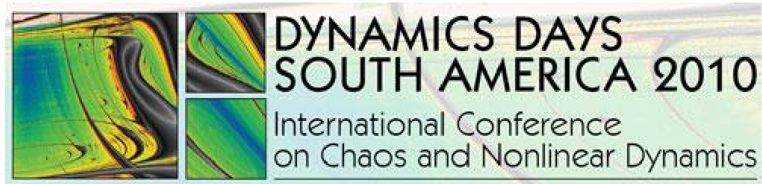

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STABILITY OF A RESTRICTED THREE-BODY PROBLEM WITH THE SPHERICAL PLUMMER POTENTIAL

Steklain, A.F.[1] *and Letelier, P.S.*[2]

[1]Federal University of Technology, Curitiba, Brazil, steklain@utfpr.edu.br
[2]UNICAMP, Campinas, Brazil, letelier@ime.unicamp.br

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The halo of a galaxy is a near-spherical mass distribution around the bulge. The visible mass of the halo is less than the mass of the bulge, but some recent works appoint that dark matter can contribute significantly to the halo mass. For our galaxy, the mass ratio between halo and bulge is of order 10^{-1} [1]. The mass distribution of the halo is in general given by the Plummer potential [2]:

$$\Phi_P = -\frac{Gm_h}{\sqrt{r^2+b^2}}, \tag{1}$$

where G is the gravitational constant, m_h the total mass of the halo, b the typical radius of the halo and r the distance from the center.

In this work we study the stability of a circular, planar restricted three-body problem, say a galaxy, a star (at a fixed distance R from the galaxy) and a planet. We employ a potential composed by the Plummer and Newtonian potentials. This potential mimics the gravitational influence of a spherical galaxy including its halo. The equations of motion of this problem are given by

$$\ddot{x} = 2\dot{y} + \left(3 - \frac{1}{r^3} - 3\frac{m_h^*}{m_g}\frac{b^2}{1+b^2}x\right)x, \tag{2}$$

$$\ddot{y} = -2\dot{x} - \frac{y}{r^3}, \tag{3}$$

where m_g is the total mass of the galaxy and $m_h^* = m_h/\left(1+(b/R)^2\right)^{3/2}$.

This system is Hamiltonian and have only one constant of motion, the Jacobi constant, given by

$$C_J = \frac{1}{2}\left(\dot{x}^2+\dot{y}^2\right) - \frac{1}{r} - \frac{3}{2}\frac{m_h^*}{m_g}\frac{b^2}{1+b^2}x^2. \tag{4}$$

From the equations of motion we can study the stability of this system. For this purpose we employ two Dynamical System techniques, namely Poincare Sections and Lyapunov exponents [3, 5, 6]. We integrate all the trajectories numerically using a Burlisch-Stoer algorithm [7] with $C_J = -2.17$,

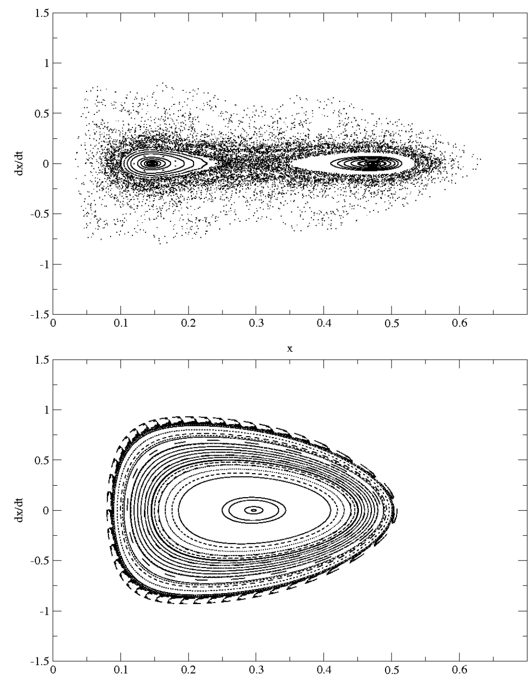

Figure 1 – PoincareSections for different values for the mass ratio m_h^*/m_g. Up: $m_h^*/m_g = 5.10^{-2}$. Down: $m_h^*/m_g = 1$.

for several values of the mass ratio m_h/m_g. Some results are presented in Figures 1 and 2.

As the mass ratio m_h/m_g increases more KAM tori are restored (as can be seen from the Poincare Sections), and the exponential divergence of near orbits is decreased (as can be seen from the Lyanpunov Exponents). Thus we find that the elevation of the gravitational influence of the halo stabilizes the system. This behavior can be explained from the decrease of the potential energy in the presence of the halo, when compared with a system where all the mass is concentrated in the bulge.

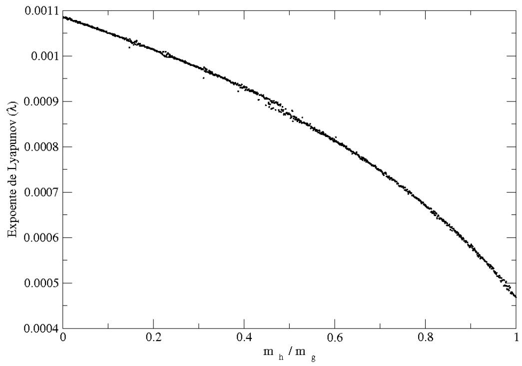

Figure 2 – Lyapunov Exponents as a function of the mass ratio m_h^*/m_g.

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