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SHADOWING OF TRAJECTORIES IN THE STANDARD MAP

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1. INTRODUCTION

Computacional solutions of mathematical systems modelling physical situations presents many erros due to truncations and roundoff. If chaos cannot ruled out than small errors are amplified with time. In this way, even the same initial condition results in diferent states after some time if we iterate the initial condition using single precision and double precision. At first, there is no reason to rely on numerical solutions. There is a lemma due to Bowen [2] that allows to assure that a noisy solution is a approximation of a real solution, that is to say a solution of the system. This approach works very well when the phase space of the system presents hyperbolic structure. However, the most of physical situations of interest are modelling by dynamical systems that presents non-hyperbolicity. The lemma due to Bowen is know by shadowing. In 1990, Grebogi and collegues [1] showed that even in systems were phase space do not present hyperbolic structure, it is possible to shadow noisy trajectories by true trajectories for long time. The aim of this work is to understand how shadowing allow to say something about validity of numerical solutions and to know the unstable periodic orbits structure for standard map. To accomplish this, we study unstable periodic orbits and manifolds in standard map.

2. ANALYSIS

Unstable periodic orbits are the most fundamental blocks in dynamical systems theory. They form a Cantor set and have zero Lebesgue measure. For this reason, it is impossible to found them by generation of random numbers. The method we use for those task is by Schmelcher and Diakonos (1997) of stability transformation:

$$\mathbf{r}_{n+1} = \mathbf{r}_n + \Lambda_k[\mathbf{f}(\mathbf{r}_n) - \mathbf{r}_n], \qquad (1)$$

where $\mathbf{f}(\mathbf{r}_n)$ is the map evaluated at the point $\mathbf{r}_n$ and Λ_k is a matrix that assures the stabilization. The map we investigate is know as standard map:

$$M : \begin{cases} x_{n+1} = x_n + y_{n+1} & \pmod 1 \\ y_{n+1} = y_n + \frac{K}{2\pi}\sin 2\pi x_n & \pmod 1 \end{cases} \qquad (2)$$

For manifolds, we choose a ball of initial conditions centered at $\mathbf{r}_p^*$ - a point of some period p for $p = 1, \dots$ - and iterate many initial points foward in time to obtain the unstable manifolds and iterate backward in time to obtain the stable manifolds.

3. RESULTS

Phase space of standard map are formed by disjoints regions of regular and chaotic motion. In principle, the interest is to understand shadowing in chaotic sea.

Some periodic points in phase space are showed in Figure 1.

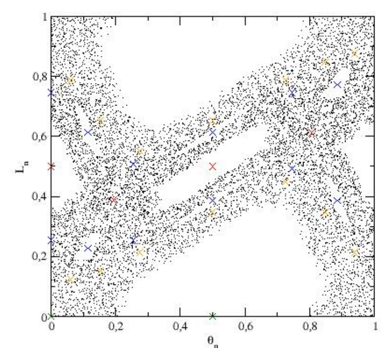

Figure 1 – Two fixed points at $(0, 0)$ and $(0, 1/2)$, respectively, and points of period $p = 2, 3, 4$. The unstable periodic orbits was found with Schmelcher and Diakonos method.

The stable and unstable manifolds are showed in figure 2, where is possible to see homoclinic tangency that ocurr with stable and unstable manifolds at some points. In regions like this shadowing cannot be done and it is not possible to rely in numerical solutions.

To choose a radius of the ball of initial conditions we evaluate the distance of the first iterate in original map and in linearized map as shown in figure 3.

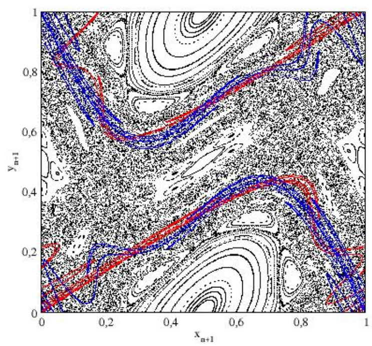

Figure 2 – Stable (blue) and unstable (red) manifolds of the $(0, 0)$ fixed point for $K = 1.5$.

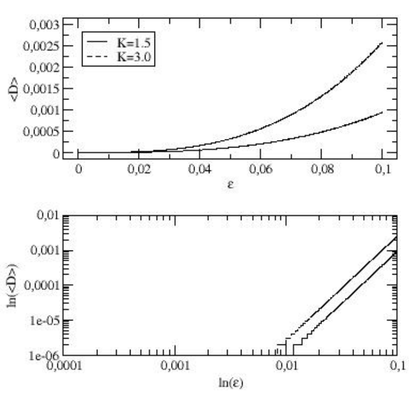

Figure 3 – Calculation of maximum distance of linearized map and original map for the first iterate.

By figure 3 it is possible to see that the average distance between the first iterate on original map and the first iterate on linearized map scalles with

$$\langle D \rangle \sim \epsilon^{\gamma}, \tag{3}$$

where γ is scalling factor and ϵ is the radius of initial condition ball.

4. CONCLUSIONS

We found regions of tangencies ocurr in phase space of standard map for some values of nonlinearity parameter K. In this regions, there is no guarantee that numerical solutions are valid. We also test Schmelcher and Diakonos method for standard map and found unstable periodic orbits. With the knowledge of unstable periodic orbits it is possible to determine stable manifolds and unstable manifolds, because those curves cross in points of some period.

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