

MINKOWSKI SPHEROPOLYHEDRA FOR THE SIMULATION OF GRANULAR MATERIALS

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Discrete element methods (DEM) are basic tools for the research on granular materials. They not just allow for the accounting of velocities and local stresses that are hard to measure in real experiments, but also they give deep insights into the microscopical origin of collective granular phenomena [Poeschel2005]. Two main elements have been widely used to model three-dimensional grains: spheres (or bundles of) and polyhedra. Spheres are much easier to program, their interactions are well known and their elastic forces allow for a simple expression for potential energy. Polyhedra, in contrast, are much harder to program, and their elastic interactions are too complex to be derived from a simple mathematical expression for the potential energy, but they capture the complex shape and related phenomena that soil grains exhibit.

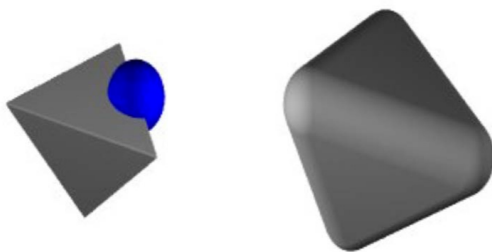


Figure 1 – How to built a spheropolyhedron.

Hereby we present a novel element, called *spheropolyhedron*, that join the advantages of these two previous approaches. It is just the Minkowski opening of a polyhedron with a sphere (Figure 1); therefore spheropolyhedra capture the irregular shape of soil grains but their contours are spheres, cylinders and planes. Since interactions among these contours are well known, the elastic forces and potentials among spheropolyhedra are easy to compute. First proposed by Liebling and Pourin [Pourin2005a, Pourin2005b]

for some simple shapes, they have been extended for irregular and even concave shapes in 2D by Alonso-Marroquin [Alonso2008], and combined with 2D Voronoi tessellations by Galindo-Torres et. al. [Galindo2009a]. Later on, they have been extended by Galindo-Torres et. al. [Galindo2009b] for any polyhedron in 3D (concave or convex), even obtained from three-dimensional Voronoi constructions.

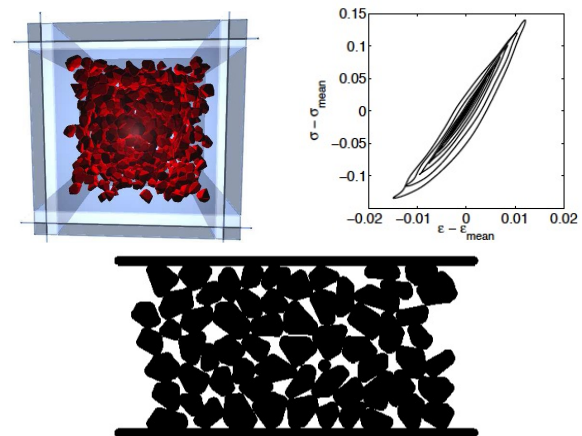


Figure 2 – Some applications. A triaxial test on Minkowski spheropolyhedra obtained from a three-dimensional Voronoi tessellation (upper left). Hysteresis loops from a sample of 2D Minkowski-Voronoi spheropolygons under cyclic loading (upper right). A fully-saturated shear band of Minkowski-Voronoi spheropolygons (bottom).

In this contribution we summarize how spheropolygons and spheropolyhedra are constructed and how they can be employed for the simulation of granular materials [Galindo2010]. Examples are the cyclic loading of a 2D sample, the modeling of a rough surface, the triaxial test of a dry sand and the time evolution of a 2D fully-saturated shear band (Figure 2). Efficiency tests and comparisons are also included. These examples illustrate how these novel discrete elements can be useful for the simulation of granular mate-

rials, specially when the elastic energy needs to be exactly computed.

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