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DIGITAL MODULATION USING HIGH PERIOD UNSTABLE PERIODIC ORBITS

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1. INTRODUCTION

Transmission of digital information relying on sequences generated by chaotic maps presents some interesting features related to the possible enhancement of communication system security against unauthorized access. However, the efficiency of such digital modulation schemes in additive white Gaussian noise - AWGN channels is inferior to that obtained with more conventional modulation techniques commonly based on periodic signals [3].

One alternative that seems to represent a good trade-off between the aforementioned characteristics is the use of high period Unstable Periodic Orbits – UPO from chaotic systems as symbols to be transmitted [1].

A preliminary study is presented in this work comparing two digital modulation strategies for binary transmission. The first strategy is the Modified Maximum Likelihood Chaos Shift Keying – ML-CSK with two logistic-like unimodal maps as proposed in [4]. The second strategy called Maximum Likelihood Unstable Periodic Orbits Shift Keying – ML-UPOSK is a modification of the first one towards the use of high period UPO instead of chaotic sequences. In both methods, following previous work on the subject [3], the Viterbi algorithm is used in the process of detecting the noise contaminated transmitted symbols in order to enhance the overall communication system efficiency.

2. COMMUNICATION SYSTEM

In Fig.1 an overview of the communication system is depicted. A binary message is used to produce a flow of two symbols injected into an AWGN channel, such that to every “zero” in the message the sequence s_1 is transmitted, and to each “one” in the message the sequence s_2 is transmitted. Both symbols s_1 and s_2 are sequences of real values between -1 and 1.

The noise contaminated flow of symbols is then processed using two Viterbi algorithms with parameters corresponding to each one of the possible transmitted symbols. The result from each Viterbi algorithm is the probability of the most probable sequence of real values corresponding to the received symbol. The decision to identify if a “zero” or a “one” was received is based on the comparison between these estimated probabilities. This demodulation process could be cathegorized as non-coherent demodulation because in the receiver there are no copies of the sub-systems used to generate symbols s_1 and s_2.

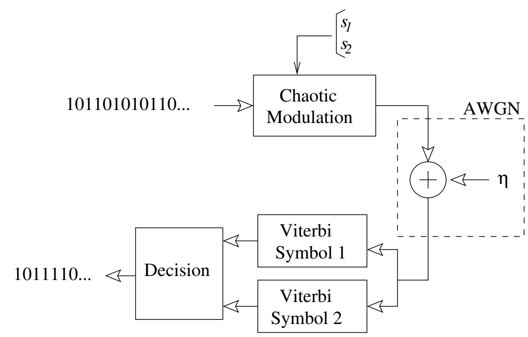

Figure 1 – Communication system overview.

3. LOGISTIC MAP HIGH PERIOD UPO GENERATION

In order to generate the symbolic sequences s_1 and s_2, the procedure shown in Fig.2 was used. This was based on the method proposed in [2], and used in [3], to obtain valid chaotic sequences from the Bernoulli Shift Map while properly accounting for finite computer precision.

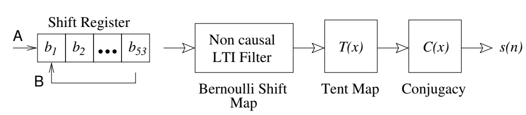

Figure 2 – Chaotic and UPO sequences generation.

In the simulation program real numbers are represented using 53 bits, which limits the capacity of reproducing more than 53 points of the Bernoulli Shift Map by using the conventional procedure of iterating its corresponding chaotic map. In the method shown in Fig.2, a binary number of 53 bits, stored in a shift register, is fed to a non-causal linear filter with a simple impulse response [2] such that a real number is generated at each time step, and the contents of the digital register is shifted. If a new randomly chosen bit is stored at the first position of the shift register at each time step, a chaotic sequence is produced (path “A” in the figure). If the shift register is otherwise rotated, a periodic signal is produced with period ranging from 1 to 53, depending on the initial binary number stored in the shift register (path “B” in the figure).

It is important to note that the Logistic Map is conjugated to the Tent Map, which is related to the Bernoulli Shift Map, such that:

$$T^2(x) = T(B(x)), \tag{1}$$

$$L(x) = -\frac{\cos\left[\pi(T(x)+1\right]}{2}, \tag{2}$$

where $B(x)$ denotes the Bernoulli Shift Map; $T(x)$ represents a symmetric Tent Map; and $L(x)$ is a Logistic Map. By using the relations above, one is able to generate both chaotic and periodic sequences of the afore mentioned unimodal maps relying on the procedure depicted in Fig.2. This leads to the possibility of generating high period UPO of the Logistic map by using the path “B” in Fig.2 together with equation (2).

4. SIMULATION RESULTS

Two different systems were simulated. In both cases symbols of length equal to 10 real values were used, and the noise contamination was implemented by considering the energy associated, at each time step, to each symbol and the corresponding desired relation Energy per bit to noise power spectrum density E_b/N_0.

In the first system (ML-CSK) it was used as symbol s_1 chaotic sequences from the Logistic map with values obtained by iterating $x_{k+1} = L(x_k)$, and as symbol s_2 it was employed chaotic sequences from the related map $x_{k+1} = -L(x_k)$. In the second system (ML-UPOSK) period 10 unstable periodic orbits were used as symbols s_1 and s_2. In order to choose the best pair of UPOs, in the sense of obtaining the smallest Bit Error Ratio – BER for a given signal-to-noise ratio, approximately 900 randomly chosen *unique* combinations of period-10 UPOs were tested with $E_b/N_0 = 10\,\text{dB}$.

After selecting one of the best pairs of UPOs, the system efficiency was simulated for different values of E_b/N_0. The result is shown in Fig.3, where the greater efficiency of the ML-UPOSK system is evident.

Among other interesting results found in this preliminary work is the fact that it was not observed a strong correlation between orthogonality β of the randomly chosen pairs s_1 and s_2 of UPOs; defined as

$$\beta = \frac{|\langle s_1, s_2\rangle|}{\|s_1\|\|s_2\|}; \tag{3}$$

and the communication system performance. This can be seen in Fig.4.

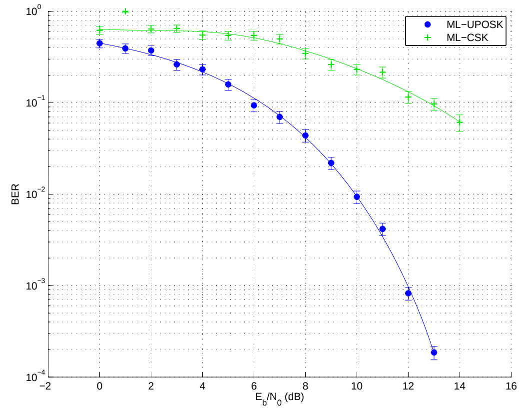

Figure 3 – Communication system efficiency.

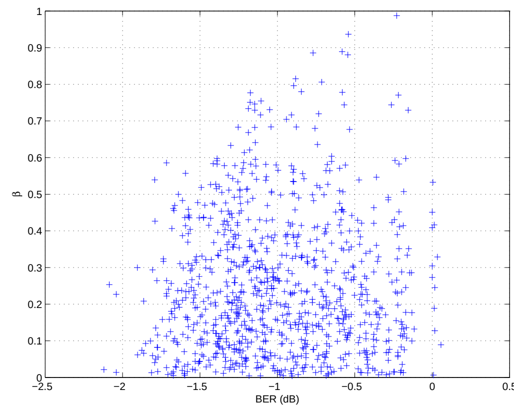

Figure 4 – Relation between orthogonality of UPOs and resulting BER for $E_b/N_0 = 10\,$dB.

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