DYNAMICS DAYS SOUTH AMERICA 2010
International Conference on Chaos and Nonlinear Dynamics

INPE – National Institute for Space Research
São José dos Campos – SP – Brazil – July 26-30, 2010

STATIONARY SHAPES OF ROTATING MAGNETIC FLUID DROPLETS

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keywords: Fluid Dynamics, Plasma and Turbulence; Formation and Dynamics of Patterns.

An effectively two-dimensional (2D) version of the problem of rotating fluid droplets can be examined when a fluid drop is placed in the spatially confined environment of a rotating Hele-Shaw cell. The rotating Hele-Shaw problem is a variation of the traditional viscosity-driven Saffman-Taylor instability [1], in witch a cell composed of two narrowlly spaced parallel plates rotates, and the competition between centrifugal and capillary forces results in interface destabilization and pattern formation [2].

A suggestive variant of the usual rotating Hele-Shaw problem considers that the spinning fluid is a magnetic fluid, a superparamagnetic liquid witch promptly responds to even modest magnetic stimuli. Two different kind of magnetic fluids can be considered: ferrofluids [3] and magnetorheological (MR) fluids [4]. In contrast to ferrofluids witch are colloidal suspensions containing nanometersized magnetic particles, MR fluids consist of much larger, micronsized particles. Ferrofluids generally behave like Newtonian liquids, while MR fluids can respond elastically to deformations due to the presence of long magnetic particle chains, presenting a magnetic field-dependent yield stress [5]. The properties of these kinds of fluids turn out to be very handy since it introduces the possibility of controlling the rotating fluid interface by tuning an external magnetic field.

In this work we study theoretically the family of steady shapes witch arise when a magnetic liquid droplet is confined in a rotating Hele-Shaw cell, and subjected to an azimuthal magnetic field produced by a current carrying wire. As in Refs. [6, 7] we apply a vortex-sheet formalism at an equilibrium situation and derive the nonlinear ordinary differential equation for the in-plane curvature of the static droplet interface

$$\begin{aligned}\kappa(r, r\cos\psi) = a + b\,r^2 \quad &+ \quad \frac{c}{r^2}\left[1 + d\,(\cos\psi)^2\right] \\ &- \quad e\,r + \frac{f}{r},\end{aligned} \tag{1}$$

where r is the polar radius and ψ is the angle between the radius vector and the tangent vector at the interface. The coefficients b, c, d, e and f are related to dimensionless physical parameters, and a is a constant of integration. The physical contributions taken into account in Eq. (1) include the rotating driven centrifugal force, capillary forces due to surface tension, the magnetic force produced by the external applied field and also a magnetic field-dependent yield stress.

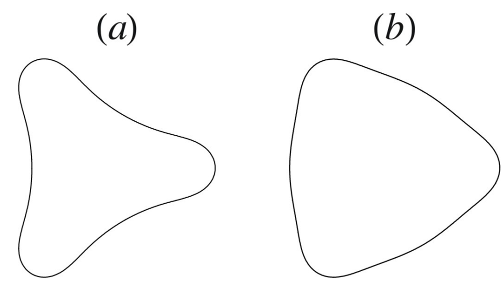

Figure 1 – Fully nonlinear stationary patterns of magnetic fluid droplets confined in a rotating Hele-Shaw cell under the influence of an applied azimuthal magnetic field. Two different types of fluids are considered: (a) ferrofluids and (b) MR fluids.

Various stationary patterns are obtained by solving Eq. (1) for different sets of parameters and by imposing the commensurability of the droplet interface. Two distinct scenarios are contrasted: first, the magnetic fluid is assumed to be a Newtonian ferrofluid of negligible yield stress (Fig. 1(a)), and then it is taken as a viscoelastic magnetorheological fluid by the inclusion of a magnetic-induced yield stress (Fig. 1(b)). The influence of distinct material properties of the fluids on the ultimate morphology of the emerging stationary patterns is then scrutinized.

Curiously, some of the exact steady structures we found are similar to the advanced time patterns obtained by existing time-evolving numerical simulations [8]. Impelled by this fact, we have employed a second-order perturbative mode-coupling approach to gain analytical insight about the stability of such shape solutions and its connection with the time evolution of the unperturbed droplet. We present the evolution of the interface perturbation in terms of its Fourier amplitudes

$$\dot{\zeta}_n = \lambda(n)\,\zeta_n + \sum_{n'\neq 0}\Big[F(n,n')\,\zeta_{n'} + G(n,n')\,\dot{\zeta}_{n'}\Big]\,\zeta_{n-n'}, \tag{2}$$

where n labels the harmonic modes of the perturbation ζ, the overdot represents a total derivative with respect to time, $\lambda(n)$ denotes the linear growth rate, and the functions $F(n,n')$ and $G(n,n')$ are second order mode-coupling terms.

In this context, the approximate interface evolution obtained with a weakly nonlinear analysis can be compared with the exact stationary solutions. We discuss to what extent a few perturbative orders can account for the entire exact pattern and hence, a certain truncation in the number of Fourier modes involved can be regarded as a good representation of the exact shapes.

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