DYNAMICS DAYS SOUTH AMERICA 2010
International Conference on Chaos and Nonlinear Dynamics

INPE – National Institute for Space Research
São José dos Campos – SP – Brazil – July 26-30, 2010

Intrinsic stickiness in open integrable billiards: border effects

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keywords: Applications of Nonlinear Sciences, Chaos in Hamiltonian Systems.

Using the example of an integrable billiard model we show here that the *rounded shape* of the open set (the escape point or hole) induces stickiness and chaotic decays for the escape times statistics and self-similar structures for the escape times and emission angles. Rounded borders are common in experiments with semiconductors devices [1] and quantum cavities [2], where the borders have a shape very similar to those shown in Fig. 1, which is the model used here (Type I and II). What is the effect of rounded open sets on the dynamics ? Recent works in this direction analyzed the effect of the *width* of the open set on the escape rates of particles in open billiards [3]. The ETs statistic is defined by

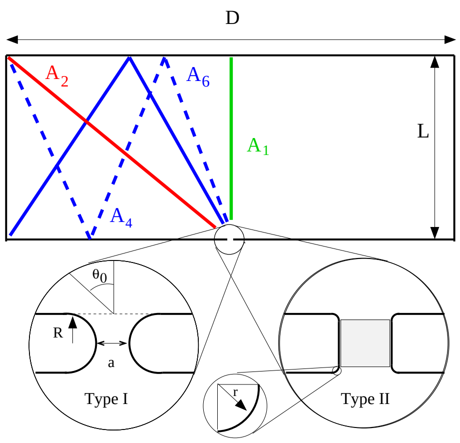

Figure 1 – The rectangular billiard, with dimension $L \times D$, showing borders Type I (with radius R) and II (radius $r \approx R/20$) considerer here. The escape point lies exactly in the middle of the billiard and has constant aperture $a = 1 \times 10^{-4}$. Initial angle θ_0 and, schematically, the shortest escape trajectories (P, E, A_2 and A_4) are shown. In all simulations we use $L = 4$ and $D = 10$.

[4] $Q(\tau) = \lim_{N\to\infty} \frac{N_\tau}{N}$, where N is the total number of trajectories which escape the billiard and N_τ is the number of trajectories which escape the billiard after the time τ. For systems with stickiness the ETs statistic decays as a power law $Q(\tau) \propto \tau^{-\gamma_{esc}}$, where $\gamma_{esc} > 1$ is the scaling exponent. For hyperbolic chaotic systems and long times the ETs statistic decays exponentially.

In the simulations particles start from the escape point with an initial angle θ_0 towards the inner part of the billiard. We use 10^5 initial conditions distributed uniformly in the interval $0.10 \leq \theta_0 \leq 1.54$. Results for $Q(\tau)$ are shown

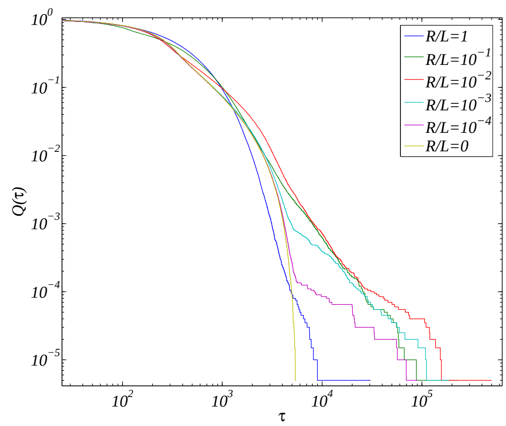

Figure 2 – (Color online) Behavior of $Q(\tau)$ for different values of the ratio R/L.

in Fig. 2 for $R/L = 0, 1/10000, 1/1000, 1/100, 1/10, 1$. First observation is that for $R/L = 0$, $Q(\tau)$ did not have a power law. This is the integrable case and the maximal ET found is $\tau \sim 5.7 \times 10^2$. For very small rounding effects, $R/L = 1/10000$, the qualitative behaviour of $Q(\tau)$ starts to change (when compared to the case $R/L = 0$) for times $\tau \geq 5.7 \times 10^2$, i. e. for those trajectories which stay longer inside the billiard. The fitted escape exponent is $\gamma_{esc} \sim 0.6$. By increasing the border to $R/L = 1/1000$, we observe a power law decay in Fig. 2 for times $\tau \geq 5.5 \times 10^2$. The escape exponent is $\gamma_{esc} \sim 1.3$. For $R/L = 1/100$ and $\tau \geq 3 \times 10^3$ we obtain $\gamma_{esc} \sim 1.8$ and for $R/L = 1/10$ and $\tau \geq 1 \times 10^3$ we obtain $\gamma_{esc} \sim 2.1$.

Figure 3 shows the ETs (τ) and escape angles θ_f plotted as function of the initial incoming angle θ_0 and for different ratios R/L. They were generated by using 500×500 points in the intervals $0.10 \leq \theta_0 \leq 1.54$ and $0.00355 \leq R/L \leq 1.0$ $[-8.0 \leq \log(R/L) \leq 0.0]$. In Fig. 3(a) each color rep-

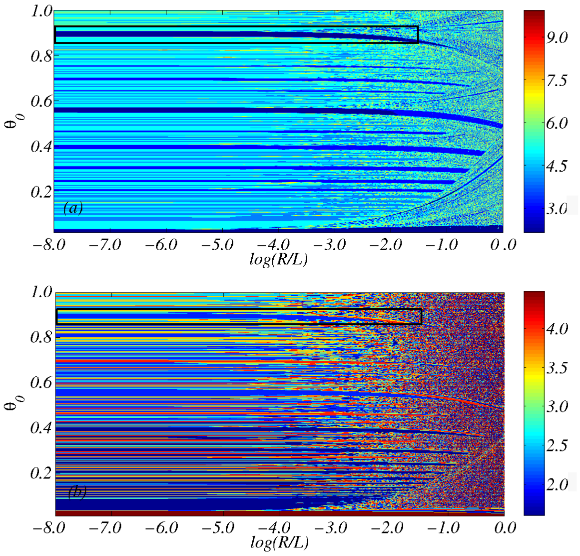

Figure 3 – (Color online) (a) Log of the escape time and (b) emission angle θ_f as a function of $\log{(R/L)}$ and θ_0.

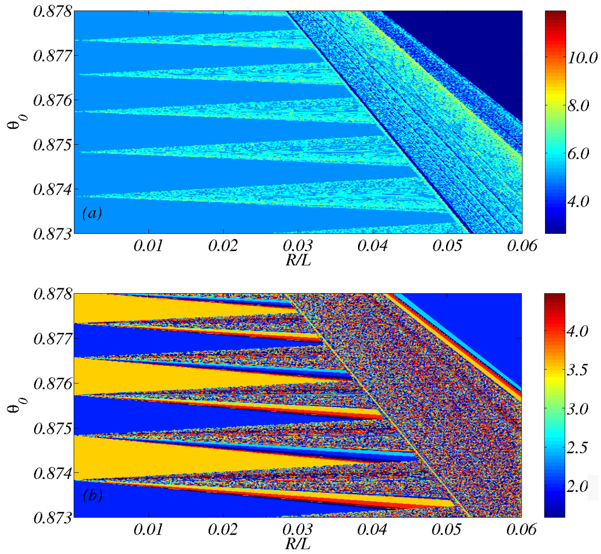

Figure 4 – (Color online) Magnifications from Figs. 3(a)-(b), respectively.

resents a given value of the ETs written as $\log{(\tau)}$. Horizontal stripes with different colors are evident for a significant range of R/L values. Each stripe is defined by a bunch of initial conditions which leave to the same ET and consequently have the same color. For example, for some specific initial angles ($\theta_0 \sim 0.39, 0.56, 0.69, 0.89$) we observe dark blue stripes which correspond to very short ETs. For $R/L = 0$ some of these angles are $\theta_0^{(n)} = \arctan\left[\frac{D}{2nL}\right]$, where $n = 1, 2, 3, \ldots$. As n increases the ETs from the trajectories A_{2n} increase, the corresponding stripes assume other colors (light blue $\rightarrow$ green $\rightarrow$ yellow) [see Fig. 3(a)] and their widths decrease.

The emergence of the power law behavior becomes more evident if we compare Fig. 4(a) with the emission angle behavior shown in Fig. 4(b). The light blue background observed in Fig. 4(a), which corresponds to *one* ETs, has two colors (blue and orange) in Fig. 4(b), which correspond to *two* emission angles ($\sim$ 2.1 and $\sim$ 3.5). Stripes remembering "backgammon" like stripes are visible. The sequence of backgammon stripes in Fig. 4(a), and the corresponding multicolor backgammon stripes from Fig. 4(b), are born at the boundary between the blue and orange escape angles at $R/L \sim 0$. Inside the backgammon stripes the range of allowed ETs increases very much (observe the number of yellow and red points). The dynamics involved in the emission angles inside the backgammon stripes is also impressive, showing that tiny changes, or errors, in the initial angle may drastically change the emission angle. The key observation here is that the dynamics *inside* the backgammon stripes is the consequence of trajectories which collide with the *inner part* of the semicircle from the escape point, generating the power law behavior for $Q(\tau)$ (see Fig. 2).

The ETs from the long living trajectories present characteristic of sticky motion for $R/L = 1/1000, 1/100, 1/10$. In other words, sticky motion and long living trajectories start to occur for *very* small rounding borders. Visually such borders are almost negligible. Take for example the border in Fig. 1, it has a ratio $R/L \sim 1/143$. Therefore rounding borders of around 0.1% from the whole billiard size are sufficient to generate the sticky motion and change the dynamics.

The authors thank CNPq and CAPES for financial support and FINEP (under project CTINFRA).

References

[1] C. Marlow, et al., "Unified model of fractal conductance fluctuations for diffusive and ballistic semiconductor devices", Phys. Rev. B, Vol. 73, No. 195318, May 2006.

[2] Y. Takagaki, et al., "Magnetic-field-controlled electron dynamics in quantum cavities", Phys. Rev. B, Vol. 62, No. 10255, October 2000.

[3] E. G. Altmann, and T. Tél, "Poincaré recurrences and transient chaos in systems with leaks", Phys. Rev. E, Vol. 79, No. 016204, January 2009; C. P. Dettmann, and O. Georgiou, "Survival probability for the stadium billiard", Physica D, Vol. 238, No. 2395, October 2009; L. A. Bunimovich, and C. P. Dettmann, "Peeping at chaos: Nondestructive monitoring of chaotic systems by measuring long-time escape rates", Europhys, Vol. 80, No. 40001, October 2007;

[4] E. G. Altmann, A. Motter, and H. Kantz, "Stickiness in mushroom billiards", Chaos, Vol. 15, No. 033105, June 2005;