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SPATIAL HIGH ORDER NUMERICAL SOLUTION FOR FLUID DYNAMICS APPLICATIONS

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1. INTRODUCTION

Over the past several years, the Computational Aerodynamics Laboratory of Instituto de Aeronáutica e Espaço (IAE) has been developing CFD solvers for two and three dimensional systems [1]. One research area of the development effort is aimed at the implementation of high-order methods suitable for problems of interest to the Institute, i.e., external high-speed aerodynamics. The group have implemented essentially non-oscillatory (ENO) and weighted essentially non-oscillatory (WENO) schemes in the past [2]. However, both classes of methods were found to be very demanding on computational resources for resolution orders greater than three, in 2-D, or anything greater than 2nd order, in 3-D. This fact motivated the consideration of the spectral finite volume method (SFV), as proposed by Wang [3], as a more efficient alternative. The numerical solver is currently implemented for the solution of the 2-D Euler equations in a cell centered finite volume context for triangular meshes, with a lower upper symmetric Gauss-Seidel (LU-SGS) scheme for time integration.

2. NUMERICAL FORMULATION

In the present work, the 2-D Euler equations are solved considering the finite volume context, as in Ref. [2]. The authors assume that the computational mesh is always composed of triangular elements. For a given order of spatial accuracy, k, using the SFV method, each SV_i element must have at least

$$m = \frac{k(k+1)}{2} \tag{1}$$

degrees of freedom. This corresponds to the number of control volumes that SV_i shall be partitioned into. Once the cell-averaged conservative variables vector, q, is available for all CVs within SV_i, a polynomial, $p_i(x,y) \in P^{k-1}$, with degree $k-1$, can be reconstructed to approximate the $q(x,y)$ function inside SV_i, i.e.,

$$p_i(x,y) = q(x,y) + O(h^{k-1}), \qquad (x,y) \in SV_i, \tag{2}$$

where h represents the maximum edge length of all CVs within SV_i. Note that this polynomial approximation is valid within SV_i and some numerical flux coupling is necessary across SV boundaries. As the numerical flux integration in the present paper is based on one of the forms of a Riemann solver, this is the mechanism which introduces the upwind and artificial dissipation effects into the method, making it stable and accurate. The authors have used the Roe method [4] to compute the numerical flux. A first order implicit LU-SGS time-march algorithm is used for temporal discretization since we are interested in steady-state problems.

Table 1 – Polynomial base functions.

Reconstruction Order	e
linear	$[\,1\ x\ y\,]$
quadratic	$[\,1\ x\ y\ x^2\ xy\ y^2\,]$
cubic	$[\,1\ x\ y\ x^2\ xy\ y^2\ x^3\ x^2y\ xy^2\ y^3\,]$

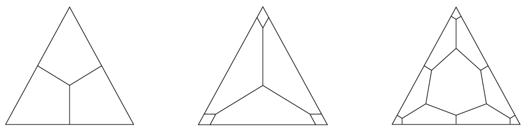
3. DATA RECONSTRUCTION

For the SFV method reconstruction one needs to partition a SV following Eq. (1) and use the polynomial expansion base vector as defined in Table 1. The partitions used in this work are presented in Fig. 3. Moreover, for the Euler equations, it is necessary to limit some reconstructed properties in order to maintain stability and convergence of the simulation, if the resulting flowfield contains discontinuities. A Total Variation Bounded (TVB) limiter technique is employed.

4. NUMERICAL RESULTS

4.1. Wedge Flow

The computation of the supersonic flow field past a wedge with half-angle $\theta = 10$ deg is considered. The computational mesh has 816 nodes and 1504 volumes and it is shown in Fig. 2, along with the density contours obtained with 4th-order



(a) (b) (c)

Figure 1 – Triangular spectral volume partitions for (a) linear, (b) quadratic and (c) cubic reconstructions.

SFV method. For comparison purposes, the second, third and fourth order SFV methods were utilized along with WENO schemes. The numerical solutions of the SFV method are in good agreement with the analytical solution. In Fig. 3 we compare the numerical solutions of the SFV and WENO schemes, in terms of pressure coefficient values, with the analytical one. Note that the SFV scheme is the one that better approximates the jump in pressure on the leading edge.

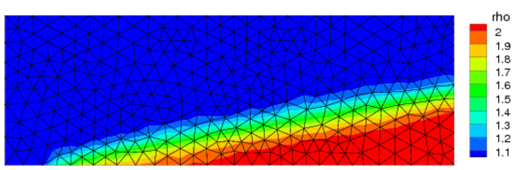


Figure 2 – Supersonic wedge flow unstructured mesh with density distribution obtained with 4th order SFV.

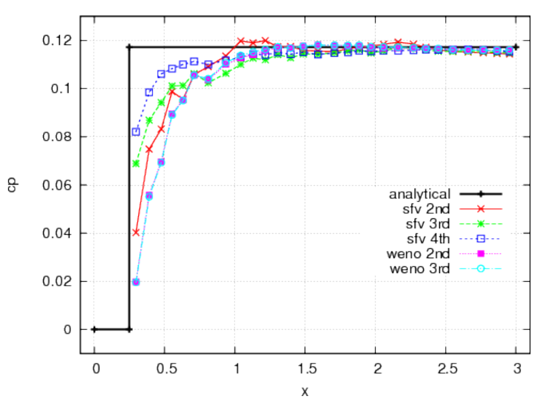
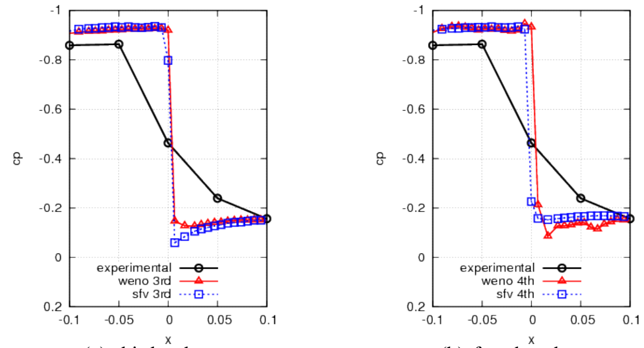


Figure 3 – Supersonic wedge flow analytical and numerical wall pressure coefficient distributions.

4.2. NACA 0012 Airfoil

For the NACA 0012 airfoil simulation, the flow conditions are freestream Mach number of $M_\infty = 0.8$ and 0 deg. angle-of-attack. The mesh has 8414 elements and 4369 nodes. Figure 4 shows the Cp plots of the numerical simulations for both WENO and SFV methods of 3rd and 4th order orders. Their agreement with the experimental data, in terms of shock position and pressure coefficient (Cp) values, is very reasonable.



(a) third order (b) fourth order

Figure 4 – Experimental (black line), SFV (blue line) and WENO (red line) Cp distributions at shock position for NACA 0012 airfoil.

5. REMARKS

The method behavior for resolution orders greater than two is shown to be in good agreement with both experimental and analytical data. Furthermore, the results obtained show that the current method can yield solutions with the same or better quality, at a much lower computational resource usage, than other high order schemes, as indicated by the comparison with the WENO scheme. The method seems suitable for the aerospace applications of interest to IAE in the sense that it is compact, given the fact that the stencil for polynomial reconstruction is always known, geometrically flexible, by supporting unstructured meshes, and computationally efficient.

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