TRAJECTORY OPTIMIZATION USING ATMOSPHERIC MANEUVERS

Walkiria SCHULZ[1]
Antônio Fernando Bertachini de Almeida PRADO[1]
Rodolpho Vilhena de MORAES[2]

(1) *National Institute for Space Research (INPE), 12227-010 – S. José dos Campos – Brazil.*
(2) *DMA-FEG-UNESP, 12516-410 – Guaratinguetá – Brazil.*

***ABSTRACT** - The current space exploration capacity is very limited by the excessive amount of fuel necessary to deliver and to transport loads in space. This situation has motivated the development of techniques for the accomplishment of orbital maneuvers using natural forces (e.g. atmospheric maneuvers) to substitute, at least partly, the propulsive forces. Several control methods of vehicles crossing the atmosphere have being studied to assure the maintenance of the acceleration and of the heating inside limits previously defined. In this scenario, the present work proposes analysis of missions for vehicles that use the terrestrial atmosphere to accomplish orbital changes. A set of analytical solutions for the variation of the orbital elements due to a single passage by the atmosphere is presented, based on previous results by King-Hele [King 1987]. These equations are tested by numerical integration.*

1 - INTRODUCTION

Among all types of purely propulsive maneuvers used to modify satellites' orbits (including size, forms and/or orientation), the costliest is the maneuver to introduce a variation in the orbital plane. The high consumption of a purely propulsive plane change maneuver has motivated the search for alternatives techniques to cause the target changes in a more economical way. One of the most interesting technique uses aerodynamic and propulsive forces combined. Pioneering works on this field are due to London and Nyland, as mentioned in Bruce's article [Bruc 65].

In the present communication, a description of the problem, a numeric analysis and a proposal of an analytical solution for Δa and Δi are presented. The results reflect the several possibilities of application of this theory.

In this paper, the main goal is to develop analytical and numerical tools to compare two approaches to change the inclination of a satellite. The first one is to perform a single impulse in the circular orbit to obtain the plane change desired. The second is more elaborate, and uses an impulse in the satellite to decrease its orbit, so it can reach the atmosphere and perform the inclination using natural effects instead of fuel consumption. Attention is given to the balance between the cost of correcting the decay in the semi-major axis and the fuel economy in the inclination change. The cost to send the satellite to and from the atmosphere is not considered at this point of this research.

2 - EQUATIONS OF MOTION INSIDE THE ATMOSPHERE

The motion of a space vehicle in a gravitational field, subject to aerodynamic forces, is given by:

$$\frac{d\vec{r}}{dt} = \vec{v} \tag{2.1}$$

$$\frac{d\vec{v}}{dt} = \frac{\vec{A}}{m} + \vec{g}(\vec{r}, t) \tag{2.2}$$

In these equations, $\vec{r}$ is the position vector, $\vec{v}$ is the vehicle speed relative to the atmosphere, m is its mass, $\vec{A}$ is the aerodynamic force and $\vec{g}$ is the gravitational acceleration.

The aerodynamic force should be decomposed (as it is done conventionally) in two parts: drag $\vec{F}_D$ and lift $\vec{F}_L$ (Fig. 1).

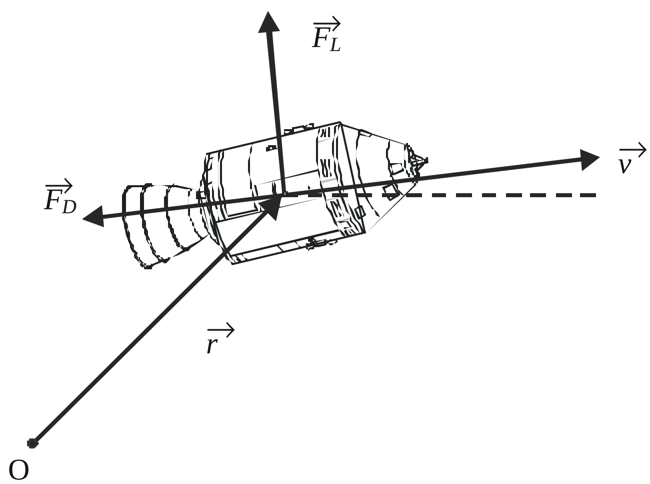

Fig. 1: State variables and aerodynamic forces.

The magnitude of each force is given by [Vinh 81]:

$$F_D = \frac{1}{2}\rho S v^2 C_D \tag{2.3}$$

$$F_L = \frac{1}{2}\rho S v^2 C_L \tag{2.4}$$

The atmospheric density ρ is given by the *U.S. Standard Atmosphere 1976*. C_D and C_L are the drag and lift coefficients which, in turn, depend on the surface area S. These coefficients are functions of the vehicle geometry and the angle of attack (i.e. angle between the vehicle's longitudinal axis and the speed relative to the atmosphere). The lift also depends on the bank angle (that is the angle between the lift plane and the plane formed by the position and velocity vectors).

3 - ANALYTICAL SOLUTION

Besides the possible changes in the Keplerian elements that belong to the same orbital plane, a non-coplanar transfer requires variations in the inclination i and/or in the longitude of the ascending node Ω. In the case of a passage through the atmosphere, part (or the totality of) the plane change is accomplished by aerodynamic forces. The vehicle should be capable of producing lift. The relationship between aerodynamic change and variation of the orbital plane is still not very well established.

Exact closed-form solutions for optimal aero-assisted orbital transfers are not found in the literature, except for very simple and ideal cases. Meanwhile, numeric solutions were obtained through direct and indirect methods.

The present work aims to develop a new relationship between the variation in semi-major axis as a result of the passage through the atmosphere, and the variation on inclination under the same circumstances. This analytical solution is based on the closed-form equations found in [King 87]:

$$\Delta a = -\delta a^2 \rho \exp[\beta(a_0 - a - a_0 e_0)] \int_0^{2\pi} \Big\{ 1 + 2e\cos(E) + \frac{3}{2} e^2 \cos^2(E) + \\ + e^3 \cos^3(E) + O(e^4) \Big\} \exp(B\cos(E))\, dE \tag{3.1}$$

$$\Delta i = -\left(\frac{a}{\mu F}\right)^{1/2} \frac{a^2 w \delta}{4} \rho \,\mathrm{sen}\, i \exp(-B) \int_0^{2\pi} \big[1 + \cos(2\omega)\cos(2E) + \\ - 4e\cos^2(\omega)\cos(E) + O(e^2) \big] \exp(B\cos(E))\, dE \tag{3.2}$$

In these equations, δ is a function of the satellite's aerodynamic characteristics, a is the semi-major axis, ρ is the atmospheric density, β is a function of the satellite's altitude, e is the orbital eccentricity, E is the true anomaly, $B = bae$, μ is the gravitational constant, F is a parameter that changes with the orbit inclination, w is the earth rotation angular speed, i is the orbit inclination and ω is the perigee argument.

Based on these equations we expanded the trigonometric function of E in Taylor series up to 6[th] order and performed the integrations involved. In this way, we were able to find the analytical expressions showed below:

$$\Delta a = -\delta a^2 \rho \exp[\beta(a_0 - a - a_0 e_0)] \Bigg\{ \left[1 + 2e + \frac{3}{2} e^2 + e^3 \right] E + \\ - \left[\frac{B}{2} + e(1+B) + 0{,}75 e^2 (2+B) + \frac{e^3(3+B)}{2} \right] \frac{E^3}{3} + \\ + \left[\frac{B(1+3B)}{24} + \frac{e(1+7B+3B^2)}{12} + 0{,}0625 e^2 (8 + 13B + 3B^2) + \\ + \frac{e^3(21+19B+3B^2)}{24} \right] \frac{E^5}{5} + O(E^7) \Bigg\} \exp(B) \tag{3.3}$$

$$\Delta i = -\left(\frac{a}{\mu F}\right)^{1/2} \frac{a^2 w\delta}{4} \rho \operatorname{sen} i \exp(-B)\Big\{\big[2(1-2e)\cos^2\omega\big]E + \\ + \left[-\frac{B}{2} + 2e(1+B)\cos^2\omega - \frac{(4+B)\cos(2\omega)}{2}\right]\frac{E^3}{3} + \\ + \left[\frac{B(1+3B)}{24} - \frac{e(1+7B+3B^2)\cos^2\omega}{6} + \frac{(16+25B+3B^2)\cos(2\omega)}{24}\right]\frac{E^5}{5} + \\ + O(E^7)\Big\}\exp(B) \tag{3.4}$$

The purpose of this solution is to find equations to preview if a passage through the atmosphere for a target plane change is economically feasible, since the variation in semi-major axis involve an extra fuel expenditure to correct the orbit decay. The relationship that we are looking for is $\frac{\Delta Va}{\Delta Vi}$, where ΔVa is the expression for the velocity increment required to correct the semi-major axis decay due to the atmospheric passage. This is:

$$\Delta Va = \sqrt{\mu\left(\frac{2}{r_p} - \frac{1}{a}\right)} - \sqrt{\mu\left(\frac{2}{r_p} - \frac{1}{a - \Delta a}\right)} \tag{3.5}$$

where r_p is the perigee radius and a is the semi-major axis of the orbit before the passage through the atmosphere.

ΔVi is the expression for the velocity variation required to obtain the inclination variation Δi by using an impulsive increment when the spacecraft is at the initial circular orbit. This expression is given by:

$$\Delta Vi = 2\sqrt{\frac{\mu}{r_C}} sin\left(\frac{\Delta i}{2}\right) \tag{3.6}$$

where r_C is the radius of the initial circular orbit.

With these equations it is possible to evaluate the cost/benefit related to the appropriate maneuvers. The aim is now to find a relationship between the cost of changing the semi-major axis and the inclination due to the atmospheric passage.

These expressions do not consider the influence of lift on the atmospheric passage, according to [King 87] and the numerical simulations showed in this paper.

The expressions for the other orbital elements (e, Ω and ω) where also found. They are:

$$\Delta x = -\delta a^2 \rho \exp[\beta(a_0 - a - a_0 e_0)]\Big\{\left[1 + 2e + \frac{3}{2}e^2 + e^3\right]E + \\ - \left[\frac{(1+B)}{2} + e(1+B) + e^2\frac{(5+3B)}{4} + \frac{e^3(3+B)}{2}\right]\frac{E^3}{3} + \\ + \left[\frac{(1+7B+3B^2)}{24} + \frac{e(4+7B+3B^2)}{12} + e^2\frac{(23+33B+9B^2)}{48} + \\ + \frac{e^3(24+19B+3B^2)}{24}\right]\frac{E^5}{5} + O(E^7)\Big\}\exp(B) \tag{3.7}$$

$$\Delta\Omega = -\left(\frac{a}{\mu F}\right)^{1/2} \frac{a^2 w\delta}{4} \rho \,\mathrm{sen}(2\omega)\Big\{[1-2e]E + + \left[\frac{-4-B+2Be}{2}\right]\frac{E^3}{3} + + \left[\frac{16+25B-2Be+3B^2-6B^2e}{24}\right]\frac{E^5}{5} + O(E^7)\Big\} \tag{3.8}$$

$$\begin{aligned}
\Delta\omega = &-\frac{\delta a k(\rho)}{e}\exp[-\beta a]\Bigg\{\left[1+\frac{1}{4}c^2\right]E + \\
&+\left[\left(1+\frac{1}{4}c^2\right)B + \left(e+(1+e)c\cos(2\omega)+\frac{c^2}{4}\cos(4\omega)\right)\exp(B)\right]\frac{E^2}{2} + \\
&+\left[\left(1+\frac{1}{4}c^2\right)\frac{B^2}{2} - \left(2(1+2e)c\sin(2\omega)+c^2\sin(4\omega)\right)\exp(B)\right]\frac{E^3}{3} + \\
&-\left[\begin{array}{l}\left(1+\frac{1}{4}c^2\right)\frac{B}{2} + \left((4+3B)\frac{e}{6} + [13+3B+e(50+3B)]\frac{c}{6}\cos(2\omega) + \right. \\ \left. + \frac{c^2}{24}(49+3B)\cos(4\omega)\right)\exp(B)\end{array}\right]\frac{E^4}{4} + \\
&+\left[\left(1+\frac{1}{4}c^2\right)\frac{(B^2-16)B^2}{24} + \left[\begin{array}{l}\left(\frac{5}{3}+B+\frac{22e}{3}+2Be\right)c\sin(2\omega) + \\ +\left(\frac{17}{6}+\frac{B}{2}\right)c^2 sin(4\omega)\end{array}\right]\exp(B)\right]\frac{E^5}{5} + O(E^6)
\end{aligned} \tag{3.9}$$

where $x = ae$ and c is a given function of the perigee height and the orbit inclination [King 87].

4 – NUMERICAL RESULTS

In the numerical simulation, the satellite enters and exits the atmosphere at an altitude of 200 km. In the analytical solution it is not possible to integrate "inside" the atmosphere. The integration is limited to the interval of true anomalies between the entrance and exit points. Because of this, a different kind of numerical solution was found that takes into account the true anomaly at the entrance point, and assumes that the satellite will stay inside the atmosphere for an arc of longitude twice this angle (Fig. 2). Therefore, the numerical result was found for the same arc as the analytical one.

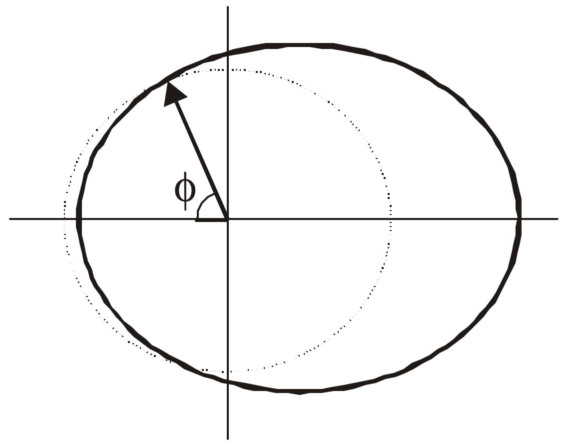

Fig. 2: Angle ϕ (true anomaly) of the atmosphere entrance.

The variations in semi-major axis and inclination due to the satellite passage through the atmosphere obtained with the analytical method, were then compared with those found through numerical simulations. The results are shown on the Tables 1 to 4.

The numerical solutions are presented in four different ways. The first one, entitled "Numerical", attends to the numerical problem resulted from an integration of the equations of motion during the time that the satellite remains under the altitude of 200 km; the second "Numerical 2 phy" are the numerical values that result from the integration of the equations of motion inside the atmosphere during the time that the satellite travels an angle that is twice the angle between the entrance and the perigee, as in the analytical solution; the third "Numerical Lift" is the same as the first numerical result, but now the lift is considered; and the last one "Numerical 2 phy, Lift" is the same as the second one with lift.

Table 1: Variation on semi-major axis (km).

Perigee height (km)	Analytical	Numerical	Numerical 2 phy	Numerical Lift	Numerical 2 phy, Lift
$i = 0°$					
200	-2.16×10^{-3}	-2.47×10^{-3}	-2.05×10^{-3}	-2.47×10^{-3}	-2.05×10^{-3}
170	-6.33×10^{-2}	-5.73×10^{-2}	-4.96×10^{-2}	-5.73×10^{-2}	-4.96×10^{-2}
130	-8.47×10^{-1}	-4.35×10^{-1}	-4.16×10^{-1}	-4.35×10^{-1}	-4.16×10^{-1}
100	-76.82	-19.11	-19.07	-19.11	-19.07
$i = 30°$					
200	-2.17×10^{-3}	-2.51×10^{-3}	-2.08×10^{-3}	-2.51×10^{-3}	-2.08×10^{-3}
170	-6.38×10^{-2}	-5.83×10^{-2}	-5.04×10^{-2}	-5.83×10^{-2}	-5.04×10^{-2}
130	-8.53×10^{-1}	-4.42×10^{-1}	-4.22×10^{-1}	-4.42×10^{-1}	-4.22×10^{-1}
100	-77.43	-19.42	-19.37	-19.42	-19.37
$i = 60°$					
200	-2.22×10^{-3}	-2.60×10^{-3}	-2.15×10^{-3}	-2.60×10^{-3}	-2.15×10^{-3}
170	-6.52×10^{-2}	-6.08×10^{-2}	-5.26×10^{-2}	-6.08×10^{-2}	-5.26×10^{-2}
130	-8.72×10^{-1}	-4.61×10^{-1}	-4.41×10^{-1}	-4.61×10^{-1}	-4.41×10^{-1}
100	-79.09	-20.26	-20.21	-20.26	-20.21

			i = 90°		
200	-2.28 x 10^{-3}	-2.74 x 10^{-3}	-2.26 x 10^{-3}	-2.74 x 10^{-3}	-2.26 x 10^{-3}
170	-6.71 x 10^{-2}	-6.44 x 10^{-2}	-5.57 x 10^{-2}	-6.44 x 10^{-2}	-5.57 x 10^{-2}
130	-8.97 x 10^{-1}	-4.88 x 10^{-1}	-4.66 x 10^{-1}	-4.88 x 10^{-1}	-4.66 x 10^{-1}
100	-81.37	-21.42	-21.38	-21.42	-21.38

In Tables 1 and 2, it should be noted that:

i) As the perigee decreases, the variation suffered by the semi-major axis increases. This is an expected result since a smaller perigee radius means that the satellite is subjected to more atmospheric influence, as it passes more time inside the atmosphere and the atmosphere density increases exponentially;

ii) The effects of the maneuver "Numerical 2 phy" are smaller than the "Numerical" one because the atmosphere causes a decay in the orbit, and the spacecraft travels an angle bigger than 2phy;

iii) The analytical equations were derived assuming that the angle phy is not too large. Thus, as the perigee height decreases (and the phy angle increases), the numerical results start to deviate from the analytical ones. Therefore, the analytical equations are valid only for perigee height above a certain limit.

Table 2: Variation on inclination

Perigee height (km)	Analytical	Numerical	Numerical 2 phy	Numerical Lift	Numerical 2 phy, Lift
			i = 0°		
200	-4.60 x 10^{-12}	-2.73 x 10^{-9}	3.71 x 10^{-11}	-2.73 x 10^{-9}	3.71 x 10^{-11}
170	-2.31 x 10^{-12}	9.15 x 10^{-10}	3.71 x 10^{-11}	9.15 x 10^{-10}	3.71 x 10^{-11}
130	-1.81 x 10^{-11}	9.15 x 10^{-10}	-1.06 x 10^{-9}	9.15 x 10^{-10}	-1.06 x 10^{-9}
100	-1.89 x 10^{-9}	-2.73 x 10^{-9}	-4.33 x 10^{-8}	-2.73 x 10^{-9}	-4.33 x 10^{-8}
			i = 30°		
200	-1.32 x 10^{-7}	-1.44 x 10^{-7}	-1.15 x 10^{-7}	-1.44 x 10^{-7}	-1.15 x 10^{-7}
170	-6.58 x 10^{-8}	-3.80 x 10^{-6}	-3.30 x 10^{-6}	-3.80 x 10^{-6}	-3.30 x 10^{-6}
130	-5.17 x 10^{-7}	-2.86 x 10^{-5}	-2.74 x 10^{-5}	-2.86 x 10^{-5}	-2.74 x 10^{-5}
100	-5.41 x 10^{-5}	-1.25 x 10^{-3}	-1.25 x 10^{-3}	-1.25 x 10^{-3}	-1.25 x 10^{-3}
			i = 60°		
200	-2.25 x 10^{-7}	-2.55 x 10^{-7}	-2.04 x 10^{-7}	-2.55 x 10^{-7}	-2.04 x 10^{-7}
170	-1.13 x 10^{-7}	-6.74 x 10^{-6}	-5.83 x 10^{-6}	-6.74 x 10^{-6}	-5.83 x 10^{-6}
130	-8.87 x 10^{-7}	-5.05 x 10^{-5}	-4.84 x 10^{-5}	-5.05 x 10^{-5}	-4.84 x 10^{-5}
100	-9.27 x 10^{-5}	-2.22 x 10^{-3}	-2.21 x 10^{-3}	-2.22 x 10^{-3}	-2.21 x 10^{-3}
			i = 90°		
200	-2.56 x 10^{-7}	-3.03 x 10^{-7}	-2.43 x 10^{-7}	-3.03 x 10^{-7}	-2.43 x 10^{-7}
170	-1.28 x 10^{-7}	-8.00 x 10^{-6}	-6.93 x 10^{-6}	-8.00 x 10^{-6}	-6.93 x 10^{-6}
130	-1.00 x 10^{-6}	-6.00 x 10^{-5}	-5.75 x 10^{-5}	-6.00 x 10^{-5}	-5.75 x 10^{-5}
100	-1.06 x 10^{-4}	-2.64 x 10^{-3}	-2.63 x 10^{-3}	-2.64 x 10^{-3}	-2.63 x 10^{-3}

These examples were obtained for an angle of attack α and bank angle σ equal to zero. Due to this, the values found for the numerical maneuvers with lift are very similar to those without lift, as this effect varies with the *sin* (2α). The results presented in the next table were obtained using $\alpha = \sigma = 0.7$ rd, in order to increase the effect of the lift.

Table 3: Variation on semi-major axis (km) for $\alpha = \sigma = 0.7$ rd.

Perigee	Analytical	Numerical	Numerical 2 phy	Numerical Lift	Numerical 2 phy, Lift
			$i = 0^o$		
200	-7.60×10^{-3}	-7.82×10^{-3}	-6.33×10^{-3}	-7.82×10^{-3}	-6.33×10^{-3}
170	-2.23×10^{-1}	-2.01×10^{-1}	-1.74×10^{-1}	-2.01×10^{-1}	-1.74×10^{-1}
130	-2.99	-1.53	-1.47	-1.53	-1.46
100	270.92	-67.52	-67.37	-58.73	-58.62
			$i = 60^o$		
200	-7.83×10^{-3}	-2.28×10^{-3}	-6.70×10^{-3}	-8.88×10^{-3}	-7.18×10^{-3}
170	-2.30×10^{-1}	-2.14×10^{-1}	-1.85×10^{-1}	-2.29×10^{-1}	-1.98×10^{-1}
130	-3.08	-1.63	-1.55	-1.74	-1.66
100	278.95	-51.79	-71.43	-66.34	-66.22

Table 4: Variation on inclination for $\alpha = \sigma = 0.7$ rd.

Perigee	Analytical	Numerical	Numerical 2 phy	Numerical Lift	Numerical 2 phy, Lift
			$i = 0^o$		
200	-1.62×10^{-11}	-2.73×10^{-9}	3.71×10^{-11}	2.56×10^{-5}	2.05×10^{-5}
170	-8.14×10^{-12}	9.15×10^{-10}	-3.27×10^{-10}	6.84×10^{-4}	5.92×10^{-4}
130	-6.39×10^{-11}	-2.73×10^{-9}	-3.24×10^{-9}	5.18×10^{-3}	4.96×10^{-3}
100	-6.69×10^{-9}	-1.37×10^{-8}	-1.55×10^{-7}	2.00×10^{-1}	2.00×10^{-1}
			$i = 60^o$		
200	-7.94×10^{-7}	-9.00×10^{-7}	-7.21×10^{-7}	2.63×10^{-5}	2.10×10^{-5}
170	-3.98×10^{-7}	-2.38×10^{-5}	-2.05×10^{-5}	7.02×10^{-4}	6.07×10^{-4}
130	-3.13×10^{-6}	-1.78×10^{-4}	-1.71×10^{-4}	5.31×10^{-3}	5.09×10^{-3}
100	-3.27×10^{-4}	-7.92×10^{-3}	-7.90×10^{-3}	2.05×10^{-1}	2.05×10^{-1}

From these results, it is visible that the variation in inclination has a stronger dependence on the lift than the variation on the semi-major axis, as expected.

A different type of result, also obtained with numerical simulations, is the ratio between velocity variation in semi-major axis (ΔVa) and inclination (ΔVi), as a function of the perigee radius (r_p). Fig. 3 shows these plots.

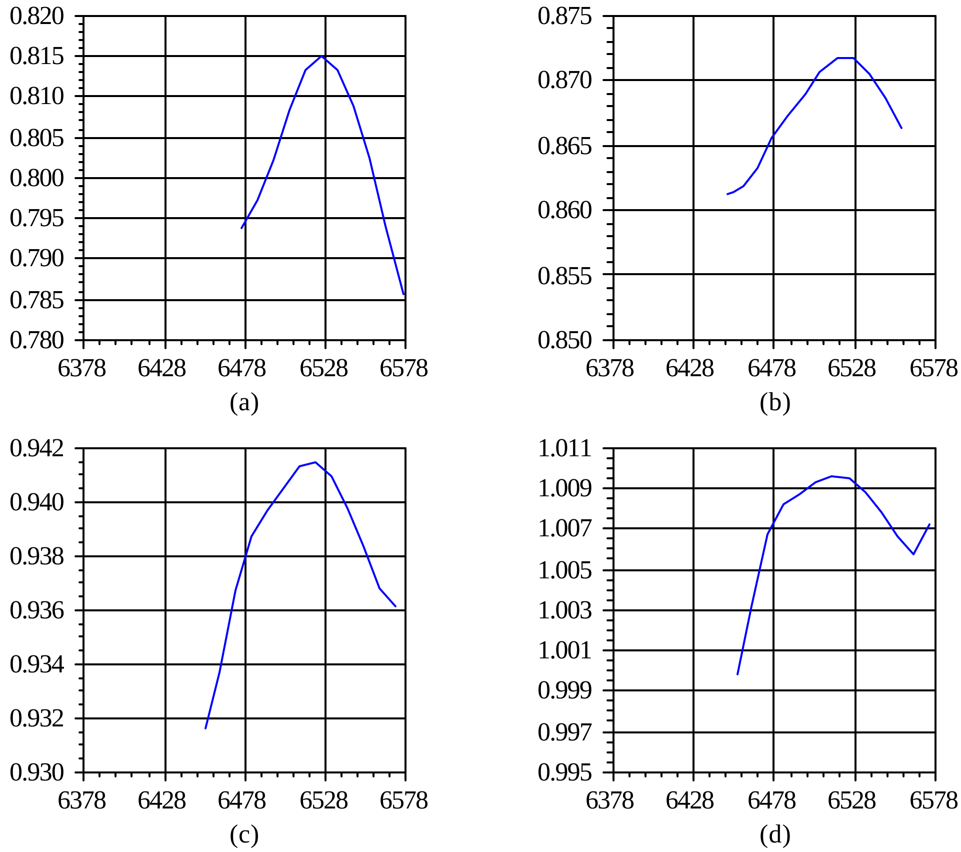

Fig. 3: $\Delta Va/\Delta Vi$ as a function of r_p (km) for r_C = 7000 (a), 8000 (b), 9000 (c) and 10000 km (d)

In Fig.3 it is important to notice that the value 1 for the relation $\Delta Va/\Delta Vi$ is the limit between a profitable and a non-profitable maneuver. In other words, when this value is bigger than 1 it means that the Δv required to correct the atmosphere effect on the semi-major axis will overcome the Δv needed to make a plane change outside the atmosphere, and therefore the maneuver is not suitable. On the other hand, when $\Delta Va/\Delta Vi < 1$, as in the cases (a), (b), (c) and part of (d), the maneuver is desirable as the atmosphere will do the plane change without large loses on the semi-major axis.

Another remarkable result is the influence of the initial orbit radius r_C. We can see that for smaller values of r_C the atmospheric maneuver is advantageous. This occurs because the plane change maneuver is practically costless, as it takes place as far as possible from the gravity center.

The variation in r_p shows a behavior close to parabolic, decreasing in the top 50 km of the atmosphere and increasing from 100 to 150 km. This means that, if the maneuver has to be performed in the top 50 km of the atmosphere, the efficiency of the maneuver increases when we increase the number of passages to accomplish a given change in inclination. On the other hand, if the constraints (like the ones related to heating) allow the passage to be realized deeper than 150 km, the efficiency increases when we reduce the number of passages. Therefore, in order to answer the question related to the optimum number of passages it is necessary to take into account other constraints.

5 - CONCLUSIONS

The analytical expressions developed here can be considered in accordance with the numerical results when the perigee height is larger and they are useless for values of perigee below a limit. The same occurs to the expressions of e, ω and Ω. These results are not show here due to the limitation in space.

As expected, the results for bank angle and angle of attack equal to zero lead to very similar values for both the case with and without lift. The same does not occur when these angles have different values. The influence of the lift force on the variation of the semi-major axis is small, but for the variation on the inclination it is bigger. These results, although expected, are quantified in the present research.

The numerical study of the relation $\Delta Va/\Delta Vi$ showed that if the maneuver has to be performed in the top 50 km of the atmosphere, the efficiency of the maneuver increases when we increase the number of passages to accomplish a given change in inclination. On the other hand, if it is to be undertaken between 100 km and 150 km, the efficiency increases when we reduce the number of passages.

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