Preliminary analysis on the variation of the maximum Lyapunov exponent h when $H_S(\omega)$ is a lowpass Nth-order FIR filter with cut-off frequency ω_c.

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